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Doubly Coprime Fractional Representations of Generalized Dynamical Systems

Wang Feiyue
(Department of Mechanics Zhejiang University, Hangzhou)
Mark J. Balas
(Robotics and Automation Laboratory Department of ECSE
Rensselaer Polytechnic Institute Troy, New York)

Abstract

Explicit formulas for doubly coprime fractional representations of the transfer matrix of a generalized dynamical system are given in terms of a stabilizable and detectable state—space realization of the transfer matrix. These formulas establish a possible way to use the fractional representation approach in the synthesis of the generalized dynamical systems.

1. Introduction

Both the fractional representation approach and the generalized dynamical system theory have received a great deal of attention over the past few years (see [2][5][6] and the references therein). It has been shown that fractional representation approach is a powerful tool in the analysis and synthesis of linear feedback system. A connection between state—space and doubly coprime fractional representations has been given recently in [3] by Nett, Jacobson and Balas. However, little has been done to apply this powerful method into the correspon-ding problems in the generalized dynamical systems. In this note we extend the result in [3] to the generalized dynamical system. The extension establishes a way to use the fractionaal representations in the generalized dynamical system.

2. Preliminaries

Let $\Omega\subset C$ denote any closed superset of the closed right—half

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complex plane which is symmetric with respect to the real axis. Let H denote the ring of proper rational functions which are analytic in Ω. A rational matrix $X \in H(s)^{p\times m}$ is said to be Ω—stable if it has entries in H.

A pair of real matrices (E, A) is said to be Ω-Hurwitz if matrix $(sE-A)^{-1}$ exists and is Ω-stable. A triple of real matrices $(E, A, B) \in R^{n\times n}\times R^{n\times n}\times R^{n\times m}$ is said to be Ω-stabilizable if there exist a $K \in R^{m\times n}$ such that $(E, A-BK)$ is Ω-Hurwitz. Similarly, a triple of real matrices $(C,E,A) \in R^{p\times n}\times R^{n\times n}\times R^{n\times n}$ is said to be Ω-detectable if there exists a matrix $F \in R^{n\times p}$ such that $(E,A-FC)$ is Ω-Hurwitz.

3. Main Results

Consider a generalized dynamical system described by the equations

$$E\dot{x} = Ax + Bu \tag{1}$$

$$y = Cx \tag{2}$$

where $E,A \in R^{n\times n}$, $B \in R^{n\times m}$, $C \in R^{p\times n}$ are real matrices. As done by Rosenbrock [4] we henceforth make the standard assumption that $|sE-A| \neq 0$. The transfer matrix of the system is $G(s)=C(sE-A)^{-1}B$. Our objective is to derive the doubly coprime factorizations of $G(s)$. Three factorizations obtained are given in the following theorems.

Theorem 1 Given the system (1)—(2), suppose the triples (E,A,B), (C,E,A) are Ω-stabilizable and detectable, respectively. Select matrices $K \in R^{m\times n}$, $F \in R^{n\times p}$ such that both $(E,A-BK)$ and $(E,A-FC)$ are Ω-Hurwitz. Define

$$N(s) = CH_C^{-1}B,\ D(s) = I - KH_C^{-1}B \tag{3}$$

$$U(s) = KH_0^{-1}F,\ V(s) = I + KH_0^{-1}B \tag{4}$$

$$\overline{N}(s) = CH_0^{-1}B, \overline{D}(s) = I - CH_0^{-1}F \tag{5}$$

$$\overline{U}(s) = KH_C^{-1}F, \overline{V}(s) = I + CH_C^{-1}F \tag{6}$$

where

$$H_C(s) = H(s) + BK,\ H_o(s) = H(s) + FC,\ H(s) = sE - A \tag{7}$$

then

i) all eight matrices described by (3)—(6) are Ω-stable

ii) $D(s)$ and $\overline{D}(s)$ are nonsingular iii) $G(s)=N(s)D(s)^{-1}=\overline{D}(s)^{-1}\overline{N}(s)$

iV)

$$\begin{bmatrix} V(s) & U(s) \\ -\overline{N}(s) & \overline{D}(s) \end{bmatrix}\begin{bmatrix} D(s) & -\overline{U}(s) \\ N(s) & \overline{V}(s) \end{bmatrix}=\begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$$

Remarks The above theorem is an straightforward extension of the theorem given in [3] where $E=I$, i.e, the normal dynamical system was considered. As said in [3], this result is readily extended to the case $G(s)=C(sE-A)^{-1}B+W$, where $W\epsilon\ H^{p\times m}$, by 1) adding $WD(s)$ to the expression for $N(s)$, 2) adding $\overline{D}(s)W$ to the expression for $\overline{N}(s)$, 3) subtracting $U(s)W$ from the expression for $V(s)$, and4) subtracting $W\overline{U}(s)$ from the expression for $\overline{V}(s)$.

Proof i) follows immediately from the definition of an Ω-Hurwitz pair of matrices, ii) Consider equations

$$D(s)=I-KH_C^{-1}B=I-K(H+BK)^{-1}B=(I+KH^{-1}B)^{-1}$$

and

$$|H_C|=|H+BK|=|H|\,|I+H^{-1}BK|=|H|\,|I+BKH^{-1}|=|H|\,|I+KH^{-1}B|$$

it follows that

$$|D(s)|=\frac{|H(s)|}{|H_C(s)|}$$

which indicates that $|D(s)|\neq 0$.

Similarly, one shows $|D(s)|=\dfrac{|H(s)|}{|H_o(s)|}\neq 0$.

iii) Write

$$\begin{aligned} N(s)D^{-1}(s) &= CH_C^{-1}B(I-KH_C^{-1}B)=CH_C^{-1}(I-BKH_C^{-1})^{-1}B \\ &= C((I-BKH_C^{-1})H_C)^{-1}B=C(H_C-BK)^{-1}B \\ &= CH^{-1}B=G(s) \end{aligned}$$

the matrix identity $B(I-AB)^{-1}=(I-BA)^{-1}B$ is used in the above second step.

In a similar fashion, one shows $G(s)=D(s)^{-1}\overline{N}(s)$.

iV) We only need to verify three equalities, the fourth being contained in iii). Now,

$$U(s)N(s)+V(s)D(s)=(KH_o^{-1}F)(CH_C^{-1}B)+(I+KH_o^{-1}B)(I-KH_C^{-1}B)$$

$$= KH_o^{-1} FCH_C^{-1}B + I + KH_o^{-1}B - KH_C^{-1}B - KH_o^{-1} BKH_C^{-1}B$$

$$= I + KH_o^{-1}(FC + H_C - H_o - BK)H_C^{-1}B$$

$$= I + KH_o^{-1}(FC + BK - FC - BK)H_C^{-1}B = I$$

Also

$$V(s)\overline{U}(s) = (I + KH_o^{-1}B)(KH_C^{-1}F) = K(I + H_o^{-1}BK)H_C^{-1}F$$

$$= KH_o^{-1}(H_o + BK)H_C^{-1}F = KH_o^{-1}(H_C + FC)H_C^{-1}F$$

$$= KH_o^{-1}(I + FCH_C^{-1})F = KH_o^{-1}F(I + CH_C^{-1}F)$$

$$= U(s)\overline{V}(s)$$

Finally, one shows $\overline{N}(s)\overline{U}(s) + \overline{D}(s)\overline{V}(s) = I$ by manipulations similar to those above.

The next two results were motivated by the state and state derivative feedback $u = -K_1x - K_2\dot{x}$.

Theorem 2.a Suppose there exist matrices K_1, $K_2 \epsilon R^{m\times n}$, $F \epsilon R^{n\times p}$ such that 1) matrix $E + BK_2$ is nonsingular, 2) $(E + BK_2, A - BK_1)$ and $(E, A - FC)$ are Ω-Hurwitz, 3) $\lim_{s\to\infty} K_2(sE - A + FC)^{-1}B = 0$ and $\lim_{s\to\infty} K_2(sK - A + FC)^{-1}F = 0$. Then Theorem 1 still holds if one replaces K in the expression (3)—(7) by $K_1 + sK_2$.

Remarks the condition 1) in the above theorem is the necessary and sufficient condition for the generalized dynamical system (1) to be normalizable[8]. Conditions 1) and 2) are equivalent to that all uncontrollable modes of (E, A, B) lying in $C - \Omega$[1,8]. Conditon 3) is satisfied when E is nonsingular (i.e., the normal dynamical system).

The proof of the above theorem is entirely analogous to that of theorem 1.

The following result is dual to the theorem 2.a.

Theorem 2.b Suppose there exist matrices $K \epsilon R^{m\times n}$, F_1, $F_2 \epsilon R^{n\times p}$ such that 1) $(E, A - BK)$ and $(E + F_2C, A - F_1C)$ are Ω Ω-Hurwitz, 2) Matrix $E + F_2C$ is nonsingular, and 3) $\lim_{s\to\infty} C(sE - A + BK)^{-1}F_2 = 0$ and $\lim_{s\to\infty} K(sE - A + BK)^{-1}F_2 = 0$. Then theorem 1 still holds if one replaces F in the expression (3)—(7) by $F_1 + sF_2$.

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广义动力系统的双互质分式表示

王 飞 跃

（浙江大学力学系，杭州）

Balas, J. Mark

（伦塞利尔工学院计算机与系统工程系，美国）

摘 要

利用传递矩阵的稳定与可测的状态空间实现，本文给出了广义动力系统传递矩阵的双互质分式表示的显式公式。这些公式提供了在广义系统的综合中应用分式表示方法的一条可能途径。