第9卷第4期
1992年8月
控制理论与应用
CONTROL THEORY AND APPLICATIONS
Vol. 9, No. 4
Aug., 1992

Stability of the Convex Combination of Two Polynomial Square Matrices Arising in Robustness Problems

ZHAO Keyou

(Department of Electrical Engineering, Qingdao University·Shandong, 266071, PRC)

Abstract: This short article will consider the robust stability of the convex combination of two polynomial square matrices. Necessary and sufficient conditions, and an algorithm will be presented to check this stability.

Key words: polynomial matrices; stability; eigenvalues

1 Introduction

Robust stability of a polynomial family has been studied a lot recently. Kharitonov[1] has shown that a family of interval polynomials is Hurwitz if and only if four specially constructed extreme polynomials are Hurwitz. For a more general family of polynomials other than interval polynomials, it is not sufficient to check all the vertex polynomials. In fact, for a polytope of polynomials, the well-known Edge Theorem[2] states that we have to check all the edge polynomials. A so-called edge is just a convex combination of two extreme polynomials; and its stability can be carried out using the classical root locus method or even more simply using the result given by Fu and Barmish[3]. In this short article we will extend the result in [3] to a polynomial matrix case, and give a criterion for checking the stability of a convex combination of two polynomial square matrices. The motivation for this attempt also stems from robust stabilization for MIMO systems; e.g., one compensator simultaneously stabilizes a convex combination of two extreme plant models. Up to now, there exist no good methods for checking this kind of robust stability.

2 Notations and Preliminaries

If H is a square constant matrix, the determinant and spectrum of H are denoted by $|H|$ and $\mathrm{sp}(H)$ respectively, and the i-th eigenvalue of H is denoted by $\lambda_i(H)$. H is stable if $\mathrm{sp}(H)$ belongs to C^-, the open left half of the complex plane. A polynomial matrix $H(s)$ is called stable if all the zeros of $|H(s)|$ lie in C^-. A family of polynomial matrices is stable if every its member is stable. Let $\mathrm{conv}\{A,B\}$ denote $\{\beta A+(1-\beta)B, 0\leqslant\beta\leqslant1\}$, the convex combination of A and B where A and B are two matrices with same dimensions; and $A\oplus A=:A\otimes I+I\otimes A$, the Kronecker sum of A and itself where $\otimes$ is the symbol of the Kronecker product of two matrices (see [4]).

Manuscript received Sept. 20, 1990, revised Mar. 11, 1991.

Given two polynomial matrices

$$A(s) = Is^n + A_{n-1}s^{n-1} + \cdots + A_1 s + A_0, \tag{1}$$

$$B(s) = Is^n + B_{n-1}s^{n-1} + \cdots + B_1 s + B_0, \tag{2}$$

where $A_i, B_j\ i,j=0,1,\cdots,n-1$ are $m\times m$ constant matrices, I the $m\times m$ unit matrix; also we define the two $mn\times mn$ matrices

$$\mathbf{A} =: \begin{bmatrix} 0 & I & 0 & \cdots & 0 \\ 0 & 0 & I & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & I \\ -A_0 & -A_1 & -A_2 & \cdots & -A_{n-1} \end{bmatrix}, \tag{3}$$

$$\mathbf{B} =: \begin{bmatrix} 0 & I & 0 & \cdots & 0 \\ 0 & 0 & I & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & I \\ -B_0 & -B_1 & -B_2 & \cdots & -B_{n-1} \end{bmatrix}. \tag{4}$$

The following lemma 1 is simple and clear; lemma 2 is based on the continuity of the eigenvalues of $\beta(\mathbf{A}\oplus\mathbf{A})+(1-\beta)(\mathbf{B}\oplus\mathbf{B})$ with the variable $\beta\in[0,1]$, and a fact[4] that $\lambda_k(H\oplus H)=\lambda_i(H)+\lambda_j(H)$ where k,i and j range the numbers of all eigenvalues of H.

Lemma 1 conv$\{A(s),B(s)\}$ is stable if and only if conv$\{\mathbf{A},\mathbf{B}\}$ is stable.

Lemma 2 conv$\{\mathbf{A},\mathbf{B}\}$ is stable if and only if

i) one of $\mathbf{A}$ and $\mathbf{B}$ is stable, and

ii) conv$\{(\mathbf{A}\oplus\mathbf{A}),(\mathbf{B}\oplus\mathbf{B})\}$ is a nonsingular family, i. e., every its member is nonsingular.

Combining the two lemmas above, we have

Lemma 3 Suppose that $A(s)$ is stable, then conv$\{A(s),B(s)\}$ is stable if and only if every member in conv$\{(\mathbf{A}\oplus\mathbf{A}),(\mathbf{B}\oplus\mathbf{B})\}$ is nonsingular.

It is important that lemma 3 turns the stability of conv$\{A(s),B(s)\}$ into the nonsingularity of conv$\{(\mathbf{A}\oplus\mathbf{A}),(\mathbf{B}\oplus\mathbf{B})\}$.

3 The Main Result and Algorithm

Having the preliminaries above, now we are in a position to state our main result.

Theorem 1 Suppose that $A(s)$ is stable, then conv$\{A(s),B(s)\}$ is stable if and only if $(\mathbf{A}\oplus\mathbf{A})^{-1}(\mathbf{B}\oplus\mathbf{B})$ has no eigenvalues in $(-\infty,0]$ where $A(s),B(s)$, $\mathbf{A}$ and $\mathbf{B}$ were defined in (1)~(4).

Proof By lemma 3, it is sufficient to prove that every member in conv$\{(\mathbf{A}\oplus\mathbf{A}),(\mathbf{B}\oplus\mathbf{B})\}$ is nonsingular if and only if $(\mathbf{A}\oplus\mathbf{A})^{-1}(\mathbf{B}\oplus\mathbf{B})$ has no eigenvalues in $(-\infty,0]$.

In fact, from

$$|\beta(\mathbf{A}\oplus\mathbf{A})+(1-\beta)(\mathbf{B}\oplus\mathbf{B})| = |(1-\beta)(\mathbf{A}\oplus\mathbf{A})|\,\left|\frac{\beta}{1-\beta}I+(\mathbf{A}\oplus\mathbf{A})^{-1}(\mathbf{B}\oplus\mathbf{B})\right|$$

and the nonsingularity of $(A \oplus A)$ we derive that $(A \oplus A)^{-1}(B \oplus B)$ has no eigenvalues in $(-\infty, 0]$ if and only if $|\beta(A \oplus A)+(1-\beta)(B \oplus B)| \neq 0$ for any $\beta \in [0,1]$, that is, every member in $\mathrm{conv}\{(A \oplus A), (B \oplus B)\}$ is nonsingular.

The above result suggests an algorithm which is easily implemented using various ready-made software packages.

Step 1 Determine whether one of the two given vertex polynomial matrices is stable. If yes, then go to step 2; otherwise the family is not stable.

Step 2 Derive the matrices A and B based on (3) and (4).

Step 3 Derive the matrices $A \oplus A$ and $B \oplus B$.

Step 4 Suppose that $A(s)$ is stable, calculate the eigenvalues of $(A \oplus A)^{-1}(B \oplus B)$, and see if there exist any eigenvalues within $(-\infty, 0]$. If yes, then the family is not stable; otherwise it is stable.

4 Conclusions

In this short article we considered the robust stability of the convex combination of two polynomial matrices. Sufficient and necessary conditions, and an algorithm have been obtained for checking this stability. For the discrete time case the corresponding problem has meaning also, but may be more difficult.

References

[1] Kharitonov, V. L.. Asymptotic Stability of an Equilibrium Position of a Family of Systems of Linear Differential Equations. Differential'nye Uravneniya, 1978, 14(11): 1483—1485

[2] Bartlett, A. C., Hollot, C. V. and Huang, L.. Root Locations of an Entire Polytope of Polynomials: It Suffices to Check the Edges, Mathematics of Control, Signals and Systems, 1988, 1: 61—71

[3] Fu, M. and Barmish, B. R.. Stability of Convex Combinations of Polynomials and Matrices Arising in Robustness Problems. In Proceedings of Conference on Information Sciences and Systems, Baltimore: John Hopkins University, 1987

[4] Brewer, J. M.. Kronecker Products and Matrix Calculus in System Theory. IEEE Trans. Circuits and Systems, 1978, 25(9): 772—781

鲁棒性问题中出现的两个多项式方阵凸组合的稳定性

赵克友

(青岛大学电气工程系·山东,266071)

摘要：本文考虑两个多项式方阵凸组合的鲁棒稳定性.给出判别这一稳定性的充分必要条件和一个算法.

关键词：多项式矩阵;稳定性;特征值

本文作者简介

赵克友 1945年生.1968年毕业于山东大学数学系.后从事电气产品的研制,电站及电网的设计维护与管理等技术工作.1978年回母校控制论专业任助教,讲师,副教授(资格).1986年调青岛大学,先后任数学系和电气工程系副教授,并曾赴美国访问进修.研究兴趣为控制理论,应用数学,微机应用及电气技术等方面.最近的研究课题为不确定系统理论与H_∞最优控制.