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Optimal Control of Bilinear Systems with Time—delay and Its Application to a Distillation Column*

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Abstract: In this paper, by using the block—pulse functions(BPFs) and the method of parameter optimization, a detailed algorithm to solve the optimal control problem of multivariable bilinear systems with arbitrary initial conditions and delays in state and control variables is presented. This algorithm is successfully employed to solve a temperature optimal control problem of a distillation column, whose dynamic behaviour is described by a bilinear model with delay in the control variable. In addition, the simulation of the other example also shows that this algorithm is satisfactory.

Key words: optimal control; bilinear control; time—delay system; block—pulse functions; Distillation column

1. Introduction

The optimal control of linear systems with time—delay has been researched by some authors. However, the dynamic behaviour of many industrial processes is reasonably described by a class of important nonlinear time—delay models—— multivariable bilinear time—delay models. Therefore, the control of bilinear time—delay systems is of great importance to industry. But the optimal control of bilinear time—delay systems is of great importance to industry. But the optimal control of bilinear time—delay systems is very difficult(in fact, it is impossible to obtain the accurate solution of this control problem), since it is necessary to solve the bilinear two—point boundary—value problems including both the time—delay terms and time—advance terms. So far, few have been found concerning this optimal control problem.

In this paper, by using the block—pulse functions(BPFs)[1−2] and the method of parameter optimization, a detailed algorithm to solve the optimal control problem of multivariable bilinear systems with arbitrary initial conditions and delays in state and control variables is presented, which is simple and convenient for applications to industrial processes. This algorithm is successfully employed to solve a temperature optimal control problem of a industrial de—ethane column, whose dynamic behaviour is described by a bilinear model with time—delay in the control variable. In addition, the simulation of the other example also shows that this algorithm is satisfactory.

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2. Operation properties of BPFs

2.1 Definition of BPFs

The block—pulse functions $\varphi_i(t), i=1,\dots,m$, are defined in the interval $[0,T)$ by

$$\varphi_i(t) = \begin{cases} 1, & \text{for } (i-1)T/m \leqslant t < iT/m, \\ 0, & \text{otherwise}, \end{cases} \tag{1}$$

where m is a positive integer and T/m is defined as the step—length of BPFs.

An arbitrary function $f(t)$ which is absolutely integrable in the interval $[0,T)$ can be approximated by BPFs:

$$f(t) \doteq \sum_{i=1}^{m} f_i\varphi_i(t) = F^T\Phi(t) = \Phi^T(t)F, \tag{2}$$

where the superscript T means transpose, and

$F=[f_1 \quad f_2\cdots f_m]^T$, F is defined as the coefficient matrix of BPFs of function $f(t)$.

$$\Phi(t) = [\varphi_1(t)\ \varphi_2(t)\dots\varphi_m(t)]^T,$$

$$f_i(t) = \frac{m}{T}\int_{(i-1)T/m}^{iT/m} f(t)\varphi_i(t)\mathrm{d}t.$$

2.2 Main properties of BPFs

(a) The disjoint property

$$\varphi_i(t)\varphi_j(t) = \begin{cases} \varphi_i(t), & \text{for } i=j, \\ 0, & \text{for } i \neq j. \end{cases} \tag{3}$$

(b) The translation property

Suppose $\tau=(q+\delta)T/m$; q, $1\leqslant q\leqslant m$, is a positive integer; $0\leqslant\delta<1$; and

$$D_q(\delta) = \begin{bmatrix} 0 & I_{m-q}(\delta) \\ 0 & 0 \end{bmatrix}_{m\times m}, \quad I_{m-q}(\delta) = \begin{bmatrix} 1-\delta & \delta & & 0 \\ & \ddots & \ddots & \\ & & \ddots & \delta \\ 0 & & & 1-\delta \end{bmatrix}_{(m-q)\times(m-q)},$$

then the translation property can be described by

$$\Phi(t-\tau) \doteq D_q(\delta)\Phi(t), \text{for } t\in[0,T). \tag{4}$$

(c) The integral property

$$\int_0^t \Phi(t')\mathrm{d}t' \doteq P\Phi(t), \tag{5}$$

where P is integral operation matrix, which is defined as

$$P = \frac{T}{m}\begin{bmatrix} \frac{1}{2} & 1 & \cdots & 1 \\ & \frac{1}{2} & \ddots & \vdots \\ & & \ddots & 1 \\ 0 & & & \frac{1}{2} \end{bmatrix}_{m\times m}.$$

(d) The product property

$$\Phi(t)\Phi^T(t)F = F_D\Phi(t), \tag{6}$$

where $F=[f_1,\cdots,f_m]^T$ and $F_D=\text{diag}(f_1,\cdots,f_m)$.

2.3 **Corollary**

Suppose $\tau=(q+\delta)T/m$; q, $1\leqslant q\leqslant m$, is a positive integer; $0\leqslant\delta<1$ and

$$\tilde{y}(t)=\begin{cases}k(t),\text{for } -\tau\leqslant t<0\\ y(t),\text{for } 0\leqslant t<T,\end{cases}$$ where $k(t)$ and $y(t)$ are n—dimension vectors.

then we have

$$\tilde{y}(t-\tau)\doteq[KS_q(\delta)+YD_q(\delta)]\Phi(t), \tag{7}$$

where Y and K are respectively coefficient matrices of BPFs of $k(t)$ and $y(t)$ vectors. And matrix $S_q(\delta)$ is defined as

$$S_q(\delta)=\begin{bmatrix} I_{q\times q} & \vdots & & 0 & \\ \cdots & & \cdots & \cdots & \cdots \\ & \vdots & \delta & 0\cdots\cdots & 0 \\ & \vdots & 0 & \cdots & 0 \\ & \vdots & \vdots & & \vdots \\ 0 & \vdots & 0 & \cdots & 0 \end{bmatrix}_{m\times m}$$

, where $I_{q\times q}$ is the qxq identity matrix.

3. Optimal control of bilinear systems with time—delay

Consider the MIMO bilinear time—delay system, which is described by

$$\begin{aligned}\dot{x}(t) &= Ax(t)+Bx(t-\tau_1)+Cu(t)+Du(t-\tau_2)+\sum_{j=1}^{r}N_{1j}x(t)u_j(t)\\ &+\sum_{j=1}^{r}N_{2j}x(t-\tau_1)u_j(t)+\sum_{j=1}^{r}N_{3j}x(t-\tau_1)u_j(t-\tau_2)+\sum_{j=1}^{r}N_{4j}x(t)u_j(t-\tau_2),\end{aligned} \tag{8}$$

where $x(t)\in R^{n\times 1}$ is a state vector; $u(t)\in R^{r\times 1}$ is a control vector; A,B,N_{1j},N_{2j},N_{3j} and N_{4j} are $n\times n$—dimension matrices of time—invariant coefficients; C and D are $n\times r$—dimension matrices of time—invariant coefficients; τ_1 and τ_2 represent the delays in the state variables and control variables respectively. For $t\leqslant 0$, the state and control variables are assumed to be

$$\begin{cases}x(t)=k(t),\text{for } -\tau_1\leqslant t<0\\ x(t)=x(0),\text{for } t=0\end{cases}$$

and $$u_j(t)=g_j(t),\text{for } j=1,\cdots,r \text{ and } -\tau_2\leqslant t<0$$

where $k(t)\in R^{n\times 1}$ and $g_j(t)$ are known arbitrary function vectors which are absolutely integrable.

The quadratic performance index of the optimal control is defined as

$$J=\min_{u(t)}\frac{1}{2}\left\{\|x(t_f)\|^2_{H^*}+\int_0^{t_f}(\|x(t)\|^2_{Q^*}+\|u(t)\|^2_{R^*})dt\right\}, \tag{9}$$

where

$$H^*=\text{diag}(h_1^*,\cdots,h_n^*), Q^*=\text{diag}(q_1^*,\cdots,q_n^*)$$
$$h_i^* \text{ and } q_i^*\geqslant 0,\quad \text{for } i=1,\cdots,n;$$
$$R^*=\text{diag}(r_1^*,\cdots,r_r^*),\ r_j^*>0,\ \text{for } j=1,\cdots,r.$$

Let $h=t_f/m$, then we have

$$\tau_1=(q_1+\delta_1)h, \tau_2=(q_2+\delta_2)h,$$

where q_1 and q_2, $1\leqslant q_1\leqslant m$, $1\leqslant q_2\leqslant m$, are positive integers; $0\leqslant\delta_1<1$, $0\leqslant\delta_2<1$.

The control, state and initial condition vectors can be described by BPFs, respectively

$$\begin{cases} u(t) = [u_j(t)]_{r\times 1} \doteq U\Phi(t) \\ u_j(t) \doteq U_{j\cdot}^T \Phi(t) \end{cases}, \qquad \begin{cases} u(t-\tau_2) = [u_j(t-\tau_2)]_{r\times 1} \doteq \tilde{U}\Phi(t) \\ u_j(t-\tau_2) \doteq \tilde{U}_{j\cdot}^T \Phi(t) \end{cases}$$

$$\begin{cases} x(t) = [x_i(t)]_{n\times 1} \doteq X\Phi(t) \\ x_i(t) \doteq X_{i\cdot}^T \Phi(t) \end{cases}, \qquad \begin{cases} x(t-\tau_1) = [x_i(t-\tau_1)]_{n\times 1} \doteq \tilde{X}\Phi(t) \\ x_i(t-\tau_1) \doteq \tilde{X}_{i\cdot}^T \Phi(t) \end{cases}$$

$$g(t-\tau_2) \doteq G\Phi(t), \quad k(t-\tau_1) \doteq K\Phi(t)$$

$$x(0) = x(0) \otimes e^T\Phi(t) = X(0)\Phi(t)$$

where

$$U = [U_{1\cdot}, \cdots, U_{r\cdot}]^T, \qquad U_{j\cdot} = [u_{j1}, \cdots, u_{jm}]^T$$

$$\tilde{U} = [\tilde{U}_{1\cdot}, \cdots, \tilde{U}_{r\cdot}]^T, \quad \tilde{U}_{j\cdot} = S_{q_2}^T(\delta_2)G_{j\cdot} + D_{q_2}^T(\delta_2)U_{j\cdot} \triangleq [\tilde{u}_{j1}, \cdots, \tilde{u}_{jm}]^T$$

$$K = [K_{1\cdot}, \cdots, K_{n\cdot}]^T, \quad K_{i\cdot} = [k_{i1}, \cdots, k_{im}]^T$$

$$G = [G_{1\cdot}, \cdots, G_{r\cdot}]^T, \qquad G_{j\cdot} = [g_{j1}, \cdots, g_{jm}]^T$$

$$X = [X_{1\cdot}, \cdots, X_{n\cdot}]^T, \qquad X_{i\cdot} = [x_{i1}, \cdots, x_{im}]^T$$

$$\tilde{X} = [\tilde{X}_{1\cdot}, \cdots, \tilde{X}_{n\cdot}]^T, \qquad \tilde{X}_{i\cdot} = S_{q_1}^T(\delta_1)K_{i\cdot} + D_{q_1}^T(\delta_1)X_{i\cdot} \triangleq [\tilde{x}_{i1}, \cdots, \tilde{x}_{im}]^T$$

$$e^T = [1, 1, \ldots, 1]_{1\times m}$$

for $i=1, \cdots, n$; $j=1, \cdots, r$.

$\otimes$ denotes Kronecker product.

Let

$$W = I - A\otimes P^T - B\otimes (D_{q_1}(\delta_1)P)^T - \sum_{j=1}^{r} N_{1j}\otimes (\mathrm{diag}(U_{j\cdot})P)^T$$
$$- \sum_{j=1}^{r} N_{2j}\otimes (D_{q_1}(\delta_1)\mathrm{diag}(U_{j\cdot})P)^T - \sum_{j=1}^{r} N_{3j}\otimes (D_{q_1}(\delta_1)\mathrm{diag}(\tilde{U}_{j\cdot})P)^T$$
$$- \sum_{j=1}^{r} N_{4j}\otimes (\mathrm{diag}(\tilde{U}_{j\cdot})P)^T, \tag{10}$$

and

$$V = \{B\otimes (S_{q_1}(\delta_1)P)^T + \sum_{j=1}^{r} N_{2j}\otimes (S_{q_1}(\delta_1)\mathrm{diag}(U_{j\cdot})P)^T$$
$$+ \sum_{j=1}^{r} N_{3j}\otimes (S_{q_1}(\delta_1)\mathrm{diag}(\tilde{U}_{j\cdot})P)^T\}[K_{1\cdot}^T, \cdots, K_{n\cdot}^T]^T$$
$$+ D\otimes (S_{q_2}(\delta_2)P)^T[G_{1\cdot}^T, \ldots, G_{r\cdot}^T]^T$$
$$+ \{C\otimes P^T + D\otimes (D_{q_2}(\delta_2)P)^T\}[U_{1\cdot}^T, \cdots, U_{r\cdot}^T]^T, \tag{11}$$

where $\mathrm{diag}(U_{j\cdot}) = \mathrm{diag}(u_{j1}, \cdots, u_{jm})$, $\mathrm{diag}(\tilde{U}_{j\cdot}) = \mathrm{diag}(\tilde{u}_{j1}, \cdots, \tilde{u}_{jm})$.

By applying BPFs, (8) can be solved by

$$W[X_{1\cdot}^T, \cdots, X_{n\cdot}^T]^T = [x_1(0)e^T, \cdots, x_n(0)e^T]^T + V, \tag{12}$$

or

$$[X_{1\cdot}^T, \cdots, X_{n\cdot}^T]^T - [x_1(0)e^T, \cdots, x_n(0)e^T]^T$$
$$= [(\sum_{j=1}^{r}\tilde{W}_1 U_{j\cdot})^T, \cdots, (\sum_{j=1}^{r}\tilde{W}_n U_{j\cdot})^T]^T + \sum_{j=1}^{r}\tilde{V}_1 U_{j\cdot} + \tilde{V}_2, \tag{13}$$

where

$$\widetilde{W}_i = c_{ij}P^T + d_{ij}(D_{q_2}(\delta_2)P)^T, \qquad \text{for } i = 1,\cdots,n \tag{14}$$

$$\begin{aligned}\widetilde{V}_1 = {} & (N_{1j}\otimes P^T)[\text{diag}(X_{1\cdot}),\cdots,\text{diag}(X_{n\cdot})]^T + (N_{2j}\otimes P^T)[\text{diag}(\widetilde{X}_{1\cdot}),\cdots,\text{diag}(\widetilde{X}_{n\cdot})]^T \\ & + (N_{3j}\otimes P^T)[\text{diag}(\widetilde{X}_{1\cdot}),\cdots,\text{diag}(\widetilde{X}_{n\cdot})]^T D_{q_2}^T(\delta_2) \\ & + (N_{4j}\otimes P^T)[\text{diag}(X_{1\cdot}),\cdots,\text{diag}(\widetilde{X}_{n\cdot})]^T D_{q_2}^T(\delta_2).\end{aligned} \tag{15}$$

$$\begin{aligned}\widetilde{V}_2 = {} & \overrightarrow{AXP} + \overrightarrow{B[KS_{q_1}(\delta_1) + XD_{q_1}(\delta_1)]P} + \overrightarrow{DGS_{q_2}(\delta_2)} + \sum_{j=1}^{r} \overrightarrow{N_{3j}[\text{diag}(X_{1\cdot})S_{q_2}^T(\delta_2)G_{j\cdot},\cdots,} \\ & \overrightarrow{\text{diag}(X_{n\cdot})S_{q_2}^T(\delta_2)G_{j\cdot}]^T P} + \sum_{j=1}^{r} \overrightarrow{N_{4j}[\text{diag}(X_{1\cdot})S_{q_2}^T(\delta_2)G_{j\cdot},\cdots,} \\ & \overrightarrow{\text{diag}(X_{n\cdot})S_{q_2}^T(\delta_2)G_{j\cdot}]^T P}\end{aligned} \tag{16}$$

and

$$\text{diag}(X_{i\cdot}) = \text{diag}(x_{i1},\cdots,x_{im}), \quad \text{diag}(\widetilde{X}_{i\cdot}) = \text{diag}(\tilde{x}_{i1},\cdots,\tilde{x}_{im})$$
$$\text{for } i = 1,\cdots,n.$$

In Eqs. (14) and (16),

$$C = [c_{ij}]_{n\times r}, \quad D = [d_{ij}]_{n\times r}, \quad \text{for } i = 1,\ldots,n; \quad j = 1,\ldots,r.$$

$\overrightarrow{(\cdot)}$ denotes rank drawing straight of matrix $(\cdot)$. Equally, the quadratic performance index J can yield

$$J = \min_{U_{j\cdot}} \frac{1}{2}\Big\{\sum_{i=1}^{n} h_i^* x_{im}^2 + h\sum_{i=1}^{n}\sum_{l=1}^{m} q_i^* x_{il}^2 + h\sum_{j=1}^{r}\sum_{l=1}^{m} r_j^* u_{jl}^2\Big\}, \text{for } j = 1,\cdots,r. \tag{17}$$

Thus by using BPFs, a dynamic optimal control problem of the bilinear time—delay system described by (8)—(9) can be transformed into a static optimization problem described by (17) and (12) (or (13)). In the following, we shall derive from a detailed algorithm to solve this static optimization problem by applying the prediction principle, the Lagrange multiplier and gradient algorithm.

Let the Lagrange multiplier vector be described by

$$\bar{\lambda} = [\lambda_{1\cdot}^T,\cdots,\lambda_{n\cdot}^T]^T, \ \lambda_{i\cdot} = [\lambda_{i1},\cdots,\lambda_{im}]^T, \quad \text{for } i = 1,\cdots,n.$$

Form (17) and (12), the Lagrange function can be defined as

$$\begin{aligned}L = {} & \frac{1}{2}\Big\{\sum_{i=1}^{n} h_i^* x_{im}^2 + h\sum_{i=1}^{n}\sum_{l=1}^{m} q_i^* x_{il}^2 + h\sum_{j=1}^{r}\sum_{l=1}^{m} r_j^* u_{jl}^2\Big\} + [\lambda_{1\cdot}^T,\cdots,\lambda_{n\cdot}^T] \\ & \cdot \{W[X_{1\cdot}^T,\cdots,X_{n\cdot}^T]^T - [x_1(0)e^T,\cdots,x_n(0)e^T]^T - V\}.\end{aligned} \tag{18}$$

Equally, from (17) and (13), the Lagrange function can be also defined as

$$\begin{aligned}L = {} & \frac{1}{2}\{\sum_{i=1}^{n} h_i^* x_{im}^2 + h\sum_{i=1}^{n}\sum_{l=1}^{m} q_i^* x_{il}^2 + h\sum_{j=1}^{r}\sum_{l=1}^{m} r_j^* u_{jl}^2\} + [\lambda_{1\cdot}^T,\cdots,\lambda_{n\cdot}^T]\{[X_{1\cdot}^T,\cdots,X_{n\cdot}^T]^T \\ & - [x_1(0)e^T,\cdots,x_n(0)e^T]^T - [(\sum_{j=1}^{r}\widetilde{W}_1 U_{j\cdot})^T,\cdots,(\sum_{j=1}^{r}\widetilde{W}_n U_{j\cdot})^T]^T \\ & - \sum_{j=1}^{r}\widetilde{V}_1 U_{j\cdot} - \widetilde{V}_2\}.\end{aligned} \tag{19}$$

Defining $\bar{X} = [X_{1\cdot}^T,\cdots,X_{n\cdot}^T]^T$

$$Q^* = \text{Block diag}(Q_1^*,\cdots,Q_n^*), Q_i^* = \text{diag}(hq_i^*,\cdots,hq_i^*,hq_i^*+h_i^*)_{m\times m}, \quad \text{for } i=1,\cdots,n.$$

From (18) and the necessary conditions of the optimization $(\frac{\partial L}{\partial \bar{X}}=0, \frac{\partial L}{\partial \bar{\lambda}}=0)$, we have

$$\bar{\lambda} = -(W^{-1})^T Q^* \bar{X}, \tag{20}$$

$$\bar{X} = W^{-1}\{[x_1(0)e^T, \cdots, x_n(0)e^T]^T + V\}. \tag{21}$$

Equally, from (19), we have

$$\frac{\partial L}{\partial U_{j\cdot}} = hr_j^* U_{j\cdot} - \sum_{l=1}^{n} \widetilde{W}_l^T \lambda_{l\cdot} \widetilde{V}_1^T \bar{\lambda}, \quad \text{for } j = 1, \cdots, r. \tag{22}$$

The rule, by which the vector $U_{j\cdot}$ can be corrected repeatedly from iteration v to $v+1$, can be described by

$$U_{j\cdot}^{v+1} = U_{j\cdot}^{v} - \mathrm{diag}(a_{j1}, \cdots, a_{jm}) \frac{\partial L}{\partial U_{j\cdot}}\Big|_{\bar{\lambda} = \bar{\lambda}^v}^{\bar{X} = \bar{X}^v}, \quad \text{for } j = 1, \cdots, r. \tag{23}$$

where a_{ji} (for $i=1, \cdots, m$) is the "step—length" factor, by which the convergence of the above algorithm can be adjusted. Generally, a_{ji} can be chosen from $0-1$.

The algorithm to solve the above static optimization problem can be schematically shown as follows:

Step 1. Predict control vectors $U_{j\cdot}^0$ (for $j=1, \cdots, r$), set up $v=0$ and given a smaller positive figure ε.

Step 2. From (21), solve the state vector $\bar{X}^v$.

Step 3. Form(20), solve the Lagrange multiplier vector $\bar{\lambda}^v$.

Step 4. From(22), calculate the each branch of vectors $\frac{\partial L}{\partial U_{j\cdot}}$(for $j=1, \cdots, r$).

Step 5. Judge wether the absolute value of the each branch of gradient vectors $\frac{\partial L}{\partial U_{j\cdot}}$(for $j=1, \cdots, r$)is smaller than ε or not. If be, change over to step 7. Otherwise, continue step 6.

Step 6. From (23), calculate new control vectors $U_{j\cdot}^{v+1}$ (for $j=1, \cdots, r$), set up $v=v+1$ and change over to step 2.

Step . Stop calculating repeatedly. $\bar{X}^v$ and $U_{j\cdot}^v$ (for $j=1, \cdots, r$)do be the solution of the static optimization problem described by (17) and (12) (or (13)).

Let $\bar{X}^* = \bar{X}^v$(i. e. $X_{i\cdot}^* = X_{i\cdot}^v$, for $i=1, \cdots, n$) and $U_{j\cdot}^* = U_{j\cdot}^v$ (for $j=1, \cdots, r$), then we can obtain the control vector and the state vector of the original dynamic system, which are

$$u^*(t) = [U_{1\cdot}^*, \cdots, U_{r\cdot}^*]^T \Phi(t), \tag{24}$$

$$x^*(t) = [X_{1\cdot}^*, \cdots, X_{n\cdot}^*]^T \Phi(t). \tag{25}$$

Since series of BPF$_s$ $\{\varphi_i(t), i=1, \cdots, m; m\to\infty\}$ is complete [3], the control and state vectors described by (24) and (25) converge to original optinmal solution. That is

$$u_{\mathrm{optim}}(t) = \lim_{m\to\infty} u^*(t) \doteq u^*(t), \tag{26}$$

$$x_{\mathrm{optim}}(t) = \lim_{m\to\infty} x^*(t) \doteq x^*(t). \tag{27}$$

Therefore, by applying the above algorithm, we can obtain the optimal control vector ($u_{optim}(t)$) and the optimal state vector ($x_{optim}(t)$) of the original bilinear time—delay system.

4. The example and the application to a distillation column

4. 1 Illustrative example

Consider a optimal control problem of the SISO bilinear time—delay system, which is described by

$$\dot{x}(t) = -2x(t) + x(t)u(t-0.15) + 3u(t-0.15) \tag{28}$$

with $x(0) = 5$; $u(t) = 0$, for $-0.15 \leqslant t < 0$,

$$J = \min_{u(t)}\{\frac{1}{2}Hx^2(t_f) + \frac{1}{2}\int_0^{t_f}(Qx^2(t) + Ru^2(t))dt\}, \tag{29}$$

where $t_f=1$, $H=10^5$, $Q=1$ and $R=1$.

Applying the algorithm presented in this paper, let m=10 and m=20 respectively, we obtain the optimal control variable $u_{optim}(t)$ and the optimal state variable $x_{optim}(t)$ of this bilinear time—delay system, which are shown in Fig. 1 and Fig. 2 respectively. Figs. 1—2 show that the optimal control results of the bilinear time—delay system are satisfactory.

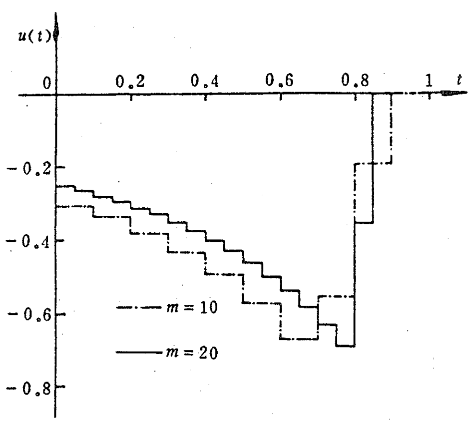

Fig. 1 The optimal control u(t)

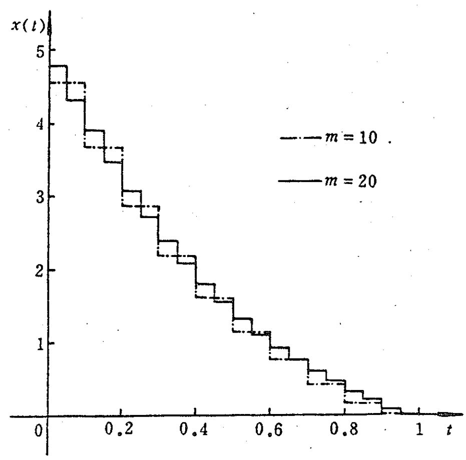

Fig. 2 The optimal staee x(t)

4. 2 The application to a distillation column

The control of distillation columns based on bilinear models always interests control engineers. In this paper, we shall apply the above algorithm to solve a temperature optimal control problem of a industrial de—ethane column, whose dynamic behaviour is a bilinear model with time—delay in the control variable. It is especially of very great importance for the temperature control in start up of the distillation column.

The model of the industrial de—ethane column can be described by[4]

$$\dot{T}(t) = 0.3848 \cdot T(t) + 0.076 \cdot T(t)u(t-0.0659) - 1.2663 \cdot u(t-0.0659), \tag{30}$$

where $T(t)$ (℃) is the temperature of the sensitive plate, which is the output variable of the system; $u(t)$ (Kml/hour) is the feed flow, which is the control variable of the system; t(hour) is time.

Assuming that the initial temperature and control variable are, respectively

$$T(0) = 5(℃); u(t) = 0, \text{for } t < 0.$$

and the quadratic performance index is

$$J = \min_{u(t)}\{\frac{1}{2}HT^2(t_f) + \frac{1}{2}\int_0^{t_f}(QT^2(t) + Ru^2(t))dt\}, \tag{31}$$

where $t_f=1$ (hour), $H=10^5$, $Q=1$ and $R=1$.

Applying the algorithm presented in this paper, let $m=10$ and $m=20$ respectively, we obtain the optimal control variable $u_{optim}(t)$ and the optimal temperature variable $T_{optim}(t)$ of this actual bilinear time—delay system, which are shown in Fig. 3 and Fig. 4 respectively.

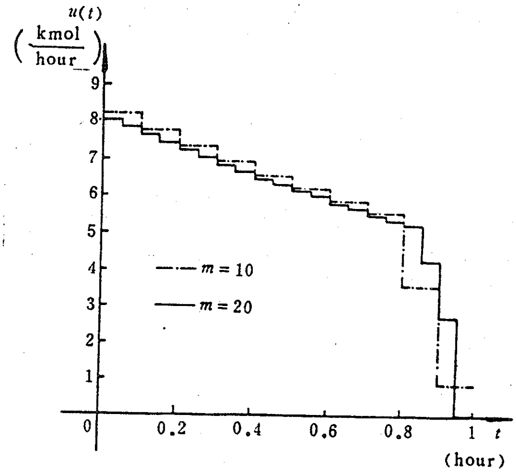

Fig. 3 The optimal control $u(t)$

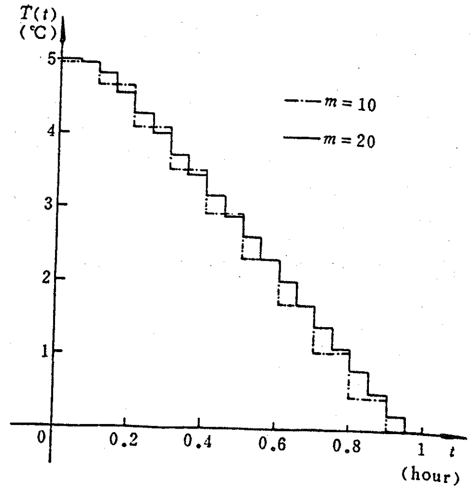

Fig. 4 The optimal temperature $t(u)$

Obviously, it has been shown that the algorithm presented in this paper is simple and convenient for solving the optimal control problem of the distillation column.

5. Conclusions

This paper presents a detailed algorithm to solve the optimal control problem of bilinear time—delay systems by using series of BPFs. Through the example and the application to a distillation column, it is shown that the algorithm proposed in this paper is simple, convenient for the applications to industrial processes and has the sufficient control accuracy (it depends on the step—length of BPFs). This algorithm provides a new method for solving the optimal control problem of bilinear time—delay systems.

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双线性时滞系统的最优控制及其在精馏塔的应用

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摘要 本文利用块脉冲函数(BPFs)和参数优化方法，提出了解决具有任意状态时滞、控制时滞及其任意初始条件的多变量双线性系统最优控制问题的具体算法. 该算法被成功地用于一类可用双线性时滞模型描述的精馏塔的温度最优控制. 同时，通过其它例子仿真，也表明这个算法是令人满意的.

关键词： 最优控制；双线性系统；时滞系统；块脉冲函数；精馏塔

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