On the Intersection Problem of Compensating Spectra and Closed-Loop Spectra

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Abstract

Using the decentralized control method, the intersection problem of compensating spectra and closed-loop spectra is considered. For centralized systems and decentralized systems respectively, we establish the necessary and sufficient conditions for a dynamic compensator to exist so that the compensating spectrum and its corresponding closed-loop spectrum do not intersect by means of the vanishing concept introduced in the present paper. In addition, the intersection of a compensating spectrum and its corresponding closed-loop spectrum is also found out when the conditions fail to hold.

Introduction

In control theory, the problem concerning designing a compensator around a given plant so that the resultant closed-loop control system meets certain design specifications such as: stability, sensitivity, and structural stability, etc., has been much discussed. Furthermore, the problem of stabilizing a plant using a stable compensator has also been studied by Youla et al. (1974). However, the further relationship between compensators and closed-loop systems for a given plant has not yet been revealed previously, except that Shaw illustrated the sensitivity of closed-loop systems to parameters of unstable compensators in 1971.

For a given plant, it is very clear that a closed-loop system will solely be determined by a compensator. If the intersection of their spectra is nonempty, it is implied that some modes of the compensator will directly affect the dynamic response of the closed-loop system. Thus this part of modes is of particular

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significance. Because of this point, we consider the following two problems in the present paper.

Do a compensating spectrum and its corresponding closed-loop spectrum always intersect——in other words, do there always exist some modes of the compensator which appear in the resultant closed-loop system——no matter how the compensator may be designed around a given centralized or decentralized plant? If so, what is their intersection?

Preliminaries

Consider the following linear time-invariant system

$$\dot{x} = Ax + Bu$$
$$y = Cx \tag{2.1}$$

with states $x \in R^n$, inputs $u \in R^n$, and outputs $y \in R^n$, where R^n denotes the n-dimensional real Euclidean space. Let

$$V \triangleq \left\{ s \in C^1;\ \text{rank}\begin{bmatrix} sI-A & B \\ C & 0 \end{bmatrix} < n+1 \right\} \tag{2.2}$$

where C^1 denotes the complex plane.
Evidently, V can only be an empty set, a finite set, or the whole complex plane.

Definition 1. V defined as above is called the set of vanishing zeros of (2.1); the system (2.1) is said to be vanishing if $V = C^1$.

Proposition 1. The system (2.1) is vanishing if and only if it has no jointly controllable and observable mode, that is,

$$C(sI-A)^{-1}B = 0 \tag{2.3}$$

Its proof is omitted here for limitation of space.

Now, we consider the linear decentralized system consisting of N control stations, described by the following model

$$\dot{x} = Ax + \sum_{i=1}^{N} B_i u_i$$

$$y_i = C_i x, \quad i = 1, 2, \cdots, N \tag{2.4}$$

where $x \in R^n$ is the state, $u_i \in R^{m_i}$ and $y_i \in R^{r_i}$ are the ith control station's input and output respectively; A, B_i, C_i are constant matrices of appropriate dimensions. Let

$$B \triangleq [B_1\ B_2\ \cdots\ B_N], \quad C^T \triangleq [C_1^T\ C_2^T\ \cdots\ C_N^T]$$

$$m \triangleq \sum_{i=1}^{N} m_i, \qquad r \triangleq \sum_{i=1}^{N} r_i$$

$K^* \triangleq \{$ block diag $(K_1, K_2, \cdots, K_N);\ K_i \in R^{m_i \times r_i},\ i = 1, 2, \cdots, N \}$

Definition 2 (Wang-Davison, 1973). The set of decentralized fixed modes of the system (2.4) is defined as

$$U \triangleq \{ s \in C^1;\ \det(sI - A - BKC) = 0, \forall K \in K^* \}$$

The following result presents an algebraic test for the existence of decentralized fixed modes.

Proposition 2 (Anderson-Clement, 1981) Given the N-control agent system (2.4), $s \in \sigma(A)$ is a decentralized fixed mode of (2.4) if and only if there exists a permutation $(i_1, \cdots, i_k, i_{k+1}, \cdots, i_N)$ of the set $(1, 2, \cdots, N)$ such that

$$\text{rank}\begin{pmatrix} sI - A & B_{i_1} & \cdots & B_{i_k} \\ C_{i_{k+1}} & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ C_{i_N} & 0 & \cdots & 0 \end{pmatrix} < n$$

Definition 3. A subset F of R^n is a proper variety in R^n, if it is a proper subset in R^n and the coordinates of each element of F, relative to a fixed basis for R^n, coincide with a zero of a finite system of real coefficient polynomial equations in n-indeterminates, and visa versa.

As well known, a proper variety in R^n must be closed and nondense.

Proposition 3. Let U denote the set of decentralized fixed modes of (2.4). Then for any finite number of points $P = (p_1, p_2, \cdots, p_t) \subset C^1/U$, the set

$$K_P = \{ p(K);\ \sigma(A + BKC) \cap P \neq \phi,\ K \in K^* \}$$

with $p(K)$ the $\sum_{i=1}^{N} m_i r_i$-dimensional vector formed by elements of K, is a union of a finite number of proper varieties in $R^{\sum_{i=1}^{N} m_i r_i}$.

Its proof is omitted here.

3. The Centralized Case

Theorem 1. Given the centralized control system (2.1), then for any positive integer q there exists a q-order dynamic compensator of the following type

$$\dot{z} = Qz + Ry$$
$$u = Sz + Ky + v \tag{3.1}$$

such that the compensating spectrum $\sigma(Q)$ of (3.1) and the spectrum of the closed-loop system made up of (2.1) and (3.1) do not intersect, if and only if the system (2.1) is not vanishing.

Proof. Sufficiency: Assume that (2.1) is not vanishing, then the set V of vanishing zeros of (2.1) must be empty or finite. Clearly, the closed-loop system composed of (2.1) and (3.1) can be described as follows:

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} A+BKC & BS \\ RC & Q \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} v$$

$$y = [C \quad 0] \begin{bmatrix} x \\ z \end{bmatrix} \tag{3.2}$$

Its state matrix is

$$\overline{A} = \begin{bmatrix} A+BKC & BS \\ RC & Q \end{bmatrix}$$

Thus the closed-loop spectrum is $\sigma(\overline{A})$. Now we are in a position to find $(Q,R,S,K) \in R^{q\times q} \times R^{q\times r} \times R^{m\times q} \times R^{m\times r}$ so that

$$\sigma(Q) \cap \sigma(\overline{A}) = \phi \tag{3.3}$$

To begin with, we take $K \in R^{m\times r}$, and choose $Q \in R^{q\times q}$ so that Q has no repeated eigenvalues and its spectrum $\sigma(Q)$ is disjoint from $V \cup \sigma(A+BKC)$. Then the following terms hold

$$\text{rank}(Q - sI) = q - 1 \tag{3.4}$$

$$\text{rank}(A + BKC - sI) = n \tag{3.5}$$

$$\text{rank}\begin{bmatrix} A - sI & B \\ C & 0 \end{bmatrix} \geq n + 1 \tag{3.6}$$

for all $s \in \sigma(Q)$. Observe that $\overline{A}$ can be rewritten as the following form

$$\overline{A}=\begin{bmatrix} A+BKC & 0 \\ 0 & Q \end{bmatrix}+\begin{bmatrix} 0 \\ I_q \end{bmatrix}R[C \quad 0]+\begin{bmatrix} B \\ 0 \end{bmatrix}S[0 \quad I_q].$$

So $\overline{A}$ can be regarded as the new closed-loop state matrix of the following two-control agent system

$$\dot{x}=\begin{bmatrix} A+BKC & 0 \\ 0 & Q \end{bmatrix}x+\begin{bmatrix} 0 \\ I_q \end{bmatrix}u_1+\begin{bmatrix} B \\ 0 \end{bmatrix}u_2$$

$$y_1=[C \quad 0]x$$

$$y_2=[0 \quad I_q]x \tag{3.7}$$

under the decentralized static output feedback $u_1=Ry_1$, $u_2=Sy_2$. Let U denote the set of decentralized fixed modes of the above system. Then, it is easy to prove

$$\sigma(Q)\cap U=\phi \tag{3.8}$$

Also by Proposition 2, (3.8) is proved. Using Proposition 3, we can find a pair $(R,S)\in R^{q\times r}\times R^{m\times q}$ so that (3.3) holds. Therefore, the sufficiency is obtained.

Necessity: Clearly, it suffices to show that (3.3) is not true for all $(Q,R,S,K)\in R^{q\times q}\times R^{q\times r}\times R^{m\times q}\times R^{m\times r}$ if the system (2.1) is vanishing. For this, we assume that (2.1) is vanishing. Then we have

$$\operatorname{rank}\begin{bmatrix} A+BKC-sI & 0 & B \\ 0 & Q-sI & 0 \\ C & 0 & 0 \end{bmatrix}=\operatorname{rank}\begin{bmatrix} A-sI & B \\ C & 0 \end{bmatrix}+\operatorname{rank}(Q-sI)\leq n+q-1$$

for all $(Q,K)\in R^{q\times q}\times R^{m\times r}$ and $s\in\sigma(Q)$, which implies from proposition 2 that $\sigma(Q)$ is contained in the set of decentralized fixed modes of (3.7), i. e.,

$$\sigma(Q)\subset\sigma\begin{bmatrix} A+BKC & BS \\ RC & Q \end{bmatrix}$$

for all $(Q,R,S,K)\in R^{q\times q}\times R^{q\times r}\times R^{m\times q}\times R^{m\times r}$. In this way, the proof of Theorem 1 is complete.

Corollary 1.1. Suppose that the system (2.1) is vanishing. Then for any fixed positive integer q, it holds:

$$\sigma(Q)\subset\sigma\begin{bmatrix} A+BKC & BS \\ RC & Q \end{bmatrix}$$

for all $(Q,R,S,K)\in R^{q\times q}\times R^{q\times r}\times R^{m\times q}\times R^{m\times r}$.

Corollary 1.2. Suppose that the system (2.1) is not vanishing.

Then for any positive integer q, the set

$$\Omega_q=\left\{p(Q,R,S,K);\ (Q,R,S,K)\in R^{q\times q}\times R^{q\times r}\times R^{m\times q}\times R^{m\times r},\right.$$

$$\left.\sigma(Q)\cap\sigma\left(\begin{bmatrix}A+BKC & BS\\ RC & Q\end{bmatrix}\right)\neq\phi\right\}$$

is a proper variety in $R^{q2+qr+qm+mr}$ where $p(Q,R,S,K)$ is a vector composed of elements of (Q,R,S,K).

Proof. Let $f\ (Q,R,S,K)$ be the resultant of two polynomials in s $\det\ (Q-sI)$ and $\det\begin{bmatrix}A+BKC-sI & BS\\ RC & Q-sI\end{bmatrix}$. Obviously, it is a real coefficient polynomial with indeterminates $p(Q,R,S,K)$. Moreover, have

$$\Omega_q=\{\ p(Q,R,S,K);\ (Q,R,S,K)\in R^{q\times q}\times R^{q\times r}\times R^{m\times q}\times R^{m\times r},f(Q,R,S,K)=0\ \}$$

By Theorem 1, it is known that there exists $(Q^0,R^0,S^0,K^0)\in R^{q\times q}\times R^{q\times r}\times R^{m\times q}\times R^{m\times r}$ such that $p(Q^0,R^0,S^0,K^0)$ does not belong to the above set, it means that Ω_q is a proper subset of $R4q+2(m+r)$.

So Ω_q is a proper variety in $R^{q2+qr+qm+mr}$

Corollary 1.3. Let the system (2.1) be jointly controllable and observable. Then there must exist a dynamic compensator (3.1) such that

(i) the closed-loop system (3.2) is stable, i. e.

$$\sigma\left(\begin{bmatrix}A+BKC & BS\\ RC & Q\end{bmatrix}\right)\subset C^{1-}$$

where C^{1-} denotes the left-half open complex plane;

(ii) the compensating spectrum and the closed-loop spectrum do not intersect, i. e.

$$\sigma(Q)\cap\sigma\left(\begin{bmatrix}A+BKC & BS\\ RC & Q\end{bmatrix}\right)=\phi.$$

4. The Decentralized Case

In this section, we come to discuss the intersection problem of decentralized compensating spectra and closed-loop spectra for the N-control agent system (2.4).

For the system (2.4), we consider the following decentralized dynamic compensator

$$\dot{z}_i = Q_i z_i + R_i y_i$$
$$u_i = S_i z_i + K_i y_i + v_i, \quad i = 1,2,\cdots,N \tag{4.1}$$

with $z_i \in R^{q_i}$ $(i = 1,2,\cdots,N)$.

The spectrum $\bigcup_{i=1}^{N} \sigma(Q_i)$ of (4.1) is called the decentralized compensating spectrum of (2.4). The order of (4.1) is denoted by $(q_1, q_2, \cdots, q_N)$. Obviously, the state matrix of the closed-loop system composed of (2.4) and (4.1) is

$$\overline{A} = \begin{pmatrix} A + \sum_{i=1}^{N} B_i K_i C_i & B_1 S_1 & \cdots & B_N S_N \\ R_1 C_1 & Q_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ R_N C_N & 0 & \cdots & Q_N \end{pmatrix} \tag{4.2}$$

Then the closed-loop spectrum is $\sigma(\overline{A})$.

Define a subset P of $\overline{N} \times \overline{N}$ as follows:

$$P \triangleq \{ [(i_1, \cdots, i_s), (j_1, \cdots, j_t)] \in \overline{N} \times \overline{N}; \quad (i_1', \cdots, i_{N-s}') \cap (j_1', \cdots, j_{N-t}') = \phi,$$
$$1 \le s, t \le N, s + t = N + 1 \}$$

where $\overline{N}$ denotes the power set of $(1,2,\cdots,N)$, i.e. the set consisting of all subsets of $(1,2,\cdots,N)$; and $(i_1', \cdots, i_{N-s}')$ is the complementary set of $(i_1, \cdots, i_s)$ in $(1,2,\cdots,N)$. For $[(i_1, \cdots, i_s), (j_1, \cdots, j_t)] \in \overline{N} \times \overline{N}$, we define a centralized control system $S_{[(i_1, \cdots, i_s), (j_1, \cdots, j_t)]}$ as follows:

$$\dot{x} = Ax + [B_{i_1} \ B_{i_2} \ \cdots \ B_{i_s}] u$$
$$y = \begin{pmatrix} C_{j_1} \\ C_{j_2} \\ \vdots \\ C_{j_t} \end{pmatrix} x \tag{4.3}$$

With the above preparation, we now state the following main result.

Theorem 2. Given the system (2.4) and a positive integer set $(q_1,\cdots,q_N)$. Then, there exists a $(q_1,\cdots,q_N)$-order decentralized compensator (4.1) such that the decentralized compensating spectrum and the closed-loop spectrum do not *intersect* if and only if for all $[(i_1,\cdots,i_s),(j_1,\cdots,j_t)]\in p$ the subsystem $S_{[(i_1,\cdots,i_s)(j_1,\cdots,j_t)]}$ is not vanishing.

Proof Sufficiency: Assume that the system $S_{[(i_1,\cdots,i_s),(j_1,\cdots,j_t)]}$ is not vanishing for all $[(i_1,\cdots,i_s),(j_1,\cdots,j_t)]\in P$. Then it is easy to see that neither is the system $S_{[(i_1,\cdots,i_s),(j_1,\cdots,j_t)]}$ for all $[(i_1,\cdots,i_s),(j_1,\cdots,j_t)]\in P'$, where

$$P'\triangleq\{[(i_1,\cdots,i_s),(j_1,\cdots,j_t)]\in\overline{N}\times\overline{N};(i'_1,\cdots,i'_{N-s})\cap(j'_1,\cdots,j'_{N-t})=\phi,\ 1\le s,t\le N,s+t>N\}$$

It is desired to construct a $(q_1,\cdots,q_N)$-order decentralized compensator so that the decentralized compensating spectrum and the closed-loop spectrum do not intersect.

For this, we first take $K\in K^*$, and choose $Q_i\in R^{q_i\times q_i}$ $(i=1,\cdots,N)$ so that

(1) Q_i has no repeated eigenvalue, i. e.,

$$\text{rank}(Q_i-sI)=q_i-1,\quad s\in\sigma(Q_i),i=1,\cdots,N$$

(2) $\sigma(A+BKC)\cap\left[\bigcup_{i=1}^{N}\sigma(Q_i)\right]=\phi$

(3) $V_{[(i_1,\cdots,i_s),(j_1,\cdots,j_t)]}\cap[\bigcup_{k\in(i_1,\cdots,i_s)\cap(j_1,\cdots,j_t)}\sigma(Q_k)]=\phi$

for all $[(i_1,\cdots,i_s),(j_1,\cdots,j_t)]\in P'$, where $V_{[(i_1,\cdots,i_s),(j_1,\cdots,j_t)]}$ denotes the set of vanishing zeros of the system $S_{[(i_1,\cdots,i_s),(j_1,\cdots,j_t)]}$ and is empty or finite by the above assumption.

Now we rewrite the closed-loop state matrix $\overline{A}$ as follows:

$$\overline{A}=\begin{pmatrix}A+BKC & 0 & \cdots & 0\\ 0 & Q_1 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & Q_N\end{pmatrix}+\begin{pmatrix}0 & \cdots & 0 & B_1 & \cdots & B_N\\ I_{q_1} & \cdots & 0 & 0 & \cdots & 0\\ \vdots & \ddots & \vdots & \vdots & & \vdots\\ 0 & \cdots & I_{q_N} & 0 & \cdots & 0\end{pmatrix}$$

$$\cdot\begin{pmatrix} R_1 & \cdots & & 0 \\ \vdots & \ddots R_N & S_1 & \vdots \\ & & \ddots & \\ 0 & \cdots & & S_N \end{pmatrix}\begin{pmatrix} C_1 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ C_N & 0 & \cdots & 0 \\ 0 & I_{q_1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & I_{q_N} \end{pmatrix}$$

$$= A^* + \sum_{i=1}^{N} B_i^* R_i C_i^* + \sum_{i=N+1}^{2N} B_i^* S_{i-N} C_i^*,$$

which, clearly, can be regarded as the new closed-loop state matrix of a $2N$-control agent system S^*: $(C_i^*, A^*, B_i^*;\ i=1,\cdots,2N)$ under the decentralized static output feedback $(R_1,\cdots,R_N,S_1,\cdots,S_N)$. In this way, the remaining problem is to find a decentralized static output feedback for the system S^* so that the closed-loop spectrum is disjoint from $\bigcup_{i=1}^{N}\sigma(Q_i)$ under the feedback. But by Proposition 3, it suffices to verify only that the system S^* does not contain any element in the set $\bigcup_{i=1}^{N}\sigma(Q_i)$ as its decentralized fixed mode. Also by Proposition 2, the verification is completed if we can show that the following inequality

$$\operatorname{rank}\left(\begin{array}{cccc:cccccc} A+BKC-sI & 0 & \cdots & 0 & 0 & \cdots & 0 & B_{i_1} & \cdots & B_{i_s} \\ & & & & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & Q_1-sI & \cdots & 0 & \vdots & & \vdots & & & \\ & & & & I_{j_1'} & \ddots & I_{j_{N-t}'} & \vdots & & \vdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & & \vdots & & & \\ 0 & 0 & \cdots & Q_N-sI & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \hdashline C_{j_1} & 0 & \cdots & 0 & 0 & \cdots & & & & 0 \\ \vdots & \vdots & & \vdots & \vdots & & & & & \vdots \\ C_{j_t} & 0 & \cdots & 0 & & & & & & \\ 0 & 0 & \cdots & 0 & & & & & & \\ \vdots & \vdots & & \vdots & & & & & & \\ & I_{i_1'} & \ddots & I_{i_{N-s}'} & & & & & & \\ \vdots & \vdots & & \vdots & \vdots & & & & & \vdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots & & & & 0 \end{array}\right) \geq n + \sum_{i=1}^{N} q_i$$

i. e.

$$\operatorname{rank}\left(\begin{array}{cccc:cccccc} A+BKC-sI & B_{i_1} & \cdots & B_{i_s} & 0 & \cdots & & & & 0 \\ C_{j_1} & 0 & \cdots & 0 & \vdots & & & & & \vdots \\ \vdots & \vdots & & \vdots & & & & & & \\ C_{j_t} & 0 & \cdots & 0 & 0 & \cdots & & & & 0 \\ \hdashline 0 & \cdots & & 0 & Q_1-sI & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & & & \vdots & & & & \vdots & & \vdots \\ & & & & \vdots & \ddots & \vdots & I_{j'_1} & \ddots & I_{j'_{N-t}} \\ & & & & & & & \vdots & & \vdots \\ & & & & 0 & \cdots & Q_N-sI & 0 & \cdots & 0 \\ & & & & 0 \cdots I_{i'_1} & \cdots & 0 & 0 & \cdots & 0 \\ & & & & \vdots & \ddots & \vdots & \vdots & & \vdots \\ & & & & 0 \cdots I_{i'_{N-s}} & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & & 0 & & & & & & \end{array}\right) \geq n+\sum_{i=1}^{N} q_i \tag{4.4}$$

holds for all $s\in \bigcup_{i=1}^{N} \sigma(Q_i)$ and $[(i_1,\cdots,i_s),(j_1,\cdots,j_t)]\in \overline{N}\times \overline{N}$. Below, we come to check the above inequality.

To begin with, it is easy to see from the conditions satisfied by $(Q_1,\cdots,Q_N)$ that the rank of the submatrix in the right-under corner in (4.4) is at least $\sum_{i=1}^{N} q_i-1$ and that of the submatrix in the left-upper corner is at least n for all $s\in \bigcup_{i=1}^{N}(Q_i)$ and $[(i_1,\cdots,i_s),(j_1,\cdots,j_t)]\in \overline{N}\times \overline{N}$. When $[(i_1,\cdots,i_s),(j_1,\cdots,j_t)]\in P'$, by the definition of P' we obviously have

$$(i_1,\cdots,i_s)\cup(j_1,\cdots,j_t)=(1,\cdots,N),$$

which results in the left-upper submatrix in (4.4) being equivalent to

$$\begin{pmatrix} A-sI & B_{i_1} & \cdots & B_{i_s} \\ C_{j_1} & 0 & \cdots & 0 \\ \cdots & \vdots & & \vdots \\ C_{j_t} & 0 & \cdots & 0 \end{pmatrix}$$

In view of the preceding assumption and Condition (3), the above matrix's rank at least equals $n+1$ for all $s \in \bigcup_{i=1}^{N} \sigma(Q_i)$. So in this case, (4.4) is true for all $s \in \bigcup_{i=1}^{N} \sigma(Q_i)$. When $[(i_1, \cdots, i_s), (j_1, \cdots, j_t)] \in P'$, i. e., $s+t \leq N$ or

$$(i'_1, \cdots, i'_{N-s}) \cap (j'_1, \cdots, j'_{N-t}) \neq \phi,$$

we distinguish the following two cases to consider.

In the first case that $s+t \leq N$ and $(i'_1, \cdots, i'_{N-s}) \cap (j'_1, \cdots, j'_{N-t}) = \phi$, that is,

$$(i'_1, \cdots, i'_{N-s}) \cup (j'_1, \cdots, j'_{N-t}) = (1, \cdots, N)$$

$$(i'_1, \cdots, i'_{N-s}) \cap (j'_1, \cdots, j'_{N-t}) = \phi$$

the number of unit matrices in (4.4) is just N, which implies that the right-under corner's submatrix in (4.4) has rank $\sum_{i=1}^{N} q_i$ for all $s \in \bigcup_{i=1}^{N} \sigma(Q_i)$. Thus, (4.4) holds for all $s \in \bigcup_{i=1}^{N} \sigma(Q_i)$. In the second case that $(i'_1, \cdots, i'_{N-s}) \cap (j'_1, \cdots, j'_{N-t}) \neq \phi$, it is not hard to see that the rank of the right-under corner's submatrix in (4.4) is at least $\sum_{i=1}^{N} q_i + (q_{i_0} - 1)$ for all $s \in \bigcup_{i=1}^{N} \sigma(Q_i)$, where $i_0 \in (i'_1, \cdots, i'_{N-s}) \cap (j'_1, \cdots, j'_{N-t})$. Therefore, (4.4) still holds for all $s \in \bigcup_{i=1}^{N} \sigma(Q_i)$. To sum up, the sufficient part of Theorem 2 is proved.

Necessity: Assume that there exists $[(i_1^0, \cdots, i_s^0), (j_1^0, \cdots, j_t^0)] \in P$ such that the system $S_{[(i_1^0, \cdots, i_s^0), (j_1^0, \cdots, j_t^0)]}$ is vanishing. Then, we shall show that

$$\left[\bigcup_{i=1}^{N} \sigma(Q_i) \right] \cup \sigma(\overline{A}) \neq \phi,$$

for all $(Q_i, R_i, S_i, K_i) \in R^{q_i \times q_i} \times R^{q_i \times r_i} \times R^{m_i \times q_i} \times R^{m_i \times r_i}$ $(i = 1, \cdots,$

N). Now because of $(i_1^0,\cdots,i_s^0)\cup(j_1^0,\cdots,j_t^0)=(1,\cdots,N)$ and the above assumption, have

$$\text{rank}\begin{pmatrix} A+BKC-sI & B_{i_1^0} & \cdots & B_{i_s^0} \\ C_{j_1^0} & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ C_{j_t^0} & 0 & \cdots & 0 \end{pmatrix} = \text{rank}\begin{pmatrix} A-sI & B_{i_1^0} & \cdots & B_{i_s^0} \\ C_{j_1^0} & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ C_{j_t^0} & 0 & \cdots & 0 \end{pmatrix} \leq n,$$

for all $s\in C^1$ and $K\in K^*$.

In addition, it is clear from $(N-s)+(N-t)<N$ and $(i_1^{0\prime},\cdots,i_{N-s}^{0\prime})\cap(j_1^{0\prime},\cdots,j_{N-t}^{0\prime})\neq\phi$ that the rank of the right-under corner's submatrix in (4.4) with $[(i_1,\cdots,i_s),(j_1,\cdots,j_t)]=[(i_1^0,\cdots,i_s^0),(j_1^0,\cdots,j_t^0)]$ is at most $\sum_{i=1}^{N} q_i-1$ for all $Q_i\in R^{q_i\times q_i}(i=1,\cdots,N)$ and all $s\in\sigma(Q_k)$, where

$k\in(1,\cdots,N)/[(i_1^{0\prime},\cdots,i_{N-s}^{0\prime})\cup(j_1^{0\prime},\cdots,j_{N-t}^{0\prime})]=(i_1^0,\cdots,i_s^0)\cup(j_1^0,\cdots,j_t^0)$. In this way, when $[(i_1,\cdots,i_s),(j_1,\cdots,j_t)]=[(i_1^0,\cdots,i_s^0),(j_1^0,\cdots,j_t^0)]$, (4.4) may not be true for each $(Q_i,K_i)\in R^{q_i\times q_i}\times R^{m_i\times r_i}$ $(i=1,\cdots,N)$ and $s\in\sigma(Q_i)$, $i\in(i_1^0,\cdots,i_s^0)\cap(j_1^0,\cdots,j_t^0)$, whichleades to

$$\bigcup_{k\in(i_1^0,\cdots,i_s^0)\cap(j_1^0,\cdots,j_t^0)}\sigma(Q_k)\subset\sigma(\overline{A})$$

for all $(Q_i,R_i,S_i,K_i)\in R^{q_i\times q_i}\times R^{q_i\times r_i}\times R^{m_i\times q_i}\times R^{m_i\times r_i}$ $(i=1,\cdots,N)$. As a result, the necessary part is concluded and the proof of Theorem 2 is complete.

Corollary 2.1. If the N-control agent spstem (2.4) is strongly connected (cf. [2]), then for any positive integer set $(q_1,\cdots,q_N)$ there must exist a $(q_1,\cdots,q_N)$-order decentralized compensator (4.1) such that the decentralized compensating spectrum and the closed-loop spectrum do not intersect.

Corollary 2.2. Given the system (2.4) and a positive integer set $(q_1,\cdots,q_N)$. If the system $S_{[(i_1,\cdots,i_s),(j_1,\cdots,j_t)]}$ is vanishing for some $[(i_1,\cdots,i_s),\ (j_1,\cdots,j_t)]\in P$, then for arbitrarily designed $(q_1,\cdots,q_N)$-order decentralized compensator of the form (4.1),

$$\bigcup_{k\in(i_1,\cdots,i_s)\cap(j_1,\cdots,j_t)}\sigma(Q_k)\subset\sigma(\bar{A}).$$

Corollary 2.3. Given the system (2.4) and a positive integer set $(q_1,\cdots,q_N)$. Suppose that the system $S_{(\bar{i},\bar{j})}$ is not vanishing for all $(\bar{i},\bar{j})\in P$. Then, the set

$$\Omega(q_1,\cdots,q_N)=\Big\{p(Q_i,R_i,S_i,K_i;i=1,\cdots,N);(Q_i,R_i,S_i,K_i)\in R^{q_i\times q_i}\times R^{q_i\times r_i}\times R^{m_i\times q_i}\times R^{m_i\times r_i}\ (i=1,\cdots,N),\Big[\bigcup_{i=1}^{N}\sigma(Q_i)\Big]\cap\sigma(\bar{A})\neq\phi\Big\}$$

is a proper variety in $R^{\sum_{i=1}^{N}(q_i^2+q_ir_i+q_im_i+r_im_i)}$, where $\bar{A}$ is defined as in (4.2).

Corollary 2.4. Suppose that the system (2.4) has no unstable decentralized fixed modes, and that the subsystem $S_{(\bar{i},\bar{j})}$ of (2.4) is not vanishing for each $(\bar{i},\bar{j})\in P$. Then, there must exist a decentralized dynamic compensator of the form (4.1) such that the following two conditions are satisfied simultaneously:

(i) the closed-loop system is stable, i. e.,

$$\sigma(\bar{A})\subset C^{1-};$$

(ii) the decentralized compensating spectrum and the closed-loop spectrum do not intersect, i. e.,

$$\Big[\bigcup_{i=1}^{N}\sigma(Q_i)\Big]\cap\sigma(\bar{A})=\phi.$$

Remark 1. The application of Corollary 2.3 shows that when the system $S_{(\bar{i},\bar{j})}$ is not vanishing for all $(\bar{i},\bar{j})$ P, compensating spectra and closed-loop spectra do not intersect for almost all dynamic compensators of the form (4.1).

Remark 2. If each single channel system (C_i,A,B_i) of (2.4) is not vanishing, then neither is the system $S_{(\bar{i},\bar{j})}$ for all $(\bar{i},\bar{j})\in P$.

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关于补偿谱和闭环谱的相交性问题

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摘　要

本文使用分散控制方法，讨论并解决了控制系统的补偿谱和闭环谱的相交性问题．文中引进了消失性的概念，得到了关于集中系统和分散系统存在使补偿器和闭环谱不相交的动态补偿器的充分必要条件．此外，该条件不成立时，给出了使用任何动态补偿器所得到的补偿谱与闭环谱的交．