第6卷 第1期 控制理论与应用 Vol.6, No.1
1989年3月 CONTROL THEORY AND APPLICATIONS Mar., 1989

Stochastic Optimal Control Using Input-Output Model with Time Delay

Wu Jie
(Radio Auto—Control Research Institute, South China University of Teohnology, Guangzhou)
Ma Yuxu
(Guangzhou Petrochrmical Works)

Abstract

Owing to the large computational load and the time delay not included in the system model, the stochastic optimal control has not been applied to the process control successfully, even though it has many advantages. From a new point of view, a discrete stochastic optimal control algorithm is presented in the paper, where the input-output plant model with time delay is used and solution of Riccati equation and spectral factorization is not required. The algorithm requires less computational load, therefore, it is easy to implement with microcomputer. The analysis of some closed-loop properties is given in the paper.

1. Introduction

The design of optimal controllers for the stochastic LQG problem is well established, using frequency and time domain theory [1,2,3]. The treatment in the frequency-domain using the Wiener-Newton approach requires a time consuming spectral factorization. The algorithms using time-domain theorr heory rest on the so-called certainty equivalence hypothesis and the solution of the Riccati equation is necessary. At present situation, LQG controllers have been rarely applied in industrial control, except in minimum variance and generalized minimum variance forms which are based on only singlestage cost function minimization. This may be due to the large computational load of solving the Riccati equation and performing the spectral factorization for LQG controllers

Manuscript received Nov. 29, 1987, revised July. 23, 1988.

Kucera[4] proposed a controller based on an infinite-tage cost function derived totally in the polynomial domain. The major difficulty with such a method is the computational requirements of solving a spectral factization and two Diophantine equations. Astrom[5,6] has discussed the use of LQG regulators, where the solution of a staedy state Riccati equation and spectral factorization were required. The work of Peterka[7], who employed a finite-inteval cost function in deriving an optimal self-tunner, is also related to the LQG controllers.

In this prper, the authers propose a new version of discrete stochastic optimal control algorithm. The plant is assumed to be a discrete input-output model and time delay is considered in the plant model, because we can obtain the industrial plant's input-output model easier than its staedy-state model, and the plant always possesses dead time property in process control as well.

The control strategy is designed to minimize the value of an N—stage cost function. The optimal control algorithm here is easy to put into effect in industrial process with microcomputer, as the solution of the Riccati equation and spectral factorization are avoided.

2. Plant Model and Minimum Variance Prediction

The input—output model of a plant may be represented in discrete time as

$$A(z^{-1})y(t)=z^{-k}B(z^{-1})u(t)+C(z^{-1})\xi(t) \quad (1)$$

where $y(t)$ is the measured variable at time t, $u(t)$ the control signal, $A(z^{-1})$, $B(z^{-1})$ and $C(z^{-1})$ are polynomials of the form

$$A(z^{-1})=1+a_1z^{-1}+a_2z^{-2}+\cdots+a_nz^{-n}$$

$$B(z^{-1})=b_0+b_1z^{-1}+b_2z^{-2}+\cdots+b_nz^{-n}$$

$$C(z^{-1})=1+c_1z^{-1}+c_2z^{-2}+\cdots+c_nz^{-n}$$

in the backward shift operator z^{-1}, k is the integer time delay of the plant and ξ(t) is an uncorrelated sequence of random variables with zero mean; below the polynomial $C(z^{-1})$ is assumed to unity.

By using the equation

$$1=AE+z^{-k}F \quad (2)$$

Astrom[1] obtained the result that the minimum variance predictor over k steps is given by

$$\hat{y}(t+k/t)=BEu(t)+Fy(t) \quad (3)$$

The minimum uariance control law was derived by minimizing the criterion

$$J = E\{y^2(t+k)\}$$

Therefore, minimum variance control is considered as one-stage cost function stochastic optimal control.

In order to derive an N-stage cost function control law, the authers extend (2) to the following form[9]

$$1 = AE_N + z^{-k-N}F_N \qquad (N=0,1,2,\cdots) \tag{4}$$

where polynomials E_N and F_N are of degrees $N+k-1$ and $n-1$, respectively.

Using (1) and (4), the optimal predictor over $N+k$ steps is given by

$$\hat{y}(t+k+N/t) = BE_N u(t+N) + F_N y(t) \tag{5}$$

3. Optimal Control Law Using Input-Output Approach

In space-state regulation it makes sense to express the criterion in terms of varinace of space—state variables and control variable, i.e.

$$J = E\left\{\frac{1}{2} x_N^\tau F x_N + \frac{1}{2}\sum_{k=0}^{N-1}(x_k^\tau Q x_k + u_k^\tau R U_k)\right\} \tag{6}$$

For input—output approach, the criterion may be to minimize the variance of the output and control variable, therefore, we introduce the loss functson

$$J = E\{e_N^\tau R e_N + u_N^\tau Q u_N\} \tag{7}$$

where R, Q are diagonal matrices of order $N+1 \times N+1$, here, $N+1$ is the stage of the cost function. e_N and u_N are $N+1$ vectors and are defined by

$$e_N = [w(t+k),\cdots,w(t+k+N)]^\tau - [y(t+k),\cdots,y(t+k+N)]^\tau = W - Y \tag{8}$$

$$u_N = [u(t),\cdots,u(t+N)]^\tau \tag{9}$$

where W is the setpoit vector.

Using (4) in (1) gives

$$y(t+k+N) = G_N u(t+N) + F_N y(t) + E_N \xi(t+k+N) \tag{10}$$

where $G_N = BE_N$ is a polynomial in z^{-1}, it may be written as

$$G_N = g_{N0} + g_{N1}z^{-1} + \cdots + g_{NN}z^{-N} + \cdots + g_{Nr}z^{-r}$$

$$= g_{N0} + g_{N1}z^{-1} + \cdots + g_{NN}z^{-N} + G_N' z^{-N-1} \tag{11}$$

The G_N' is also a polynomial in z^{-1} and of dimension $r-N-1$.

Then, as N takes the value $0,1,2,\cdots,N$ respectively, the output vector Y may be written as

$$Y=\begin{pmatrix} g_{00} & 0 & \cdots & \\ g_{10} & g_{11} & \cdots & \\ \cdots & & & \\ g_{N0} & g_{N1} & \cdots & g_{NN}\end{pmatrix}\begin{pmatrix} u(t) \\ u(t+1) \\ \vdots \\ u(i+N)\end{pmatrix}+\begin{pmatrix} G_0'u(t-1)+F_0y(t) \\ G_j'u(t-1)+F_1y(t) \\ \cdots \\ G_N'u(t-1)+F_Ny(t)\end{pmatrix}+$$

$$+\begin{pmatrix} E_0 & & & \\ & E_1 & & \\ & & \ddots & \\ & & & E_N\end{pmatrix}\begin{pmatrix} \xi(t+k) \\ \xi(t+k)H) \\ \vdots \\ \xi(t+k+N)\end{pmatrix}=G\mathbf{u}+F+H\xi \qquad (12)$$

where G is a matrix of degree $N+1\times N+1$, H a polynomial matrix in z^{-1} and F and ξ are $N+1$ vectors. Substituting (12) into (7) yields

$$J=E\{(W-Gu-F-H\xi)^{\tau}R(W-Gu-F-H\xi)+u^{\tau}Qu\} \qquad (13)$$

Because $\xi(t+k),\cdots,\ \xi(t+k+N)$ are a sequence of independent random variables and are independent of vector u and $y(t),\ y(t-1),\ \cdots$, then (13) can be written as

$$J=E\{(W-F)^{\tau}R(W-F)+2(F-W)^{\tau}RGu+u^{\tau}G^{\tau}RGu+u\ Qu+\xi\ H\ RH\xi\}$$

The optimal controller is chosen to minimize the above cost function J. This can be done by differentiating J to u and the optimal control u^* is given by

$$u^*=(G\ RG+Q)^{-1}G\ R(W-F) \qquad (14)$$

Although the optimal control vector u^* is obtained here, it is worth mentioning that this control law must be used in the receding-horizon sense to ensure that the same control law applied for all time [8].

Define

$$(G^{\tau}RG+R)^{-1}G^{\tau}R=\begin{pmatrix} s_1 & s_2 & \cdots & s_{N+1} \\ \times & \times & \cdots & \times \\ \cdots & & & \cdots \\ \times & \times & \cdots & \times\end{pmatrix}\Big/D \qquad (15)$$

where $$D=|G\ RG+Q| \qquad (16)$$

Then solving for first control signal $u(t)$ yields

$$u^*(t)=(s_1\ s_2\cdots s_{N+1})(W-F)/D \qquad (17)$$

Notice that

$$F=\begin{pmatrix} G_0'u(t-1)+F_0y(t) \\ \vdots \\ G_N'u(t-1)+F_Ny(t) \end{pmatrix}$$

hence

$$u^*(t)=\left[\sum_{i=0}^{N}s_{i+1}w(t+k+i)-\left(\sum_{i=0}^{N}F_is_{i+1}\right)y(t)-\left(\sum_{i=0}^{N}G_i's_{i+1}\right)u(t-1)\right]\Big/D \tag{18}$$

It is convenient to rewrite (18) as

$$u(t)=\left[\sum_{i=0}^{N}s_{i+1}w(t+k+i)-\left(\sum_{i=0}^{N}F_is_{i+1}\right)y(t)\right]\Big/\left[D+\left(\sum_{i=0}^{N}G_i's_{i+1}\right)z^{-1}\right] \tag{19}$$

It is thus obvious that the optimal controller takes the form of a feedback control law. The analysis of closed-loop properties can be made below.

4. Closed-Loop Properties

The block diagram of the feedback system can be drawn as Fig.1

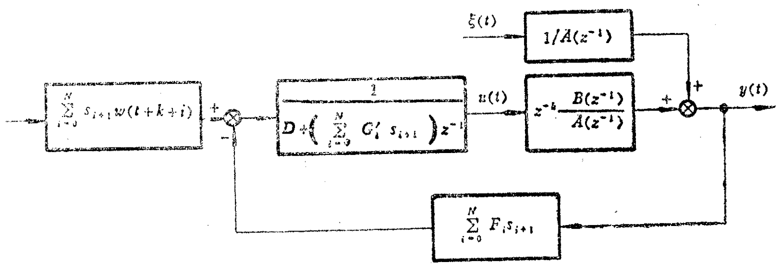

Fig.1 The block-diagram of the closed-loop system

according to (19) and (1). It can be seen that the indivadual elements of this block-diagram are all causal.

From Fig.1, the system output $y(t)$ is given by

$$y(t)=z^{-k}\frac{B}{A}\cdot\frac{\sum_{i=0}^{N}s_{i+1}w(t+k+i)-\left(\sum_{i=0}^{N}F_is_{i+1}\right)y(t)}{D+\left(\sum_{i=0}^{N}G_i's_{i+1}\right)z^{-1}}+\frac{1}{A}\xi(t) \tag{20}$$

Using equation (4) and (11), we can obtain the closed-loop transfer function as follows

$$y(t)=\frac{z^{-k}B\left[\sum_{i=0}^{N}s_{i+1}w(t+k+i)\right]+\left[D+\left(\sum_{i=0}^{N}G_i's_{i+1}\right)z^{-1}\right]\xi(t)}{\left(\sum_{i=0}^{N}s_{i+1}z^{i}\right)B+\left[D-\left(\sum_{j=0}^{N}\sum_{i=0}^{j}s_{j+1}g_{ji}\right)\right]A} \qquad (21)$$

This is not a cancellation law and the closed-loop characteristic equation is

$$\left(\sum_{i=0}^{N}s_{i+1}z^{i}\right)B+\left[D-\left(\sum_{j=0}^{N}\sum_{i=0}^{j}s_{j+1}g_{ji}\right)\right]A=0 \qquad (22)$$

when analyzing the control system, it is important to calculate the stead-state value of the output of the system. It is usually desired that stead—state error dose not exist between the setpoint and the output in industrial control applications.

To calculate the stead-state value of the output, consider the case

$$w(t+k+i)=w(t) \qquad (i=0,1,2,\cdots,N)$$

and

$$\xi(t)=0$$

using the final-value theorem, the stead-state output is then given by

$$y(\infty)=\frac{B(1)\left(\sum_{i=0}^{N}s_{i+1}\right)w(\infty)}{\left(\sum_{i=0}^{N}s_{i+1}\right)B(1)+\left[D-\left(\sum_{j=0}^{N}\sum_{i=0}^{j}s_{j+1}g_{ji}\right)\right]A(1)} \qquad (23)$$

It is clear that the condition existing no stead-state error is determined by

$$D=\sum_{j=0}^{N}\sum_{i=0}^{j}s_{j+1}g_{ji} \qquad (24)$$

Now, let's discuss what the equation (24) means. It is known that from Section 3, g_{ji} is determined by the parameters of the system model and s_i is dependent on G and matrices R, Q, D is calculated according to (16). Therefore, equation (24) means that one of the $2(N+1)$ elements of R and Q can not be selected arbitarily, and should be determined by (24). In practice, the elements of the matrices R, Q can be selected in advance and then (24) is used to calculate D. This is equivalent to determine one of the matrices R or Q by (24).

5. Simulation Results

Previous section have formulated a discrete optimal control algorithm using input-output approach. Some closed-loop properties have been given and further properties of this algorithm will be illustrated in simulation. Below, the weighting matrices in the criterion are unity matrices.

The simulated plant had the second-order discrete time model

$$(1-1.489z^{-1}+0.5488z^{-2})y(t)=z^{-4}(0.139+0.1076z^{-1})u(t)$$

where the time delay k is 4. In designing the LQG controllers, it is assumed that the time delay is 2 instead of the actual value 4. Fig. 2(a) and (b) show the cases that the cost function stage $N+1$ is 4

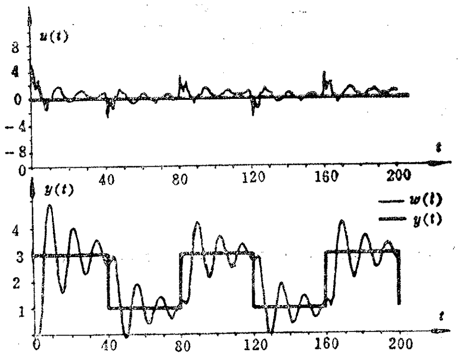

Fig.2(a) The case of N+1=4

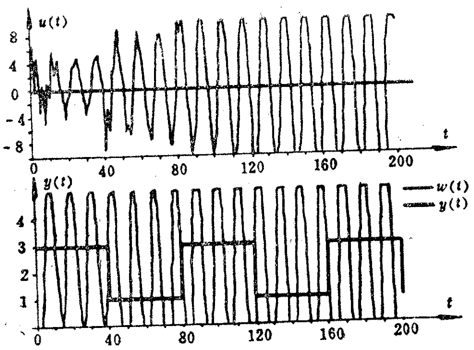

Fig.2(b) The case of N+1=9

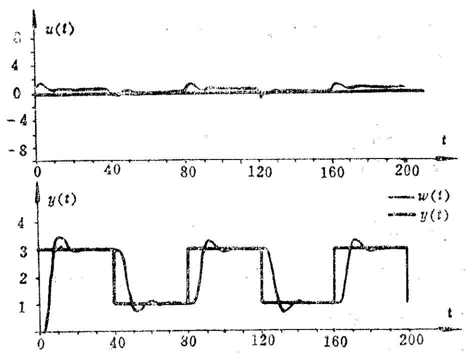

Fig.2(c) The case of N+1=1

and 9 respectively. It can be seen from the figures that as the stage of the cost function becomes larger, the properties of the closed loop can be improved. The resulting performance with minimum variance control (i. e. $N+1=1$) is shown in Fig. 2(c).

6. Conclusions

After extending the Diophantine equation to the general case, a new version of LQG control algorithm is presented. Because we can obtain the industrial plant's input-output model easier than its steady-state model, and the industrial plant always posesses dead time property in process control as well. So, the input-output model with dead time is used above. The main advantage of the algorithm here is that the solution of the Riccati equation or spectral factorization is not required, which most of previous LQG controllers always involved. Therefore, the computational load is lightened and the storage and speed requirements decrease, and this algorithm is easy to apply to the indusrial process.

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采用带时滞的输入–输出模型的随机最优控制

吴 捷

（华南理工大学无线电与自动控制研究所，广州）

马裕旭

（广州石油化工总厂）

摘 要

由于在系统模型中未引入时滞，以及计算量大等原因，尽管随机最优控制有很多优点，但未能成功地应用于过程控制．本文从新的角度，提出一种离散随机最优控制算法，该法采用带有时滞的输入－输出对象模型，而且无需求解 Riccati 方程和进行谱因子分解．本算法要求很少的计算量，因而易在微型计算机上实现．文中还对系统闭环特性进行分析．