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Servo-regulation Combined Optimum Adaptive Control

Pang Quan

(Department of 1st Mechanical Engineering, Luoyang Institute of Technology)

He Yue, Chen Kangning

(Department of Mechanical Engineering, Xian Jiaotong University)

Abstract

A minimum-variance and dynamic-optimum combined adaptive control is proposed, to accomplish the optimum adaptive control of both the stochastic-regulating and the servo-tracking simultaneously for the stochastic servo systems. The whole control algorithm with complete recursive process demands a rather modified computational effort, and discussion on its convergence shows its fairly good robustness.

Key words——Self-tuning regulators; Stochastic control theory; Adaptive control; Optimization.

1. Introduction

In dealing with the adaptive control of stochastic servo systems, we met with the stochastic-regulating and the servo-tracking problems together. Usually it's difficult to get to the optimum performances for both simultaneously, and for systems particularly with larger parameter variations no effective methods seem to be available yet.

Reference[2] proposes a separating design, which can obtain an optimum regulation but not the servo. The latter which is dominated by fixed pole-assignment usually shares no optimum performance, especially for time-varying systems when unreasonable control signal may be synthesized because of the discrepancy between the fixed pole-assignment and the varied systems[2]. Besides, the design

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requires an overall parameter estimation, which seems to be a heavy burden for fast moving systems.

In this paper, a combined adaptive control algorithm for both the minimum-variance stochastic regulation and the optimum servo-tracking control is proposed. The algorithm with only partial parameter estimate and complete recursive process demands a rather modified computation and is then more suitable to the fast moving stochastic servo systems.

2. Control Algorithm

Notations of differential operator and its coefficient vector are defined as follows

$$A = a_0 + a_1 q^{-1} + \cdots + a_{n_a} q^{-n_a}, \quad a_0 \neq 0$$

$$A' = a_1 q^{-1} + a_2 q^{-2} + \cdots + a_{n_a} q^{-n_a}$$

$$\overline{A} = [\, a_0 \; a_1 \; \cdots a_{n_a} \,]^T$$

where q^{-1} is the backward shift operator, n_a the degree of A and A is said to be monic when $a_0 = 1$.

(1) Basic control law

The stochastic process, that be controlled, is described as

$$Ay(t) = q^{-d}Bu(t) + Ce(t) \tag{1}$$

where $y(t)$ is the output signal, $u(t)$ the control signal and $e(t)$ the white noise with zero mean and unit variance. A is monic and the time delay $d \geq 1$.

A compound control is employed as shown in Fig. 1, where $u_r(t)$

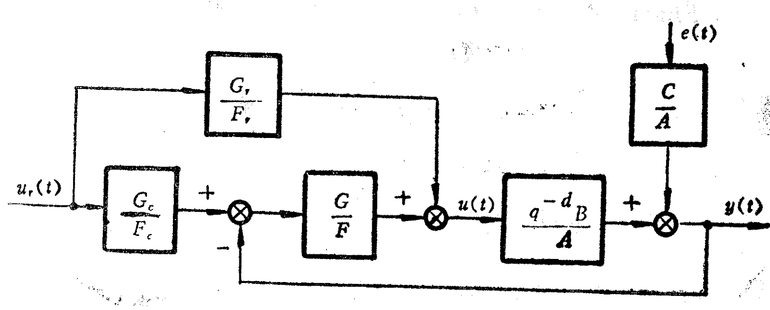

Fig. 1 Basic control structure

is the reference input signal and F_r, F_c and F are monic. Let

$$H = F_r G_c G + G_r F_c F \tag{2}$$

$$P = F_r F_c$$

the control and the closed-loop systems' output can be obtained as

$$u(t)=\frac{H}{FP}u_r(t)-\frac{G}{F}y(t) \tag{3}$$

$$y(t)=\frac{q^{-d}BH}{P(AF+q^{-d}BG)}u_r(t)+\frac{CF}{AF+q^{-d}BG}e(t) \tag{4}$$

In order to get the minimum variance control of the second item on the right side of eq. (4), the control qarameters can be decided as follows[1,8]

$$AF+q^{-d}BG=C \tag{5}$$

$$H=\frac{P(1)}{B(1)}C \tag{6}$$

where $P(1)$ and $B(1)$ are the sums of the coefficients of T and B respectively, and their combination in eq. (6) insures a unit transmission in systems' steady-state response. The condition to get unique solution from eq. (5) is $n_f=n_b+d-1$ and $n_g=n_a-1$.

Substituting (5) and (6) into (4), we have

$$y(t)=\frac{P(1)B}{B(1)P}u_r(t-d)+Fe(t) \tag{7}$$

where P dominates the servo-tracking property of systems and will be discussed later.

(2) Recursive formulation

Multiplying both sides of (5) by $y(t)$ and substituting (1) into it yields

$$FBu(t-d)+GBy(t-d)+FCe(t)=Cy(t) \tag{8}$$

Let

$$D=FC \tag{9}$$

$$\left.\begin{aligned}\tilde{u}(t-d)&=Bu(t-d)\\ \tilde{y}(t-d)&=By(t-d)\\ \tilde{v}(t-d)&=Fu(t-d)+Gy(t-d)\end{aligned}\right\} \tag{10}$$

and notice that F, C and D are monic, eq. (8) can be further deduced as

$$2y(t)-\tilde{u}(t-d)-2D'e(t)=F'\tilde{u}(t-d)+G\tilde{y}(t-d)+B\tilde{v}(t-d)-2C'y(t)+2e(t) \tag{11}$$

By substituting predicting error for white noise, i. e.

$$\hat{e}(t) = \frac{1}{2}\{ [2y(t) - \tilde{u}(t-d) - 2\hat{D}'\hat{e}(t)] - \varphi^T(t)\hat{\theta} \}$$

where

$$\left.\begin{aligned} \theta &= [f_1 \cdots f_{n_f} g_0 \cdots g_{n_g} b_0 \cdots b_{n_b} c_1 \cdots c_{n_c}]^T \\ \varphi(t) &= [\tilde{u}(t-d-1) \cdots \tilde{u}(t-d-n_f)\tilde{y}(t-d) \cdots \tilde{y}(t-d-n_g) \\ &\quad \tilde{v}(t-d) \cdots \tilde{v}(t-d-n_b) - 2y(t-1) \cdots - 2y(t-n_c)]^T \end{aligned}\right\} \quad (12)$$

the Extended Matrix Method can then be used to estimate the parameter $\hat{\theta}$ from eq. (11) directly[4]. Many applications have reported its good convergence. As the estimation of A has been omitted and the control parameters are predicted with process parameters directly, the above algorithm presents a rather modified calculation.

(3) Optimization of servo

From eq. (7) we see that the servo characteristic of the closed-loop systems is dominated by P, or say, that $y(t)$ is the function of P and should be represented as $y(t, \overline{P})$.

Consider the criterion function

$$J(\overline{P}) = \frac{1}{2T} \sum_{t=k-T}^{k} [y(t, \overline{P}) - u_r(t-d)]^2 \quad (13)$$

where T is a constant, approximating to the settle-time of closed-loop systems. Let

$$W = \frac{d}{d\overline{P}}\left\{ \frac{1}{2T} \sum_{t=k-T}^{k} [y(t, \overline{P}) - u_r(t-d)]^2 \right\} \quad (14)$$

Then, we can optimize $\overline{P}$ with the criterion function being minimized as follows[5]

$$\overline{P}_{n+1} = \overline{P}_n - I_n L_n W_n \quad (15)$$

where I_n is the step length and L_n the modification matrix, which can be determined by the DFP approach.

Taking derivative with respect of p_i $(i=1 \sim n_p)$ on both sides of (7), we have

$$\frac{dy(t, \overline{P})}{dp_i} = \frac{1}{P}[-y(t-i, \overline{P}) + \frac{B}{B(1)}u_r(t-d) + F_e(t-i)] \quad (16)$$

Substituting (16) into (14) and noticing that $Fe(t-i)$ is uncorrelated with $y(t, \overline{P})$ and $u_r(t-d)$ so that from the ergodic property of stationary stochastic process and under an enough large T

$$\frac{1}{T}\sum_{t=k-T}^{k}[y(t, \overline{P})-u_r(t-d)]Fe(t-i)=0$$

thus, we have

$$W=\frac{1}{TP}\sum_{t=k-T}^{k}[y(t, \overline{P})-u_r(t-d)]\left\{\begin{pmatrix}-y(t-1, \overline{P})\\ -y(t-2, \overline{P})\\ \vdots\\ -y(t-n_p, \overline{P})\end{pmatrix}\right.$$

$$\left.+\frac{B}{B(1)}u_r(t-d)\begin{pmatrix}1\\1\\ \vdots\\1\end{pmatrix}\right\} \tag{17}$$

(4) Combined optimum adaptive control algorithm (COACA)

Given. Predetermined servo characteristic $\overline{P}_0$ and initial parameter $\theta(0)$.

Step 1 Calculate $\tilde{u}(t-d)$, $\tilde{y}(t-d)$ and $\tilde{v}(t-d)$ from (10) by using $\hat{\theta}(t-1)$ obtained in last step, and then estimate $\hat{\theta}(t)$ from (11) by Extended Matrix Method.

Step 2 Calculate $\hat{H}$ from (6) and then $u(t)$ from (3), by using $\hat{\theta}(t)$ obtained in last step.

Step 3 If $t<k$, return to step 1;

If $t\geqslant k$, calculate W_n from (17) and then $\overline{P}_{n+1}$ from (15); substitute $\overline{P}_{n+1}$ for $\overline{P}_n$, let $k=k+T$ and $n=n+1$, and return to step 1.

3. Convergence Consideration

Only a brief discussion on convergence is given here since the paper space limitation. For more detail, see reference [6].

During the initial stage of systems' performance or, particularly, in time-varying systems when the parameter estimates have not converged at its real value, i. e., $\hat{B}\neq B$, eq. (17) becomes

$$\hat{W}=\frac{1}{TP}\sum_{t=k-T}^{k}[y(t,\ \overline{P})-u_r(t-d)]\left\{\begin{pmatrix}-y(t-1,\ \overline{P})\\-y(t-2,\ \overline{P})\\\vdots\\-y(t-n_p,\ \overline{P})\end{pmatrix}\right.$$

$$\left.+\frac{\hat{B}}{\hat{B}(1)}u_r(t-d)\begin{pmatrix}1\\1\\\vdots\\1\end{pmatrix}\right\}$$

Though $\hat{W}$ is diverted from W due to $\hat{B}$, it can be seen from the Fig. 2 obviously that the algorithm will still converge as long as the deviation between $\hat{W}$ and W is less than 90 degree. Only the descent is not the speediest as shown in Fig. 2 by line ab′ c′, comparing with the dotted line ab which represents the speediest descent. However most experiments have shown the satisfactory convergence even under a larger deviation between $\hat{B}$ and B during systems′ initial stage of performance.

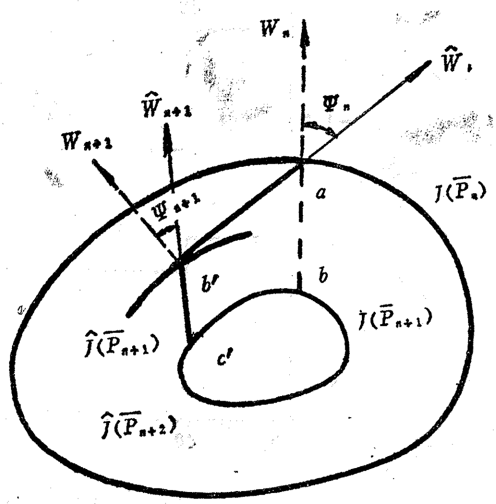

Fig. 2 Convergence analysis

4. Simulation

A non-minimum phase process is chosen for experiment as follows

$$(1-0.78q^{-1}+0.28q^{-2})y(t)=(0.08q^{-1}+0.21q^{-2})u(t)+(1-0.2q^{-1})e(t)$$

By using square wave as reference input signal and setting $T=40$, which is of half period of the signal, and the predetermined servo property $\overline{P}_0=[-0.9\ \ 0.2]^T$, the system′s dynamic response, the optimizing process of $\overline{P}$ and the descent of criterion under the combined optimum adaptive control algorithm are shown in Fig. 3. Where, after about 200 sampling steps, the system achieves both the minimum-variance stochastic regulationand the optimum servo-

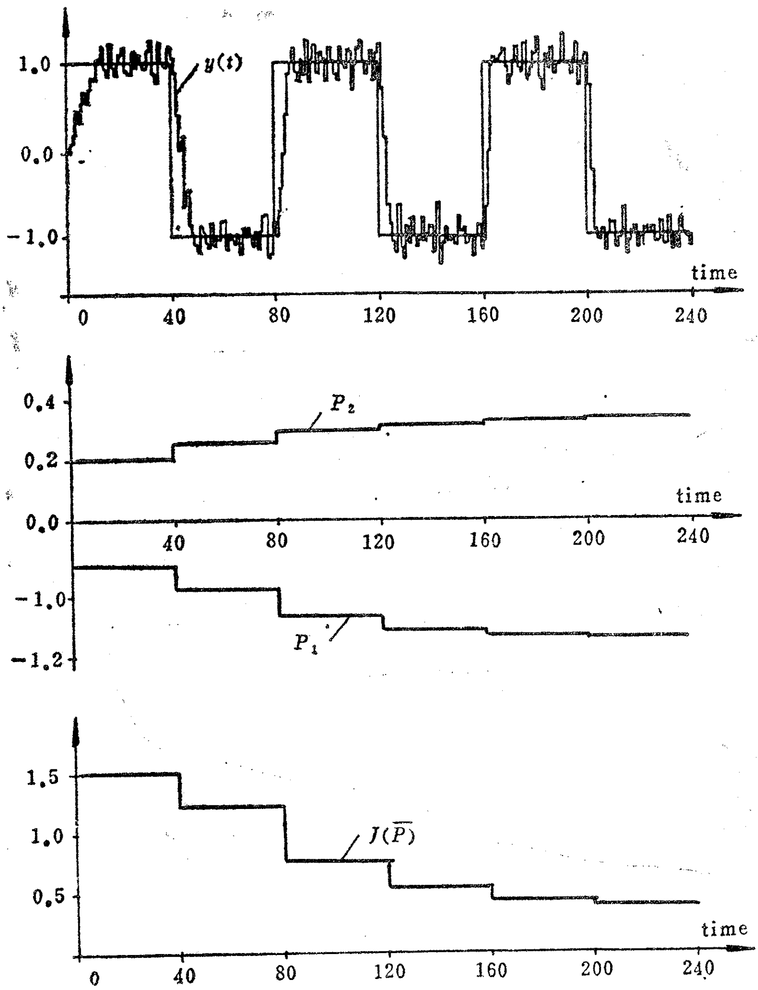

Fig. 3 Simulation result under COACA

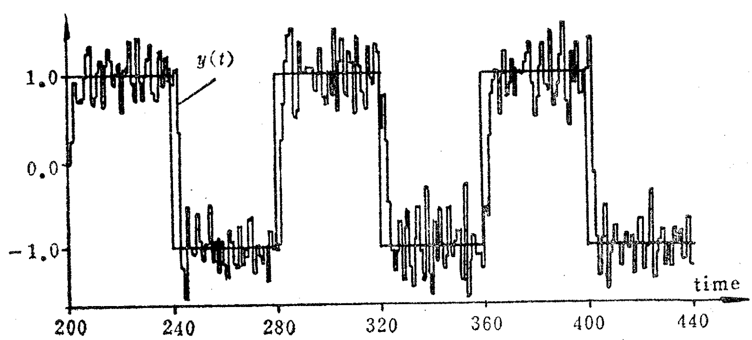

Fig. 4 Simulation result under EST

tracking control. The servo characteristic after convergence is

$\overline{P_{\infty}} = [-1.14 \quad 0.33]^T$.

Fig.4 shows the response of the system under EST control[3], where the regulating variance is clearly larger than that in Fig. 3.

5. Conclusion

The approach developed in this paper achieves both the minimum-variance stochastic regulation and the optimum servo-tracking control simultaneously for the stochastic servo systems. The approach presents a modified computation and fairly good robustness, which is then more suitable for the time-varying and fast moving stochastic servo systems.

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伺服-调节组合最优自适应控制

庞 全 何 钺 陈康宁

（洛阳工学院机械一系）（西安交通大学机械系）

摘 要

建立起最小方差和动态最优的组合自适应控制，对随机伺服系统同时实现随机调节和伺服跟踪的最优控制，整个控制算法呈递推形式，计算量较小。算法的收敛性讨论说明其具有良好的鲁棒性。

关键词：自校正调节器；随机控制理论；自适应控制；最优化。