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New Explanation of Canonical Structure of Linear Control Systems and Its Algorithm

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Abstract

By means of the concept of absolute observability the canonical structure for linear control systems is given a new explanation, which claims that the canonical structure consists of four parts, i. e. absolutely observable and controllable; absolutely observable but uncontrollable; absolutely controllable but unobservable; absolutely uncontrollable and unobservable. In our framwork the canonical structure can be obtained in elementary matrix operations.

I. Introduction

It is known to us that there are some definite structure properties, which are related to a given linear control system (C, A, B). These properties are invariant under various transformation groups, and then a canonical structure can be imposed on the given system under a transformation group. A. S. Morse [1] provided a canonical structure of $(C. A. B)$ related to the group Gs, the actions of which on (C, A, B) are defined by

$$(C,A,B)\xrightarrow{(T,K,F,G,H)}(HCT^{-1},\ T(A+BK+FC)T^{-1},\ TBG) \tag{1.1}$$

where T, H, G are nonsingular matrices and of appropriate sizes.

Recently the concept of maximal absolutely observable subsystem and an algorithm of it were advanced [2,3]. By this concept and its dual the canonical structure of system has a new significance and it

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can be obtained by a primary technique, which depends on the elementary operations. In the process some important properties of system are discovered. The system is described as follows

$$\dot{x} = Ax + Bu \qquad (1.2)$$
$$y = Cx$$

or in matrix form

$$\begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \qquad (1.3)$$

Given nonsingular matrices T, H, G in (1.1), the following system matrix is called generally algebraic equivalent to (1.3)

$$\begin{bmatrix} TAT^{-1} & TBG \\ HCT^{-1} & 0 \end{bmatrix} \qquad (1.4)$$

If H, G are unit matrices, then the action of (1.4) is named by (T) - action. Similarly, we define the (H) - action and (G) - action, as well as the (T,H) - action and so on.

Definition 1.1 For any $K \in R^{m\times l}$ the system matrix

$$\begin{bmatrix} A+BK & B \\ C & 0 \end{bmatrix} \qquad (1.5)$$

is called state - feedback equivalent to (1.3). The action is called (K) - action.

Definition 1.2 For any $F \in R^{n\times l}$ the system matrix

$$\begin{bmatrix} A+FC & B \\ C & 0 \end{bmatrix} \qquad (1.6)$$

is called dual - state - feedback equivalent to (1.3). The action is called (F) - action.

Definition 1.3 System (1.3) is called absolutely observable system (a. o. s. for short) if its observability is reserved under any (K) - action. Dually, system (1.3) is called absolutely controllable system (a. c. s. for short) if its controllability is reserved under any (F) - action.

II Canonical Structure Under Gs

It is not difficult to transform (1.3) into following block -

matrix form by a (T)-action

$$(1.3)\xrightarrow{(T)}\begin{pmatrix} A_{11} & A_{12} & \vdots & B_1 \\ A_{21} & A_{22} & \vdots & B_2 \\ 0 & C_2 & \vdots & 0 \end{pmatrix} \tag{2.1}$$

In the form (2.1) the triple (C_2, A_{22}, B_2) is regarded as a subsystem of (1.3). If the rank $C>0$ in (1.3), then there at least exists an a. o. subsystem of (1.3). E. g. in (2.1) if C_2 is nonsingular, then (C_2, A_{22}, B_2) is a. o..

Definition 2.1 Among the a. o. subsystems of a system, the subsystem is called the maximal a. o. if A_{22} has the largest size in the forms (2.1).

Lemma 2.1 (1.3) is algebraic equivalent to the following system by (T)-action:

$$\begin{pmatrix} A_{11} & A_{12} & A_{13} & \vdots & B_1 \\ A_{21} & A_{22} & A_{23} & \vdots & 0 \\ A_{31} & A_{32} & A_{33} & \vdots & B_3 \\ 0 & C_2 & C_3 & \vdots & 0 \end{pmatrix} \tag{2.2}$$

where 1) $Nc(C_2\ C_3)=\mathrm{Rank}(C)$, ($Nc(C)$: = the number of columns of C),

2) B_3 has full row rank,

3) The non-zero row vectors in B_1 and B_3 are independent, or say, B_1 and B_3 are row independent.

The proof of it refers to [2].

Definition 2.2 (2.1) is given, if (C_2, A_{22}, B_2) is an a. o. s. and there exists K_1 such that $A_{21}+B_2K_1=0$, then, say, (2.1) is of maximal absolutely observable canonical form (m. a. o. c. form for short). Dually, we have the m. a. c. canonical form.

Lemma 2.2 Assume that (2.1) is of m. a. c. canonical form and $B_1=0$, then there exists an action of Gs, such that

$$(2.1)\xrightarrow{(T,K,F)}\begin{pmatrix} A_{11} & 0 & 0 \\ 0 & \tilde{A}_{22} & B_2 \\ 0 & C_2 & 0 \end{pmatrix} \tag{2.3}$$

Proof: By the algorithm of m. a. o. canonical form in [2] there exists a (T,K)-action by which (2.1) can be transformed into one of the followinog forms:

$$\begin{pmatrix} A_{11} & * & & & * & 0 \\ 0 & A_1 & \cdots\cdots & & * & 0 \\ 0 & \begin{bmatrix} \overline{C}_2 \\ * \end{bmatrix} & A_2 & * & & \begin{bmatrix} 0 \\ \hline B_2 \end{bmatrix} \\ & \cdots\cdots & \cdots\cdots & \cdots\cdots & & \\ 0 & \begin{bmatrix} 0 \\ * \end{bmatrix} & \cdots\cdots & \begin{bmatrix} C_p \\ * \end{bmatrix} & A_p & \begin{bmatrix} 0 \\ \hline B_p \end{bmatrix} \\ 0 & 0 & 0 \cdots\cdots & & C_{p+1} & 0 \end{pmatrix} \tag{2.4}$$

$$\begin{pmatrix} A_{11} & * & * & \cdots\cdots & * & 0 \\ 0 & A_1 & * & \cdots\cdots & * & \begin{bmatrix} 0 \\ \hline B_1 \end{bmatrix} \\ 0 & \begin{bmatrix} \overline{C}_2 \\ * \end{bmatrix} & A_2 & \cdots\cdots & * & \begin{bmatrix} 0 \\ \hline B_2 \end{bmatrix} \\ & & & \cdots\cdots & & \\ 0 & \begin{bmatrix} 0 \\ * \end{bmatrix} & & & A_p & \begin{bmatrix} 0 \\ \hline B_p \end{bmatrix} \\ 0 & 0 & 0 & \cdots\cdots & \overline{C}_{p+1} & 0 \end{pmatrix} \tag{2.5}$$

where $*$ represents block-matrices. In (2.4), C_i ($i=3, 4, \cdots, p$) has full column rank and $(C_{i+1}\ A_i)$ is observable. In (2.5), C_i ($i=2,3,\cdots, p$) has full column rank. The algorithm of cancelling the first block-matrices except A_{11} in (2.5) can be described in following:

$$\xrightarrow[A_{12}\overline{C}_2^{+}\times]{(-)} \begin{pmatrix} \multicolumn{3}{c}{\overset{\times A_{12}\overline{C}_2^{+}}{\longrightarrow}\ (+)} & 0 \\ A_{11} & A_{12} & * & 0 \\ 0 & A_1 & * & \\ & \begin{bmatrix} \overline{C}_2 \\ * \end{bmatrix} & & \\ 0 & & * & \end{pmatrix} \tag{2.6}$$

where $\overline{C}_2^{+}$ is the pseud-inverse of $\overline{C}_2$. On succession of executing the above algorithm as (2.6), the (2.3) is obtained. In (2.4) the pair $(\overline{C_2}\ \ A_1)$ is observable, thus there exists a F_2 such that sp $(-A_{11})\cap \mathrm{sp}(A_1+F_2\overline{C_2})=0$(if $\mathrm{sp}(-A_{11})\cap\mathrm{sp}(A_1)\neq\phi$, then let $F_2=0$). Execut operations on the system (2.4) as follows:

$$F_2\times\ (+)\begin{pmatrix} A_{11} & A_{12} & * \\ 0 & A_1 & * \\ 0 & C_2 & * \\ & \cdots\cdots & \end{pmatrix}\xrightarrow{V_1\ \times}\ (+)\ \times V_1\ (-)\begin{pmatrix} A_{11} & \widetilde{A}_{12} & * & 0 \\ 0 & \widetilde{A}_1 & * & 0 \\ 0 & & & \\ & \cdots\cdots & & \end{pmatrix}$$

where $\widetilde{A}_1=A_1+F_2\overline{C_2}$ such that $\mathrm{sp}(\widetilde{A}_1)\ \cap\ \mathrm{sp}(-A_{11})=\phi$ and V_1 is the solution of Sylvestre's matrix equation, $A_{11}V_1-V_1\widetilde{A}_1=A_{12}$. In what follows the algorithm is as same as in the case (2.5).

Proposition 2.1 Given system (2.1), by (T)-action we get

$$(2.1)\longrightarrow\left(\begin{array}{cccc|c} A_{11} & 0 & A_{13} & A_{14} & 0 \\ A_{21} & A_{22} & A_{23} & A_{24} & B_2 \\ A_{31} & A_{32} & A_{33} & A_{34} & B_3 \\ 0 & 0 & 0 & A_{44} & 0 \\ \hline 0 & 0 & C_3 & C_4 & 0 \end{array}\right) \tag{2.7}$$

where 1) $\left((C_3\ C_4),\ \begin{bmatrix} A_{33} & A_{34} \\ 0 & A_{44}\end{bmatrix},\ \begin{bmatrix} B_3 \\ 0\end{bmatrix}\right)$ is an m. a. o.

subsystm;

2) (A_{33},B_3) and (A_{22},B_2) are controllable pairs;

3) B_2 and B_3 are row independent;

4) (2.7) is an m. a. o. canonical form. i. e. there is $(K_1\ K_2)$ such that $(A_{31}\ A_{32})+B_3(K_1\ K_2)=(0\ 0)$.

The proof reffers to [2].

Lemma 2.3 In(2.7) B_2, B_3 are row independent, then there exists

$$G\in R^{m\times n}\ \text{such that}\ \begin{pmatrix} 0 \\ B_2 \\ B_3 \\ 0 \end{pmatrix} G=\begin{pmatrix} 0 & 0 \\ B_{21} & 0 \\ 0 & B_{32} \\ 0 & 0 \end{pmatrix} \tag{2.8}$$

Theorem 2.4 There exists an action of Gs such that

$$(2.1) \stackrel{(T,K.F,G,H)}{\longmapsto} \left(\begin{array}{cccc:cc} A_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & A_{22} & 0 & 0 & B_{21} & 0 \\ 0 & 0 & A_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & A_{44} & 0 & B_{42} \\ \hdashline 0 & 0 & C_{13} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{24} & 0 & 0 \end{array}\right) \quad (2.9)$$

where (A_{22}, B_{21}) is controllable, (C_{13}, A_{33}) is observable, and the (C_{24}, A_{44}, B_{42}) is a. o. c..

To save the space we ommit the proof, it can be understood by means of lemma 2.2 and 2.3.

The form (2.9) consists of four parts, they are absolutely unobservable aud uncontrollable; a. c. but absolutely unobservable; a. c. but absolutely uncontrollable; a. c. and a. o. simutanousely.

The form (2.9) is called a canonical structure of linear control systems.

Ⅲ. Structure Invariants of (C, A, B)

In this section two types of simplified canonical structures are advanced, one is same as Morse's [1], but we get it in a different way; the other is based on the Yokayama canonical form.

Notation: 1) $[m_i;\ i=0,1,2,\cdots,v]$ is the controllability index of Yokayoma form of (C, A, B),

2) $[l_j;\ j=0,1,2,\cdots,r]$ is the observability index of Yokayama form of (C,A,B).

Because of the space limitation, we do not give the full proofs of main results

Lemma 3.1 the triple (C,A,B) is a. o. c. only if A_{21} has full row rank in (2.2).

Proof: The key step is to transform (2.2) into following form by an action Gs

$$(2.2) \stackrel{(K,F,G,H)}{\longmapsto} \left(\begin{array}{ccc:cc} A_{11} & 0 & 0 & B_{11} & 0 \\ A_{21} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & B_{32} \\ \hdashline 0 & 0 & C_{13} & 0 & 0 \\ 0 & C_{22} & 0 & 0 & 0 \end{array}\right) \quad (3.1)$$

The details are ommited to save space.

Lemma 3.2 Assume that the system have the form (3.1) already.

1) (C,A,B) is a. o. c. if and only if both $(C_{13},0,B_{32})$ and

$$\left((0\ C_{22}),\ \begin{bmatrix} A_{11} & 0 \\ A_{21} & 0 \end{bmatrix},\ \begin{bmatrix} B_{11} \\ 0 \end{bmatrix} \right) \text{ in (3.5) are a. o. c.,}$$

2) $$\left((0\ C_{22}),\ \begin{bmatrix} A_{11} & 0 \\ A_{21} & 0 \end{bmatrix},\ \begin{bmatrix} B_{11} \\ 0 \end{bmatrix} \right) \text{ is a. o. c. if and only}$$

if (A_{21},A_{11},B_{11}) is a. o. c.

Theorem 3.1 If (C,A,B) is a. o. c., then

1) $Nc(B)=Nr(C)$(: = the number of the rows of C),

2) (C,A,B) can be decoupled as a composition of following types of subsystems by an action of Gs

(a) $$\left((0\ 0\ \cdots\ 0\ I_i),\ \begin{pmatrix} 0 & 0 & \cdots\cdots & 0 & 0 \\ I_i & 0 & \cdots\cdots & 0 & 0 \\ & & \cdots\cdots\cdots\cdots & & \\ 0 & 0 & & I_i & 0 \end{pmatrix},\ \begin{pmatrix} I_i \\ 0 \\ \\ 0 \end{pmatrix} \right),\ i=1,2,\cdots t.$$

(b) $(I_0,0,I_0)$.

Write $s_i=\text{size}(I_i)$, which are invariants.

Theorem 3.2 If the complet system (C,A,B) is given, then $[l_1, l_2,\cdots,l_r]$, $[m_1,m_2,\cdots,m_v]$ and $[s_1,s_2,\cdots,s_t]$ are invariats of (C,A,B) under Gs. The invariant factors of A_{11} in (2.9) are invariants, too.

Remark There are definite structure parts for linear systems. They have three types:

a) $\dot{x}=u$, $y=x$; b) $\dot{x}=0$, $y=x$; c) $\dot{x}=u$.

a) is the a. c. o. part; b) is the a. o. part; c) is the a. c. part. A system can always be assembled by these parts.

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线性控制系统规范结构的新解释与它的算法

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摘　要

应用绝对能观性的概念可以对线性控制系统的规范结构赋于新的解释．文中指出，规范结构由四部分组成，即绝对能观但不能控；绝对能控但不能观；绝对能观与能控；绝对不能观不能控。应用我们引入的方法，规范结构可以用矩阵的初等变换来实现．