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Servocompensator Design with Robust Stability

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Abstract: A design method for robust controller is developed and applied to a large-scale distillation plant. This method produces a controller which meets closed-loop stability, regulation, tracking and some transcient performance specifications for the approximate model on which the design is based and at the same time possesses the robustness of these performances in the face of the structured modelling uncertainty.

Key words: servocompensator design; robust stability; robust stabilization, multivariable systems; distillation column control

1. Introduction

Classically, allowance for modelling uncertainties has been attempted by ensuring adequate gain and phase margins for the approximate model in the frequency domain. It would clearly be advantageous, however, to be able to assess the effect of modelling errors in a more precise manner and, in particular, to be able to confidently assess both the stability and transient performance of the implemented system in terms of the closed-loop dynamics of the approximate model. This objective is achieved by the robust design which will be presented in this paper. We will concentrate on the situation where the step response of the plant is reliably known and hence the modelling errors are well-known (i.e. structured) and characterised in the time domain in terms of the error between the unit step responses of the plant and its approximate model. A design method is presented such that the designed controller meets stability, regulation, tracking and some transcient performance specifications for the approximate feedback system. At the same time system possesses the robustness of these specifications in case of the structured modelling uncertainty mentioned above.

2. Problem Formulation and Resolution

Let a controlled r-input m-output linear system G be described by the plant step-response matrix $\mathbf{Y}(t)$ estimated reliably from plant tests, where the element $Y_{ij}(t)$ is the response from zero initial conditions of the ith output to a unit step in the jth input. Given the data $\mathbf{Y}(t)$, suppose that an approximate model G_A of the plant is derived from $\mathbf{Y}(t)$with a step-response matrix $\mathbf{Y}_A(t)$and $\mathbf{Y}_A \neq \mathbf{Y}$. Our goal is to design an m-input r-output linear forward-path controller K in a unit feedback configuration such that the feedback system meets the required closed-loop stability, requlation, tracking and dynamic performance specifications for the approximate plant Y_A and at the same time, these specifications are guaranteed for the real plant Y.

Let Φ be the least common multiple of minimal polynomials of references and disturbances. It follows from the robust servomechanism theory (Chen, 1984) that the regulation and tracking with stability is solvable for both G_A and G only if

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(1) $r \geqslant m$, and

(2) None of G_A and G has the same zeros as those of Φ

It is assumed that conditions (1) and (2) hold throughout this paper. Thus, the regulation and tracking with stability is solvable for both G_A and G and, furher, the controller for this purpose must have the following form. $K = K_1 K_2$, where $K_1 = \Phi^{-1} I$, and K_2 simultaneously stabilizes $\Phi^{-1} G_A$ and $\Phi^{-1} G$. In view of these observations, the design problem becomes a robust stabilization problem with respect to the structured modelling uncertainty, $(\mathbf{Y}-\mathbf{Y}_a)$. Our strategy to solve such a problem is first to construct a stabilizing controller K_2 in the parametric form, $K_2 = K_2(Z)$, for $\Phi^{-1} G_A$ with the expected closed–loop poles and free parameters z_{ij}, and then to determine an optimal Z^* in the sense that it minimizes some robust stability measure.

In order to construct a stabilizing controller for $\Phi^{-1} G_A$, we write the strictly proper $\Phi^{-1} G_A$ in a left coprime polynomial matrix fraction $A^{-1}B$ such that A is row reduced with row degrees $\alpha_1 \geqslant \alpha_2 \geqslant \cdots \geqslant \alpha$ and the proper controller K_2 is also expressed in a right coprime polynomial matrix fraction $K_2 = YX^{-1}$with column degrees $\mu_1 \geqslant \mu_2 \geqslant \cdots \geqslant \mu_m$ and $\mu_1 \geqslant \beta - 1$, where β is the controllability index of $\Phi^{-1} G_A$. Then the closed–loop characteristic polynomial of the approximate feedback system is $\det(P)$, where P is given by

$$P = AX + BY. \tag{3}$$

Now, let $P_e = \operatorname{diag}(p_i)$withdegree$(p_i) = \alpha_i + \mu_i$ be a prescribed ploynomial matrix det (P_e) and rave the expected closed – loop poles in the left half of the complex plane. Setting $P = P_e$ and equating terms of the same degree on both sides of (3) yield

$$\bar{S}Z = \bar{Q}, \tag{4}$$

where $\bar{S}$, Z and $\bar{Q}$ consist of coneefficient matrices of (A,B), (X,Y) and P_e, respectively. If zero rows in $\bar{S}$ and $\bar{Q}$ are deleted,(4) is simplified into

$$SZ = Q. \tag{5}$$

It is shown in(Chen,1984) that for arbitrary Q(5) has a solution, which also means that arbitrary pole assignment can be achieved. Furthermore, partitions S and Z as

$$S = [S_1 \quad S_2], \ Z = \begin{bmatrix} Z_1^T & Z_2^T \end{bmatrix}^T, \tag{6}$$

with S_1 being nonsingular, then all solutions can be expressed as the following parameteric form

$$Z_1 = S_1^{-1}(Q - S_2 Z_2), \tag{7}$$

where Z_2 is an arbitrary real matrix.

Concerned with robust stabilization, we define the error matrix function as follows

$$E(t) = Y(t) - Y_A(t) = [E_1(t), \ E_2(t), \ \cdots, \ E_r(t)]. \tag{8}$$

Simulations are undertaken to calculate the matrix $W(t) = [W_1(t)], \cdot W_2(t), \ \cdots, \ W_r(t)$, where W_i is the response from zero initial conditions of the system $(I + KG_A)^{-1}K$ to the input vector E_i Let w_{ij} be the element of W at row i and column j.Since $(I + KG_A)^{-1} K$ is stable, then w_{ij} is bounded and continous on the infinite open interval $0 < t <$

$+\infty$ with its local maxima and minima reached at times $t_1 < t_2 < \cdots$, satisfying $\sup t_j = +\infty$ in the extended half-lime $t > 0$. Define

$$N_T(w_{ij}) = \left|w_{ij}(0^+)\right| + \sum_{k=1}^{k^*}\left|w_{ij}(t_k) - w_{ij}(t_{k-1})\right| + \left|w_{ij}(T) - w_{ij}(t_{k^*})\right|,$$

where k^* is the largest integer k such that $t_k < T$,and

$$N_\infty(w_{ij}) = \sup_{T>0} N_T(w_{ij}), \quad N_\infty(W) = \{N_\infty(w_{ij})\}.$$

Then K_2 will also stabilize $\Phi^{-1}G$(Owens et al, 1984) if

the composite system $(\Phi^{-1}G)(K_2)$ has no pole-zero cancellation, and (9)

$r(N_\infty(W)) < 1$, where $r(\cdot)$ is the spectral radius of a matrix. (10)

Obviously, r depends on the parameter matrix Z_2 in (7), and we then search for a optimal Z_2^* so that it minimizes r

$$\min_{Z_2} r[N_\infty(W)]. \tag{11}$$

Since the relationship between Z_2 and r is very complicated, the gradient of r with respect to Z_2 is not available. We have to use some multidimensional search methods such as Hooke–Jeeves and Rosenbrock ones (Bazaraa and Shetty, 1979) solve the problem (11). If the minima $r^* < 1$ with the optimal solution Z_2^*, then the controller parameters are obtained from(7), and closed-loop system simulations can be carried out to check the effectiveness of the design. Otherwise, If the minima $r^* > 1$, the robust stability is not achieved either because the given approximate model is not accurate enough to provide stability predictions for the real feedback system, or because the controller is too simple to meet the required robustness. It is thus necessary to construct a more accurate model, or to construct a more complex $K_2$with increased column degrees in an attempt to reduce r^*.

3. Application Example

The design method presented in the last section has been applied to the controller design of a large-scale three-compoment distillation plant with cascade two columns. The feed is a mixture of methanol, ethanol and propanol, which flows into the first column. The side stream product is fed into the second column for further seperation. The purpose of the operation is to make the concentrations of two top and a bottom products as high as possible. The control inputs are two relfux ratioes and a vapour flow rate at the bottom of the first column. The plant frequently operates at two different conditions, under which models have been built seperately as follows:

$$G_1(s) = \begin{bmatrix} -2.187/(1.15s+1) & 0 & 1.075/(1.35s+1) \\ 0 & -0.425/(0.86s+1) & 0.8/(1.2533s+1) \\ -6.46/(1.08s+1) & -3.4/(1.3267s+1) & 2.837/(0.95s+1) \end{bmatrix},$$

$$G_2(s)=\begin{bmatrix} -1.85/(0.82667s+1) & 0 & 1.175/(1.45s+1) \\ 0 & -0.344/(0.61667s+1) & 1.15/(1.3333s+1) \\ -4.527/(0.62667s+1) & -2.515/(1.0067s+1) & 1.84/(0.72s+1) \end{bmatrix},$$

where the unit time is 30 minutes. The problem at hand is how to design a robust controller such that the closed-loop system works well at both operating conditions. With the-method mentioned above, G_1 is regarded as the " approximate" model of the plant, or $G_A=G_1$, and G_2 as the "real" plant, or $G=G_2$. Then, the step-response amtrices $Y(t)$ and $Y_A(t)$ can be obtained, respectively, from the simulations of G_2 and G_1, and $E(t)$ is just their difference.Suppose the disturbances and tracking references are step signals. It implies that $\Phi=s$. The required left coprime rfaction of $\Phi^{-1}G_1$ is obtained as follows

$$\Phi^{-1}G_1=A^{-1}B,$$

$$A=\begin{bmatrix} 0 & 0 & s(1.08s+1)(1.3267s+1)(0.95s+1) \\ 0 & s(0.86s+1)(1.2533s+1) & 0 \\ s(1.15s+1)(1.35s+1) & 0 & 0 \end{bmatrix},$$

$$B=\begin{bmatrix} -6.46(1.3267s+1)(0.95s+1) & -3.4(1.08s+1)(0.95s+1) & 2.887(1.08s+1)(1.3267s+1) \\ 0 & -0.425(1.2533s+1) & 0.8(0.86s+1) \\ -2.187(1.35s+1) & 0 & 1.075(1.15s+1) \end{bmatrix}.$$

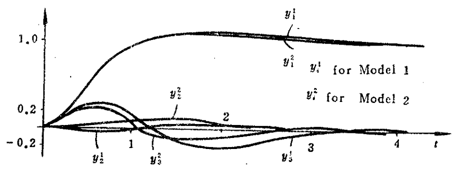
It follows that $\alpha_1=4$, $\alpha_2=\alpha_3=3$. It is easily seen that $\mu=4$. Let the controller $K_2=YX^{-1}$ with X having column degrees $\mu_1=3$, $\mu_2=\mu_3=2$, and $P_e=\mathrm{diag}\{(s+2)^7,(s+2)^5,(s+2)^5\}$ with all the closed-loop poles at -2. Then we solve (11) and obtain the minima $r^*=0.637422<1$, wich shows that the robust stability has been achieved. The resulting controller is

$$K_2=\begin{Bmatrix} -(1.1s^3+4.5s^2+6.0s+2.88)/(s^3+8.38s^2+17s+4.25), \\ (4.74s^2+21.1s+15.9)/(s^2+9.27s+15), \quad -(2.23s^2+10.6s+10.8)/(s^2+8.5s+20) \\ -(4.4s^3+18s^2+24.0s+11.5)/(s^3+8.38s^2+17s+4.25), \\ -(0.536s^2+2.40s+1.81)/(s^2+9.27s+15), \quad +(8.69s^2+41.2s+42.4)/(s^2+8.5s+20) \\ -(2.2s^3+9.0s^2+12.0s+5.76)/(s^3+8.38s^2+17s+4.25), \\ (9.65s^2+43.1s+32.5)/(s^2+9.27s+15), \quad +(5.62s^2+26.6s+27.4)/(s^2+8.5s+20). \end{Bmatrix}$$

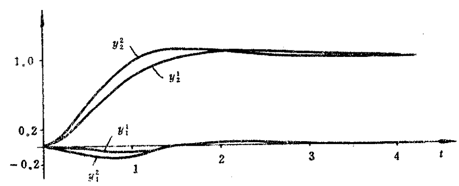
Finally, the composite controller is $K=K_2/s$. Simulation results are shown in Fig.1 and exhibit satisfactory performances.

4. Conclusion

With the assumption that the control designer has a reliable estimate $Y(t)$ of the step response matrix of a linear system G, here is a systematic technique for the design of feedback controllers based on an approximate plant model G_A with plant modeling error $(Y-Y_a)$. The data can be used in a robust stability criteria that produces the guarantee that the controller designed based on the approximate model will also stabilize the real plant despite the known modelling error. Therefore with this technique, it is not necessary for design to use or even to know the real plant model G; i.e., the technique can be used directly on data from a plant step test. It produces the greatest degree of robust stability and significantly reduces the degradation of the real transcient performance from the expected one. The example included indicates the design being highly successful and easily achieved even in the presence of substantial modelling error or at changeable operating conditions.



Response to Reference $[1.0,0.0,0.0]^T$



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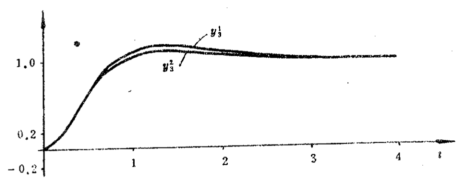

Response to Reference $[0.0,0.0,1.0]^T$

Fig.1 Closed–loop Responses to References

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具有鲁棒稳定性的伺服补偿器设计

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摘要 本文发展一种鲁棒控制器设计方法，并将该法用于一个大型精馏塔的控制．利用该法设计的控制器，能使闭环系统满足稳定性，渐近调节和跟踪，以及某些瞬态性能指标，即使在结构性建模不确定的情况下，这些性能指标仍能满足．

关键词: 伺服补偿器设计；鲁棒稳定性；鲁棒镇定；多变量系统；精馏塔控制

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