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A Sufficient Condition for Unit Feedback Linear System Decoupling with Stability

Wang Qingguo, Zhou Chunhui, Sun Youxian
(Department of Chemical Engineering, Zhejiang University, Hangzhou)

Abstract

A sufficient condition for decoupling of a linear system by cascade compensator in a unit feedback system is derived under the requirement that the decoupled system be internally stable. The condition is stated in terms of a quantity which is directly related to the transfer matrix of the given system and can be therefore checked readily. An Example is presented to illustrate the application ot the result.

1. Introduction

Linear system decoupling by appropriate compensations has been an extensively investigated problem in the system theory literature during past two decades [1-4]. An important issue associated with decoupling is the necessity to ensure the internal stability of the decoupled system. In the present paper, we consider an unit feedback linear time-invariant system, shown in Fig. 1. A sufficient condition for the existence of a proper compensator $K(s)$ is derived such tnat it will both decouple the given system $G(s)$ nally stabilize it. If it exists, a procedure of ng such a compensator is also given. The main result is and a numerical example given in Section 2. In the part of this section, we review some concepts and facts l be used in the following section.

; be strictly proper and K be proper rational transfer matrices, which are represenied by coprime polynomial action descriptions

$$G = A_l^{-1} B_l = B_r A_r^{-1}, \qquad K = C_l^{-1} D_l = D_r C_r^{-1} \qquad (1)$$

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The system in Fig,1 is called internally stable if det $[C_1A_r + D_1B_r]$ (or det $[A_1C_r + B_1D_r]$) has all its roots in C^-, the open left half the of the complex plane.

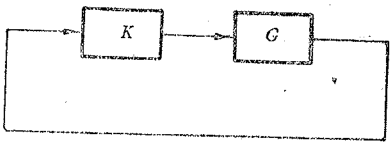

Fig.1

Lemma [6] The system in Fig. 1 is internally stable if and only if

(a) Both $[B_1^T(s)\ \ C_1^T(s)]$ and $[A_r(s)\ \ D_r(s)]$ have full rank for any $s \in C^+$, the closed right half of the complex plane.

(b) $(I+GK)^{-1}$ has all its poles is C^-.

Let P and P^* be nonsingular polynomial matrices, P^* is said to be a right strict adjoint of P[4] whenever the following conditions are satisfied: (a) PP^* is diagonal; and (b) if P' is any polynomial nonsingular matrix such that PP' is diagonal, then P^* is a left divisor of P'. A right strict adjoint of a nonsingular P can be constructed as follows. Let $(P^{-1})_i$ be the ith column of P^{-1} and let $\hat{d}_{PCi}$ be the least common denominator of all elements in $(P^{-1})_i$, then $P^* = P^{-1}\text{diag}[a_{pci}]$.

2. The Result

Let there be a given liear time-invariant square system with a m×m strictly proper rational transfer function matrix G. Consider an unit feedback system, shown in Fig. 1 where K is a m×m proper rational ransfer function matrix of a compensator. The transfer matrix of the closed loop system is

$$H = GK[I + GK]^{-1}$$

G is called unit feedback decoupleable with internal stability (UDIS) if there is a K such that H is a nonsingular diagonal matrix and the closed-loop system is internally stable. It easily follows that Theorem 1 holds.

Theorem 1 G is UDIS if and only if there is a proper K such that GK is diagonal and the condition (a) in the Lemma holds.

Let d_{gri} be the least common denominator of all elements in the ith row of G; This implies that $G=\text{diag}[d_{gri}^{-1}]\,P$ for some polynomial matrix P. We now are in a position to prove our main result.

Theorem 2 G is UDIS if (2) houlds

$$\text{Rank}\,[A_r(s)\;\; P^*(s)]=m \qquad \text{for any } s\in C^+ \tag{2}$$

where P^* is a right strict adjoint of P.

Proof Assume that (2) holds. Let a set of polynomials d_{kci}, $i=1,2,\cdots,m$ satisfy the following: (*a*) $\text{diag}^{-1}[d_{kci}]$ is stable; and (*b*) $P^*\,\text{diag}^{-1}[d_{kci}]$ is proper. Obviously such a set of polynomials always exists. Let $K=P^*\,\text{diag}^{-1}[d_{kci}]$. then K is proper and $GK=(\text{diag}^{-1}[d_{gri}])$. (PP^*) $(\text{diag}^{-1}[d_{kci}])$ is diagonal. Furthermore with the coprime fractions (1) and K being stable, it follows that $[\,B_1^T(s)\;\; C_1^T(s)]$ has full rank for any $s\in C^+$. Since $K=D_rC_r^{-1}=P^*\text{diag}^{-1}[d_{kci}]$.

$$\begin{bmatrix} P^* \\ \text{diag}[d_{kci}] \end{bmatrix} = \begin{bmatrix} D_r \\ C_r \end{bmatrix} R$$

for some polynomial matrix R which has no zero in C^+. Thus we have

$$[A_r(s)\;\; D_r(s)]=[A_r(s)\;\; P^*(s)]\begin{bmatrix} I & 0 \\ 0 & R^{-1}(s) \end{bmatrix} \tag{3}$$

Because $\begin{bmatrix} I & 0 \\ 0 & R^{-1}(s) \end{bmatrix}$ is nonsingular for any $s\in C^+$ and, by assumption, $[A_r\,(s)\;\; P*(s)]$ has full rank for any $s\in C^+$. then the left hand side of (3) also has full rank for any $s\in C^+$. It follows from Theorem 1 that G is thus UDIS.

It is noted that if the solvability condition is satisfied, a compensator which solves UDIS can be constructed by the procedure given in the proof of Theorem 2. In particular with an appropriate high order of K it is always possible to arbitrarily assign all poles of the closed-loop system except for cancellated stable pole-zeros. This is now a standard problem and will not be further discussed here.

Example Let G be given as

$$G=\begin{bmatrix} 1/(s-1) & 0 \\ 1/(s+1) & 1/(s-2) \end{bmatrix}$$

Simple calculations yield

$$P^*=\begin{bmatrix} (s+1) & 0 \\ -(s-2) & 1 \end{bmatrix},\quad A_r=\begin{bmatrix} (s-1)(s+1) & 0 \\ 0 & (s-2) \end{bmatrix}$$

Clearly Rank $[A_r(s)\ P^*(s)]=2$ for any $s\in C^+$. By Theorem 2, G is UDIS. Indeed, Let $K=P^*\ \mathrm{diag}^{-1}[d_{kci}]$ with $d_{kci}=(\alpha_i s+\beta_i)$, $i=1,2$. Then K is proper and

$$GK=\begin{bmatrix} (s+1)/(s-1)(\alpha_1 s+\beta_1) & 0 \\ 0 & 1/(s-2)(\alpha_2 s+\beta_2) \end{bmatrix}$$

This implies that the pole at $s=-1$ of G has been cancellated. Except for this one, all other poles can be arbitrarily assigned by choosing appropriate α_i and β_i.

3. Conclusion

A sufficient condition for the existence of a compensator which decouples a given system and at the same time internally stabilizes it has been established employing a polynomial matrix approach. It is given in terms of a quantity which is directly related to the transfer matrix of the system, and is readily checked by some CAD software.

Reference

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单位反馈线性系统稳定解耦的一个充分条件

王庆国　周春晖　孙优贤

（浙江大学化工系，杭州）

摘　要

本文讨论了具有内部稳定性约束的单位反馈线性系统的解耦问题，并给出了解存在的一个充分条件。该条件容易从对象传递函数矩阵来判定它是否满足，示例说明了结果的应用。

对《代谢系统与内分泌系统之数学建模》一书的简介

(THE METHEMATICAL MODELING OF METABOLIC & ENDOCRINE SYSTEMS
E.Carson, C.Colelli, and L.Finkelstein
John Wiley & Sons, 1983, 394p.)

涂　其　栵

（华南工学院自动化系，广州）

目前讨论数学建模理论的书已有不少，但讲述其应用的书却是不多。现在介绍的一本就是其中之一。原书共十章。前九章叙述建模的基础知识（理论与实际问题均在内），并举例说明。最后一章为案例分析。虽然作者认定生理学家及临床医师为主要读者，但亦不排除工程师、物理学家及应用数学家。由于叙述清晰，搞控制理论的人也不难读懂。书中有关建模的目的，模型的等级，可辨识性，有效性判定以及对一些应用中发生的问题之讨论，未见于同类书籍。因此，本书对生物医药以外的实际问题建模也有指导价值。

第一章：引言。第二章：代谢与内分泌系统。第三章：代谢系统建模的目的。第四章：建模过程。第五章：代谢系统建模方法。第六章：模型辨识的一般方法。第七章：理论上的可辨识性及其与实验数据之关系。第八章：参数辨识，实际上的可辨识性及高精度实验设计。第九章：代谢系统与内分泌系统的模型有效性判定。第十章：案例分析（糖尿病人注入胰岛素的各种闭环方案比较；活体内胰岛素分泌动力学的数学模型；碳水化合物代谢状态的临床模型；甲酮体动力学模型；非共轭胆红素代谢模型的临床应用；使用胆红素代谢模型协助解释生理机制；肝脏对半乳糖吸收过程的分布参数模型；磺溴酞动力学模型对肝胆病的检查作用；人体甲状腺激素的调节模型；甲状腺激素的临床模型。全书共394页，参考文献270篇，