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Real Decoupling Technique for Reducing the Order of Large Scale System Models

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Abstract

A theorem about the reducible dimension of large scale system models is presented. According to this theory We have developed a new algorithm to simplify large scale linear systems.

Since Davison proposed the method for reducing large scale systems by constructing an aggregated system which retains the dominant eigenvalues of the original system,[1] many aggregation methods have been developed.[2] Among these existing methods, some of these are too difficult to apply to the practical systems because of the complication in calculation, and the others can't get the perfect aggregation. Especially for the systems hev ng complex eigenvalues, many methods have a lot of difficulties in calculation of complex modal matrix.

Following Aoki's condition for perfect aggregation,[3] we have developed a new aggregation algorithm. We begin with considering what dimension reduced model is able to be obtained from a high order original system model by an easy computational method.

Aggregation of a linear time-invariant system described by Aoki is as follows.[3][4]

Let original system S_1 be

$$\dot{X} = AX + BU,\ X(0) = X_0,\ Y = HX$$

Where, $X \in R^n$, $Y \in R^r$, $U \in R^m$, $A \in R^{n\times n}$, $B \in R^{n\times m}$, $H \in R^{r\times n}$.

And let reduced system S_2 be

$$\dot{Z} = FZ + GU,\ Z(0) = Z_0,\ \widetilde{Y} = \widetilde{H}Z,$$

where, $Z \in R^l$, $\widetilde{Y} \in R^r$, $F \in R^{l\times l}$, $G \in R^{l\times m}$, $\widetilde{H} \in R^{r\times l}$, and $l < n$. If

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$$Z=CX,\ Z_O=CX_O,$$

where, C is a $l\times n$ constant aggregation matrix, and following conditions are satisfied

$$FC=CA, \qquad (1)$$

$$G=CB, \qquad (2)$$

$$\widetilde{H}C=H. \qquad (3)$$

then, S_2 is called a perfect aggregation of S_1.

The state matrix F of reduced system can be calculated by the formula

$$F=CAC^{+}=CAC^{T}(CC^{T})^{-1}, \qquad (4)$$

where, $C^{+}=C^{T}(CC^{T})^{-1}$ is a Penrose pseudo inverse of C.

But in general case, because of $C^{+}C\neq I$, the F calculated by (4) can't satisfy the condition (1). This problem has caused many discussions.

There are $l\times n$ equation in (1), but F only has $l\times l$ variables, $l<n$. Therefore, in order to avoid the contradictory equations in(1), the aggregation matrix C must meet some conditions.

Rewrite C and A in partitioned form:

$$C=[\,C_1 : \widetilde{C}_1\,],\quad A=\begin{bmatrix} A_l & A_{12} \\ A_{21} & A_{n-l}\end{bmatrix},$$

where, $C_1\in R^{l\times l}$, $A_l\in R^{l\times l}$, and C_1 is nonsingular.

From (1) we can get the condition in another form which C should satisfy

$$(C_1A_1+\widetilde{C}_1A_{21})C_1^{-1}\widetilde{C}_1=C_1A_{12}+\widetilde{C}_1A_{n-l}. \qquad (5)$$

The matrix F and G can be determined by

$$F=C_1A_lC_1^{-1}+\widetilde{C}_1A_{21}C_1^{-1} \text{ and } G=CB. \qquad (6)$$

If $\widetilde{C}_1=0$ and $A_{12}=0$, then (5) is true for any A ane C. For the general systems, $A_{12}\neq 0$, then we can construct a nonsingular T, $T\in R^{n\times n}$, which makes

$$TAT^{-1}=\widetilde{A}=\begin{bmatrix} \widetilde{A}_l & 0 \\ \widetilde{A}_{21} & \widetilde{A}_{n-l}\end{bmatrix}. \qquad (7)$$

then, the perfect aggregation can be obtained from aggregating $\widetilde{A}$ by selecting $C=[C_1 : 0]$, and

$$F=C_1\widetilde{A}_lC_1^{-1}, \qquad (8)$$

To simply the treatment, let

$$T=\begin{bmatrix} I_l & D \\ 0 & I_{n-l} \end{bmatrix},$$

where, I_l and I_{n-l} are l and $n-l$ dimensional identity matrices. From (7) we have

$$-A_l D - DA_{21}D + A_{12} + DA_{n-l} = 0. \tag{9}$$

Fq. (9) is a Riccati equation, and its solution D generally is a complex matrix, i. e. $D \in C^{l\times(n-l)}$. From (8) we get

$$F = C_1 \tilde{A}_1 C_1^{-1} = C_1 (A_l + DA_{21}) C_1^{-1} \tag{10}$$

In general, we can get the real F only from the real D, so we begin considering what dimension reduced model can guarantee that D is real.

Definition If there exists a real matrix D, i. e. $D \in R^{l\times(n-l)}$, which satisfies eq. (9), we call that the system S_1 is able to be reduced to an l-dimensional system by real decoupling technique.

The principal theorem about the reducible dimension is mentioned as follows.

Theorem If system S_1 has

$$M^{-1}AM = J = \begin{bmatrix} J_l & 0 \\ 0 & J_{n-l} \end{bmatrix}$$

where, $M = \begin{bmatrix} M_l & M_{12} \\ M_{21} & M_{n-l} \end{bmatrix}$, and $M_{12}M_{n-l}^{-1} \in R^{l\times(n-l)}$ is real, then S_1 is able to be reduced to an l-dimensional system by real decoupling technique.

It must be noted that the modal matrix M is generally complex and the Jordan matrix J can't be parted as above form for any dimension l.

Proof
$$\begin{bmatrix} A_l & A_{12} \\ A_{21} & A_{n-l} \end{bmatrix}\begin{bmatrix} M_l & M_{12} \\ M_{21} & M_{n-l} \end{bmatrix} = \begin{bmatrix} M_l & M_{12} \\ M_{21} & M_{n-l} \end{bmatrix}\begin{bmatrix} J_l & 0 \\ 0 & J_{n-l} \end{bmatrix} \tag{11}$$

Without loss of generality, through properly exchanging the rows and columns of A, a nonsingular M_{n-l} can be got.

Partially extending (11), we have

$$A_l M_{12} + A_{12} M_{n-l} = M_{12} J_{n-l} \tag{12}$$

$$A_{21} M_{12} + A_{n-l} M_{n-l} = M_{n-l} J_{n-l}. \tag{13}$$

Removing J_{n-l} from (12) and (13), we get

$$A_l M_{12} M_{n-l}^{-1} - M_{12} M_{n-l}^{-1} A_{21} M_{12} M_{n-l}^{-1} A_{12} - M_{12} M_{n-l}^{-1} A_{n-l} = 0. \qquad (14)$$

Comparing (14) with (9), we have $D = -M_{12} M_{n-l}^{-1}$.

So there must be a real matrix D which satisfies (9).

Corollary 1. If state matrix A has a simple real eigenvalue, then S_1 can be reduced to a $n-1$ dimensional system by real decoupling.

Proof Let λ be the simple real eigenvalue of A. i. e. $\lambda \in R^1$, then, $AX = \lambda X$, $X \in R^n$.

Let $X = [x_1 \cdots x_{n-1} x_n]^T$, without loss of generality, it can considered that $x_n \neq 0$, so

$$M_{12} M_{n-l}^{-1} = [x_1 x_2 \cdots x_{n-1}]^T / x_n \in R^{n-1}.$$

Corollary 2. If the state matrix A has a pair of conjugate complex eigenvalues, then S_1 can be reduced to a $n-2$ dimensional system by real decoupling.

Proof Let $\lambda, \bar{\lambda} \in C^1$ be a pair of complex eigenvalues, $X, \bar{X} \in C^n$ be the corresponding complex eigenvactors,

$$X = [x_1 \cdots x_{n-2} x_{n-1} x_n]^T \text{ and } \bar{X} = [\bar{x}_1 \cdots \bar{x}_{n-2} \bar{x}_{n-1} \bar{x}_n]^T.$$

Then,

$$M_{12} M_{n-l}^{-1} = \begin{bmatrix} x_1 & \bar{x}_1 \\ \vdots & \vdots \\ x_{n-2} & \bar{x}_{n-2} \end{bmatrix} \cdot \begin{bmatrix} x_{n-1} & \bar{x}_{n-1} \\ x_n & \bar{x}_n \end{bmatrix}^{-1}$$

$$= \frac{1}{\mathrm{Im}(x_{n-1}\bar{x}_n)} \begin{bmatrix} \mathrm{Im}(x_1\bar{x}_n) & \mathrm{Im}(\bar{x}_1 x_{n-1}) \\ \vdots & \vdots \\ \mathrm{Im}(x_{n-2}\bar{x}_n) & \mathrm{Im}(\bar{x}_{n-2} x_{n-1}) \end{bmatrix} \in R^{(n-2)\times 2}$$

where, $\mathrm{Im}(x)$ represents the imaginary part of x.

Clearly, if we take $n-1$ or $n-2$ as the dimension of the reduced system, the real decoupling technique certainly is effective to most practical systems. According to this theory we develop a new algorithm for reducing large scale system models, we call it Nested Aggregation. This algorithm reduces the order of system model step by step by using the real decoupling technipue, until the proper order model is obtained.

We have programed this algorithm and computed a large number of example problems, the satisfactory results have been achived.

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大系统模型降阶的实解耦技术

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摘 要

本文提出了一个关于大系统模型可简化维数的定理，并按此理论发展了一种简化大型线性系统的新算法。