A Robust Adaptive Pole Placement Prediction Controller

Chen Yixuan, Wu Guangyu

(Department of Automatic Control, Harbin Institute of Technology)

Abstract

This paper proposes a new method for robust adaptive pole placement prediction control, which effectively overcomes the instability phenomena caused by the mismatch of plant model and smith predictor.

Key words——Robustness; Pole placement; Adaptive control.

1. Introduction

It is well known that smith prediction control is a valid method for system with moderately large delay[1]. However its main weakness is its sensitiveness to the variations of process parameters. If the error for plant model and smith compensator exceeds a certain degree, system performance will rapidly deteriorate, and the system may even lose its stability[2-4]. Recently Chien et al[5] presented a self-tuning controller with smith predictor for improvement, but they did not consider the influence of model mismatch in transient state when system stability can not be guaranteed. In this paper we combine pole assignment[6] with delay compensation, develop a new control method which successfully solves system instability problem under model mismatch.

2. A Robust Adaptive Pole Placement Prediction Controller

The system to be considered is a SISO one described by Fig. 1 below with expression form

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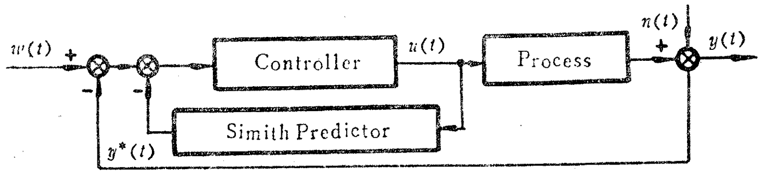

Fig.1 Block diagram of controlled system

$$Ay(t) = Bu(t-k_{\min}) + Ce(t) \tag{1}$$

and control law

$$Fu(t) = G(w(t) - y(t) - y^*(t)) \tag{2}$$

among them, y^* satisfses

$$Ay^*(t) = B(1-q^{-K_{\min}})u(t) \tag{3}$$

where $w(t)$, $y(t)$ and $u(t)$ are reference input, system output and control input, $y^*(t)$ is predictor output, $n(t)$ is disturbance noies and $k_{\min}$ is minimun expected process delay, $e(t)$ is white noise. A, B, C, F and G are polynomials of q^{-1} with orders n_a, n_b, n_c, n_f and n_g, and among them $a_0 = c_0 = 1$.

Combining equations (1), (2) and (3), we can obtain the following relationship between system input and output:

$$y(t) = \frac{q^{-k_{\min}}\hat{A}BG}{A(\hat{A}F + \hat{B}G) + q^{-k_{\min}}G(\hat{A}B - A\hat{B})}w(t) \tag{4}$$

Provided the expected closed loop characteristic polynomial is

$$V(q^{-1}) = v_0 + v_1 q^{-1} + \cdots + v_{n_v} q^{-n_v} \tag{5}$$

If models are completely matched, or $\hat{A} = A$, $\hat{B} = B$, we assign poles to equation(4)

$$AF + BG = V \tag{6}$$

We can see that F and G can be designed based on underlayed process. In this way, system control sbility will be greatly enhanced, and the larger $k_{\min}$ is, the more obvious this advantage will be.

However, plant parameters are often time-varying in practical process, which directly leads to $\hat{A} \neq A$ and $\hat{B} \neq B$. Thus model mismatch always exists and system stability can not be ensured. From equation (4) the system closed loop characteristic equation can be written as

$$\hat{A}(AF + BG) + GP = \hat{A}V + GP = 0 \tag{7}$$

where

$$P = -\hat{A}B + A\hat{B} + q^{-k_{min}}(\hat{A}B - A\hat{B}) = (1 - q^{-k_{min}})(A\hat{B} - \hat{A}B)$$

$$= p_0 + p_1 q^{-1} + \cdots + p_{n_p} q^{-n_p} \tag{8}$$

$$n_p = \deg(P) \leqslant n_a + n_b + k_{min}$$

We can see that, if models are well matched, then, $P \to 0$ and the closed loop poles are near to the expected values. But at dynamic situation when process parameters are varying and the estimates have not yet converged to their true values, if the model mismatch happened at this time is serious enough, the variation of P may make the roots outside the unit circle on the complex z-plane. That is to say, the system loses its stability.

In order to overcome this difficulty, we first introduce a theorem on system stability.

Theorem: If the system is open loop stable, when using pole placement with (6), as long as the assigned characteristic polynomial $V(q^{-1})$ is variable, we can certainly adjust the parameters $v_i(i=1,2,\cdots,n_v)$ on-line to guarantee the system stabity under model mismatch.

Proof From equation (8), P is only concerned with plant parameters, therefore the coefficients of polynomial P are bounded, i. e.

$$|p_i| \leqslant M, \quad M = \text{constant} \tag{9}$$

$$i = 1, 2, \cdots, n_p$$

It is clear that, we can adjust controller parameters $F(q^{-1})$and $G(q^{-1})$ to assure that equation (7) does not involve the roots outside unit circle. In particular we rewrite equation(7) as

$$\hat{A}(1 + r_1 q + \cdots + r_{n_v} q^{-n_v}) + GP/v_0 = 0 \tag{10}$$

where $r_i = v_i/v_0 (i=1,2,\cdots,n_v)$. Polynomial G is given, v_0 is selected to be large enough and $v_i(i=1,2,\cdots,n_v)$ are chosen in such a way that polynomial $(1 + r_1 q^{-1} + \cdots + r_{n_v} q^{-n_v})$ is a stable one, which can be realized by changing $F(q^{-1})$ only. Since the system is open loop stable, combining with (9), equation (10) or equivalently (7) will not involve the unstable roots, that is to say, the system is stable.

According to this theorem, the key is how to supervise in real-time the degree of model mismatch and then correspondingly adjust parameters $v_i(i=1,2,\cdots,n_v)$. Provided the expected placement polynomial is $V_e(q^{-1})$, we have the following adaptive law on $V(q^{-1})$:

$$V(q^{-1}) = f(V_e(q^{-1}),\ \varepsilon(t)) \tag{11}$$

where $\varepsilon(t)$ is the prediction error of ERLS estimation

$$\varepsilon(t) = y(t) - \phi^T(t)\theta(t-1) \tag{12}$$

and vectors

$$\theta = (a_1, \cdots, a_{n_a}, b_0, \cdots, b_{n_b}, c_1, \cdots, c_{n_c})^T$$

$$\phi(t) = (-y(t-1), \cdots, -y(t-n_a), u(t-k_{\min}), \cdots, u(t-k_{\min}-n_b),$$
$$\varepsilon(t-1), \cdots, \varepsilon(t-n_c))^T$$

Obviously, $V(q^{-1})$ is a function of $V_e(q^{-1})$ and $\varepsilon(t)$, $\varepsilon(t)$ is used as a judgement factor from which we know the model mismatch range. The form of (11) is not unique, but some conditions have to be satisfied:

(a) When parameter estimates have converged or prediction error $\varepsilon(t) \to 0$, $V(q^{-1}) \to V_e(q^{-1})$.

(b) When model mismatch appers, which makes the system tend to be unstable, (11) will automatically adjust $V(q^{-1})$to maintain the system stability properly.

In summary, we give the adaptive algorithm below:

Step 1 Estimate parameters of polynomials A, B and C of model (1), where $n_g \geqslant n_b$ and $n_f \geqslant n_a - k$ are required as idenfication condition

Step 2 Adjust parameters of the assigned polynomial $V(q^{-1})$ with adaptive law (11) according to the variation of $\varepsilon(t)$. Smith predictor is simultaneously tuned.

Step 3 Calculate $F(q^{-1})$ and $G(q^{-1})$ from equation (6).

Step 4 Calculate control signal $u(t)$ from equation (2).

3. Simulation Results

Recently, various different pole/zero placement approaches were presented[7-10], but our emphasis is not in that respect. Here we consider a pole placement method for open loop stable system, which simplifies the closed loop transfer function by cancelling stable $A(q^{-1})$. From (6) we take

$$V(q^{-1}) = (v_0 + v_1 q^{-1})A(q^{-1})$$
$$G(q^{-1}) = g_0 A(q^{-1})$$
$$F(q^{-1}) = (v_0 + v_1 q^{-1}) - g_0 B(q^{-1})$$

Substituting them into (4) where we have assume $\hat{A} \neq A$ and $\hat{B} \neq B$, we can get

$$y(t) = g_0 \frac{q^{-k_{min}} B(q^{-1})}{v_0 + v_1 q^{-1}} w(t)$$

where $g_0 = (v_0 + v_1)/B(1)$ gives system zero steady offset.

In simulation example, we choose the adaptive law (11) to be the following form according to conditions (a) and (b):

$$v_0 = e^{R|\varepsilon(t)|} \qquad v_1 = -0.5v_0$$

where R is a constant to be determined in experiment.

It can be seen that, when $\varepsilon(t) \to 0$, $v_0 = 1$ and system has a root $q = 0.5$. If $|\varepsilon(t)|$ becomes large when a sudden plant parameter or time delay variation happens, which means a tendency of system instability, v_0 will become large since g_0 is invariant at this transient state, and the weight on the root will add to oppose this tendency (see equation (7)). Thus the system will be stable according to the theorem proved.

Example. A first order system is now considered

$$(1 + 0.4q^{-1})y(t) = 0.2u(t-2) + e(t)$$

Process parameter and time delay change occurs at $t = 90$ when the model becomes

$$(1 + 0.4q^{-1})y(t) = 0.5u(t-3) + e(t)$$

In order to compare our algorithm with the fixed pole placement method in which the assigned polynomial $V(q^{-1})$ is set equal to the expected one $V_e(q^{-1}) = 1 - 0.5q^{-1}$ without change in overall runing process, we simultaneously list the results of two algorithms under the same simulation condition shown in Fig. 2, Fig. 3 and Fig. 4. From these experiment results we can see that our controller has a

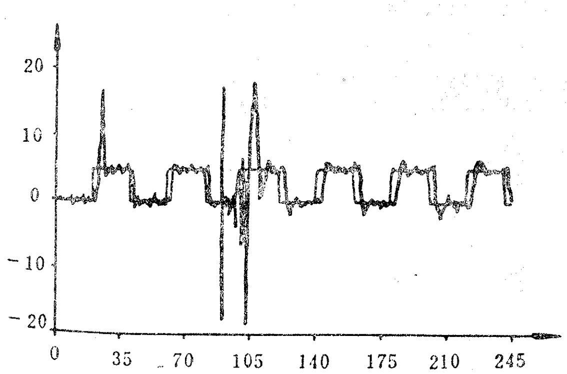

Fig. 2 Fixed pole placement controller

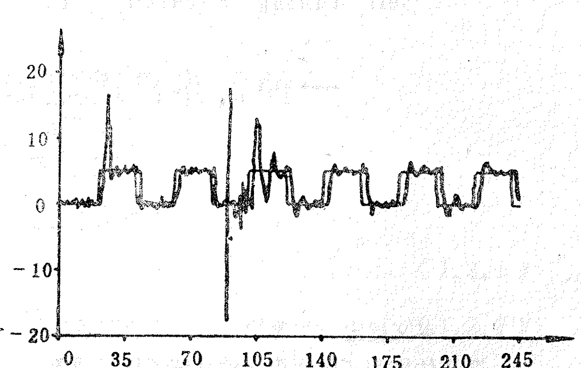

Fig. 3 Our controller

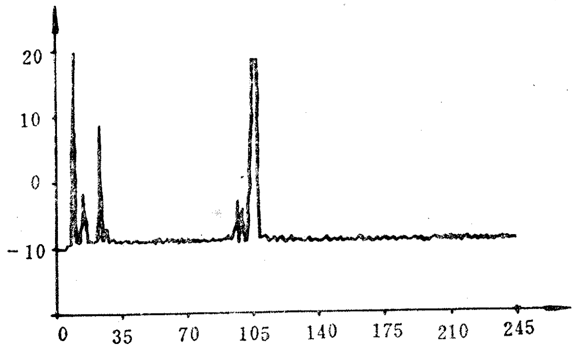

Fig. 4 Parameter v_6 of our algorithm

better robustness.

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一种鲁棒自适应极点配置预估控制器

陈怡轩 吴广玉

（哈尔滨工业大学控制工程系）

摘 要

本文提出一种鲁棒自适应极点配置预估控制新方法．它有效地克服了因对象模型与史密斯预估模型不同而引起的系统不稳定现象．

关键词： 鲁棒性；极点配置；自适应控制．