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A Note on the Identification of Distributed Parameter Systems

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Abstract

In this short paper we show that, in the identification of some varying coefficients of diffusion equation, the functional derivative formula given by Ref. 1-2 is not correct in L_p space.

1. Problem Formulation

We consider a system governed by

$$\left.\begin{array}{ll} \mathrm{div}(K(x)\mathrm{grad}Q)=S(x)Q'_t+U(x,t), & x\in G,\ t\in(0,T) \\ (K(x)\mathrm{grad}Q,N)=C(x)(Q_b-Q), & x\in BG, t\in(0,T) \\ Q(x,0)=g(x), & x\in G, \end{array}\right\} \quad (1.1)$$

where G is a regular bounded open domain with a piecewise smooth boundary in R^n, BG is the boundary of G, N is the unit normal to BG outward from G, (,) is the symbol of inner product in R^n.

In practice, K means conductivity (in heat transfer systems), transmissivity (in ground water systems) or diffusivity (in diffusion systems). In many cases, K only depends on mediums, i. e. it is only a function of x and obviously it may be discontinuous. When $K(x)$ is discontinuous the projection of vector KgradQ on any normal to the discontinuous surface must be continuous. This requirement comes from physics and the mathematical definition of this generalized solution shown in Ref. 2. We denote the class involving all piecewise continuous functions with finite bounded smooth pieces by C_p. The practical problem is how to choose $K(x)\in C_p$ such that

$$J=\frac{1}{2}\int_{T_1}dt\int_{G_1}(Q(x,t)-Q^*(x,t))^2dx=\min. \quad (1.2)$$

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where T_1 and G_1 are some subsets of (0, T) and G respectively, and Q^* is a given function which means some observation values.

Prof. J. H. Seinfeld and W. H. Chen showed, in Ref. 1, that the Frechet derivative operator of J with respect to $K(x) \in L_2$ can be expressed as

$$\delta J = \int_{T_1} dt \int_{G_1} \Delta K(\text{grad}P, \text{grad}Q)dx, \tag{1.3}$$

where $P(x,t)$ is governed by

$$\left.\begin{array}{ll} \text{div}(K(x)\text{grad}P) + S(x)P'_t = Q - Q^*, & x \in G,\ t \in (0,T) \\ (K(x)\text{grad}P, N) + C(x)P = 0, & x \in BG, t \in (0,T) \\ P(x,T) = 0, & x \in G. \end{array}\right\} \tag{1.4}$$

In Ref.1 they assume $G_1 = G$ and $T_1 = (0,T)$, but the observation values are hardly known on the whole $(0,T)$ and G in practice. We will show now that the (1.3) and (1.4) are wrong as $K(x)$ is in L_p.

2. The Difference of Variational Formulas

First of all, as $K(x)$ is a smooth function (1.1) has a classical solution $Q(x,t)$. If we take the norm of C^1 space, it is easy to get the above variational formla. Let $P(x,t)$ be some undetermined function. For any $K(x) \in C^1$ and its associated state $Q(x,t)$, we always have

$$J = \frac{1}{2} \int_0^T dt \int_G ((Q - Q^*)^2 + P(\text{div}(K\text{grad}\,Q) - SQ'_t - U))dx. \tag{2.1}$$

Now let $K + \Delta K \in C^1$ and $Q + \Delta Q$ be its associated state, then

$$\Delta J = \int_0^T dt \int_G (\Delta Q(Q - Q^*) + \frac{1}{2}\Delta Q^2 + P(\text{div}(\Delta K\text{grad}Q + K\text{grad}\,\Delta Q$$
$$+ \Delta K\text{grad}\,\Delta Q) - S\Delta Q'_t))dx$$

Using the known formula for any vector A and scalar F, that

$$\int_G (A, \text{grad}F)dx = \int_{BG} (FA, N)dx - \int_G F\text{div}A dx \tag{2.2}$$

and the boundary condition and initial condition of ΔQ, that

$$\left.\begin{array}{ll} (K\text{grad}\,\Delta Q + \Delta K\text{grad}Q + \Delta K\text{grad}\,\Delta Q, N) + C\Delta Q = 0, & x \in BG,\ t \in (0,T) \\ \Delta Q(x,0) = 0, & x \in G. \end{array}\right\} \tag{2.3}$$

we have

$$\Delta J = -\int_0^T dt\int_G (\Delta K(\text{grad}P\cdot\text{grad}Q) + \Delta Q(\text{div}(K\text{grad}P) + P'_t + Q - Q^*)$$

$$-\frac{1}{2}\Delta Q^2 + (\Delta K\text{grad}\,\Delta Q, \text{grad}P))dx - \int_0^T dt\int_{BG}\Delta Q(CP + K(\text{grad}P, N))dx$$

$$+\int_G (P\Delta Q)_{t=T}dx.$$

Let $P(x, t)$ satisfy (1.4), we have

$$\Delta J = \int_0^T dt\int_G \Delta K(\text{grad}P, \text{grad}Q)dx - \int_0^T dt\int_G ((\Delta K\text{grad}\,\Delta Q, \text{grad}P)$$

$$+\frac{1}{2}\Delta Q^2)dx. \tag{2.4}$$

Because Q is classical, as $\|\Delta K\|_{C^1}\to 0$, it is not hard to show

$$\int_0^T dt\int_G (\Delta K\text{grad}\,\Delta Q, \text{grad}P)dx = 0(\|\Delta K\|_{C^1}), \tag{2.5}$$

$$\int_0^T dt\int_G \Delta Q^2 dx = 0(\|\Delta K\|_{C^1}). \tag{2.6}$$

Then from (2.4) it implies that the linear operator of ΔK

$$\delta J = \int_0^T dt\int_G \Delta K(\text{grad}P, \text{grad}Q)dx, \tag{2.7}$$

is a Frechet derivative operator of J mapping C^1 into R^1. This result can be generalized into the case of $K\in C$ space. As $K(x)$ is discontinuous, according to the definition of generalized solution all the above calculus holds. but if we take the norm of L_2 space the left hand side of (2.5) can be the same order quantity as (2.7) when $\|\Delta K\|_{L_2}\to 0$. For example, consider a steady system with one dimension,

$$\left.\begin{aligned} &\frac{d}{dx}\left(K\frac{dQ}{dx}\right) = 0, \quad x\in(0,2),\\ &Q(0) = 1, \qquad \left.\frac{dQ}{dx}\right|_{x=2} = 1. \end{aligned}\right\} \tag{2.8}$$

Let $K(x) = 1$, $x\in(0,2)$, then $Q(x) = x+1$ is the associated state. we make a variation as

$$K + \Delta K = \begin{cases} 1, & x\in(0, 1-e),\\ 1+r, & x\in(1-e, 1),\\ 1, & x\in(1, 2),\end{cases}$$

then

$$Q + \Delta Q = \begin{cases} x+1, & x\in(0, 1-e)\\ (1+r)^{-1}(x-1+e)+2-e, & x\in(1-e, 1)\\ x+1-(1+r)^{-1}re, & x\in(1, 2)\end{cases}$$

is the unique solution in space H^1, where $r>0$ is a constant.

Let the functional is given as

$$J=\int_0^2 (Q-1)^2 dx$$

then $P(x)$ satisfies

$$\frac{d}{dx}\left(\frac{dP}{dx}\right)=Q(x)-1=x \qquad i.\ e.\ P(x)=\frac{x^3}{6}-2x.$$

$$P(0)=0, \qquad \left.\frac{dP}{dx}\right|_{x=2}=0$$

We have

$$\int_0^2\left(\Delta K\frac{dP}{dx}\cdot\frac{dQ}{dx}\right)dx=\int_{1-e}^{1} r\left(\frac{x^2}{2}-2\right)dx=re\frac{e^2-3e-9}{6}, \qquad (2.9)$$

$$\int_0^2\left(\Delta K\frac{dP}{dx}\,\frac{d\Delta Q}{dx}\right)dx=\int_{1-e}^{1} r\left(\frac{x^2}{2}-2\right)\left(\frac{1}{1+r}-1\right)dx=-\frac{r^2e}{1+r}\frac{e^2-3e-9}{6}. \qquad (2.10)$$

Since r is constant, as $e\to 0$

$$\|\Delta K\|_{L_p}=\left(\int_{1-e}^{1} r^p dx\right)^{\frac{1}{p}}=re^{\frac{1}{p}}\to 0.$$

But (2.9) and (2.10) are the same order quantities as $e\to 0$. So we can not regard (2.7) as the Frechet derivative operator of J mapping L_2 into R^1. This example also shows that in L_p space linear terms of variation should not be rashly regarded as first variation. The variation formula suitable to this example is given by principle of pulse variation (Ref. 3).

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分布参数系统辨识中的一个注记

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摘 要

在研究变系数参数的辨识问题时，J. H. Seinfeld 与 W. H. Chen 于1974年提出了一个计算泛函变分的公式，并称此公式在L_2空间也是正确的[1]。本文给出一个反例，说明此变分公式仅在C或L_∞空间中正确，而在L_p空间是错误的。尽管C与L_∞在L_p中是稠密的。这个反例的意义在于提醒人们在泛函空间推导变分公式时不应轻意地将高次项当成范数的高阶项而舍去。