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An Implicit Algorithm for Pole-assignment Self-tuning Regulator

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Abstract

This paper proposes a new implicit algorithm for pole-assignment self-tuning regulator based on that of[7]. The global convergence properties are obtained by the use of near supermartingales. Simulation results show the algorithm is effective.

1. Introduction

Interests on pole-assignment self-tuning algorithms have been enhanced in recent years. Numerous algorithms have been proposed [1][2][4][5][6]. But the applications of these algorithms are limited for their great computational efforts. In a recent paper [7], M. B. Zarrop et al introduced several valuable implicit algorithms which greatly reduced the computation. But the global convergence properties of these algorithms remained unanalyzed. This paper proposes a new implicit algorithm. On the assumption of stability, the global convergence results can be obtained by near supermartingales. Simulation results show that the performances of the algorithm are similar to that of algorithm #2 of [7].

2. Implicit Self-tuning Algorithm

Suppose the system to be controlled can be described by the following equation,

$$A(q^{-1})Y(t)=q^{-k}B(q^{-1})U(t)+C(q^{-1})e(t) \tag{2.1}$$

where

$$A(q^{-1})=1+a_1q^{-1}+\cdots+a_{na}q^{-na}$$

$$B(q^{-1})=b_0+b_1q^{-1}+\cdots+b_{nb}q^{-nb}$$

$$C(q^{-1})=1+c_1q^{-1}+\cdots+c_{nc}q^{-nc}$$

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q^{-1} is the one-step backshift operator. $C(q^{-1})$ is a stable polynomial. $Y(t)$ and $U(t)$ are the output and control, respectively. $e(t)$ is the noise.

If the following control strategy is applied to system(2.1),

$$F(q^{-1})U(t)+G(q^{-1})Y(t)=0 \tag{2.2}$$

where $F(q^{-1})$, $G(q^{-1})$ satisfy the equation,

$$A(q^{-1})F(q^{-1})+q^{-K}B(q^{-1})G(q^{-1})=C(q^{-1})T(q^{-1}) \tag{2.3}$$

then the closed-loop equation can be written as,

$$T(q^{-1})Y(t)=F(q^{-1})e(t) \tag{2.4}$$

We see that the system is poles assigned.

The conditions for eq. (2.3)having a uniqe solution is that $A(q^{-1})$, $B(q^{-1})$ are coprime and $\deg(F(q^{-1}))=n_f=n_b+k-1$, $\deg(G(q^{-1}))=n_g=n_a-1$, $\deg(C(q^{-1})T(q^{-1}))\le n_f+n_g+1$. Since $F(0)=1$, we rewrite $F(q^{-1})=1+q^{-1}\overline{F}(q^{-1})$ and denote the cofficients of $\overline{F(q^{-1})}$, $G(q^{-1})$ as,

$$\theta^*=(f_1\cdots f_{nf}\ g_0\cdots g_{ng})^T \tag{2.5}$$

The materials discussed above are well-known. Now we derive an implicit self-tuning algorithm to assign closed-loop poles for unknown parameter systems.

Combining (2.2) and (2.4), get

$$T(q^{-1})Y(t+1)=F(q^{-1})U(t)+G(q^{-1})Y(t)+F(q^{-1})e(t+1) \tag{2.6}$$

This equation can be rewritten as,

$$T(q^{-1})Y(t+1)-U(t)=\overline{F(q^{-1})}(U(t-1)+e(t))+G(q^{-1})Y(t)+e(t+1) \tag{2.7}$$

Using the past residuals $\hat{e}(t)$ instead of $e(t)$ in eq. (2.7), we have the following equation

$$\overline{Y}(t+1)=\theta^T\varphi(t)+e(t+1) \tag{2.8}$$

where
$$\overline{Y}(t+1)=T(q^{-1})Y(t+1)-U(t) \tag{2.9}$$

$$\varphi(t)=(U(t-1)+\hat{e}(t)\cdots U(t-n_f)+\hat{e}(t-n_f+1)\ Y(t)\cdots Y(t-n_g))^T \tag{2.10}$$

Then, the following implicit algorithm is obtained,

1) read in the current output value $Y(t+1)$ and form the filtered variable

$$\overline{Y}(t+1)=T(q^{-1})Y(t+1)-U(t) \tag{2.11}$$

2) estimate the controller cofficients θ^* using RLS on the model(2.8).

3) implement the controller

$$U(t+1) = -G(q^{-1})Y(t+1) - \overline{F}(q^{-1})U(t) \quad (2.12)$$

For more details, the recursive formulations are as follows,

$$\hat{\theta}_{t+1} = \hat{\theta}_t + P_t\varphi(t)(\overline{Y}(t+1) - \varphi_{(t)}^T \hat{\theta}_t)/x_t \quad (2.13)$$

$$x_t = 1 + \varphi^T(t)P_t\varphi(t) \quad (2.14)$$

$$P_{t+1} = P_t(I - \varphi(t)\varphi^T(t)P_t/x_t) \quad (2.15)$$

$$\hat{e}(t+1) = \overline{Y}(t+1) - \varphi^T(t)\hat{\theta}_{t+1} \quad (2.16)$$

$$U(t+1) = -(U(t)\cdots U(t-n_f+1)Y(t+1)\cdots Y(t-n_g+1))\hat{\theta}_{t+1} \quad (2.17)$$

where $\hat{\theta}_{t+1}$ is the estimator of θ^* at sample time $t+1$.

This algorithm is somewhat similar to algorithm #2 of [7]. The main difference between them is that the algorithm proposed in this paper merges the term $\overline{F}(q^{-1})e(t)$ with $\overline{F}(q^{-1})U(t-1)$ instead of $\overline{Y}(t+1)$. This change can aid the analysis of the convergence properties.

3. Convergence Properties

Theorem 1 For the implicit self-tuning algorithm proposed in section 2, if the following assumptions are satisfied,

3.A) $\{Y(t)\}$, $\{U(t)\}$ are bounded, i.e. the closed-loop system is stable.

3.B) $\{e(t)\}$ is a sequence of white noise with $E\{e(t)\}=0$, $E\{e^2(t)\}=\sigma^2$ $e(t)$ is independent with observations before time t $(Y(t-1)\cdots U(t-1)\cdots)$Further, $|e(t)|<M$.

3.C) $1/F(q^{-1}) - \frac{1}{2}$ is strictly positive real.

then

3.a) $$\lim_{N\to\infty}\frac{1}{N}\sum_{t=1}^{N}(\hat{e}(t)-e(t))^2 = 0 \quad a.s. \quad (3.1)$$

3.b) $$\lim_{N\to\infty}\frac{1}{N}\sum_{t=1}^{N}\hat{e}^2(t) = \lim_{N\to\infty}\frac{1}{N}\sum_{t=1}^{N}e^2(t) \quad a.s. \quad (3.2)$$

3.c) $$\lim_{N\to\infty}\frac{1}{N}\sum_{t=1}^{N}((\hat{\theta}_t-\theta^*)^T\varphi(t-1))^2 = 0 \quad a.s. \quad (3.3)$$

3.d) $$\lim_{t\to\infty}(\hat{\theta}_t-\theta^*)^T(P_t^{-1}/t)(\hat{\theta}_t-\theta^*) = 0 \quad a.s. \quad (3.4)$$

3.e) If, in addition, $\lim_{t\to\infty} P_t^{-1}/t>0$ a.s., then $\lim_{t\to\infty}\hat{\theta}_t = \theta^*$ (3.5)

The proof of the theorem is based on the results of [3] and will not be presented here since it is quite long.

The restrictive part of theorem 1 is the condition of (3.c) since many systems have non-positive real $1/F(q^{-1})-\frac{1}{2}$. Simulation results show this condition may be replaced by a weaker condition of stable $F^{-1}(q^{-1})$. Similar to algorithm #2 of [7], the regulator parameters don' t convergence to their true values for systems with unstable $F^{-1}(q^{-1})$.

4. Simulation Results

A great deal of examples were simulated on a digital computer. The results showed that the strictly positive real condition in theorem 1 was over strong. The algorithm seemed to work well in all systems with stable $F^{-1}(q^{-1})$. If deg $F(q^{-1})=1$, the strictly positive real and the stability condition are equivalent. Now we present three examples to illustrate the above conclusions.

In all examples, the initial values of the parameters are taken to be zero and the following technique is used to avoid the possible initial unstability of the system.

1) restrict $|U(t)|<5$ (4.1)

2) set the forgetting factor $\rho=0.8$ in the next ten steps when $U(t)$ is restricted (i.e. when the control $U(t)$ get from eq. (2.17) is greater than 5 and is restricted by eq.(4.1)

Example 1 $(1-0.8q^{-1})Y(t)=(0.9+0.5q^{-1})U(t-1)+e(t)$ (4.2)

This is an example of [7]. We choose $T(q^{-1})=1-0.9q^{-1}$, $\sigma^2=0.1$. The performances of the algorithm are similar to that of algorithm #2 of [7]. Fig. 1.1 shows the convergences of parameters of the regulator. Fig.1.2 and Fig.1.3 show the values of $e(t)$ and $e(t)-\hat{e}(t)$, respectively. As indicated by theorem 3, we see that $\lim_{t\to\infty}\hat{\theta}_t=\theta^*$ and $\lim_{t\to\infty}(e(t)-\hat{e}(t))=0$.

Example 2 $(1-0.5q^{-1})Y(t)=(2.5-q^{-1})U(t-1)+e(t)$ (4.3)

This is also an example of [7]. We choose $T(q^{-1})=1$, $\sigma^2=0.1$. The system has an unstable $F^{-1}(q^{-1})=1/(-2q^{-1})$. The values of regulator parameters are shown in Fig. 2.1 and Fig. 2.2. Similar to [7], we see that the parameters do not converge to their true values for systems with unstable $F^{-1}(q^{-1})$.

Example 3 $(1-0.8q^{-1})Y(t)=(1+0.7q^{-1})U(t-2)+e(t)$ (4.4)

We choose $T(q^{-1})=1+0.1q^{-1}$, $\sigma^2=0.1$. From eq. (2.3) we get $F(q^{-1})=0.9+0.336q^{-1}$ and $G(q^{-1})=0.384$. It can be proved that $1/F(q^{-1})-\frac{1}{2}$ is not strictly positive real but $F^{-1}(q^{-1})$ is stable. simulation results illustrat that the parameters convergence well as shown in Fig.3.1 in spite of the assumption (3.C) is not satisfied for this system. So we conclude that the strictly positive real assumption of theorem 3 is over strong.

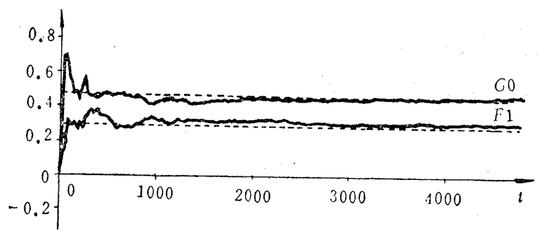

Fig. 1.1 The convergence of Parameters

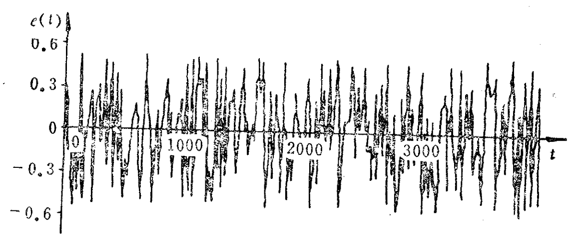

Fig. 1.2 The noise

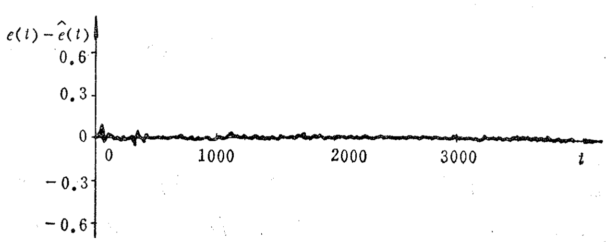

Fig. 1.3 The difference e(t) and ê(t)

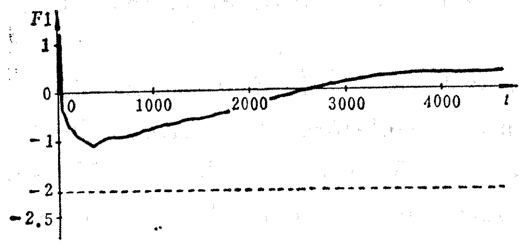

Fig. 2.1 The convergence of F1

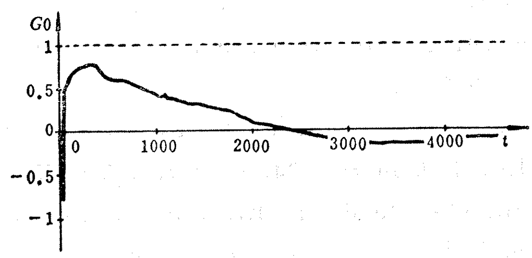

Fig. 2.2 The convergence of G0

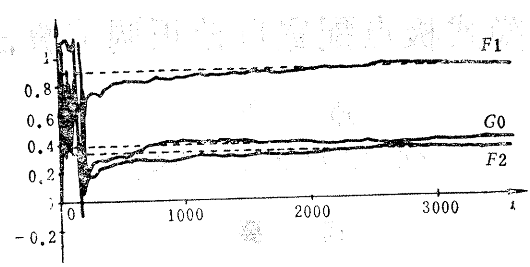

Fig. 3.1 The convergence of parameters

5. Conclusion

A new pole-assignment self-tuning algorithm is proposed. On the assumption of stability, the global convergence property is analyzed by the use of near supermartingales. Simulation results show the algorithm is effective but the problem of making the algorithm work well for systems with unstable $F^{-1}(q^{-1})$ remains unsolved.

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一种隐式极点配置自校正调节算法

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摘 要

本文在文献〔7〕的基础上，提出了一种新的隐式极点配置自校正调节算法。该算法的主要优点是：从理论上可以通过鞅论证明其大范围收敛性质，从而解决了文献〔7〕中遗留下来的问题。计算机仿真结果表明该算法在实际中是行之有效的。