A Genetic Algorithm for Optimal Control of Stretching Process of Solar Arrays on Spacecraft Based on Wavelet Approximation *

Zhang Qizhi and Ge Xinsheng
(Department of Automation, Beijing Institute of Machinery · Beijing, 100085, P.R.China)
Liu Yanzhu
(Department of Engineering Mechanics, Shanghai Jiaotong University · Shanghai, 200030, P.R.China)

Abstract: The optimal attitude control problem of spacecraft in its solar arrays stretching process is discussed. Control input rule is approximated by wavelet. The genetic algorithm of optimal control for the stretching process is proposed. The simulations show that wavelet approximation and genetic algorithm are effective tools for solving optimal control problem.

Key words: optimal control; wavelet; genetic algorithm; spacecraft

基于小波逼近的航天器太阳帆板展开过程最优控制的遗传算法

张奇志 戈新生 刘延柱
(北京机械工业学院自动化系·北京,100085) (上海交通大学工程力学系·上海,200030)

摘要: 本文讨论了航天器太阳帆板展开过程中航天器姿态的最优控制问题.在控制算法中用小波函数逼近控制输入规律.提出了太阳帆板展开过程最优控制的遗传算法.数值仿真表明,小波逼近和遗传算法联合求解最优控制问题是有效的.

关键词: 最优控制; 小波; 遗传算法; 航天器

1 Introduction

With the development of space technology, the large strectchable solar arrays have been widely applied to observe satellite on solar synchronous orbit, communication satellite on earth synchronous orbit and data relay station satellite. They are folded in the carrier. After the carrier enters its orbit, the solar arrays are going to stretch to their working state. In this process, the attitude of spacecraft is changed due to the coupling of stretching motion and attitude motion of spacecraft[1]. In order to ensure that the spacecraft locates in the designed position, it is necessary to study the control regularity in the stretching process of solar arrays. Ge Xinsheng and Liu Yanzhu have discussed the optimal control problem in the stretching process of solar arrays[2]. This idea originates from Ritz's approximation method. In practice, the Fourier basis function expansion is often used. It is developed from the traditional Fourier analysis, and extensively used in information and control fields[3,4]. In this paper, the discrete orthonormal wavelet basis functions are substituted for Fourier basis functions. Compared with Fourier basis functions, the wavelets have better characteristics of localization and multi-resolution. We can obtain an initial solution by a scaling function, then improve the solution to a higher resolution by superimposing the component of higher-order wavelet. The Gauss-Newton iteration method is often utilized to search the optimal solution of optimal control problem. Complex differential quotient operation is required in this method, and there are lots of equations. Recently, many researchers are motivated to pursue their research in the genetic algorithm (GA)[5]. GA is a global searching optimization algorithm based on the mechanics of natural genetics. Because gradient information is not required, GA is widely used in a variety of fields. In this paper, the GA with real coding is utilized to research into the optimal control problem, and a genetic algorithm for the optimal control of streching process of solar arrays on spacecraft based on wavelet approximation is proposed. Considering the characteristics of non-

* This work is supported by the National Natural Science Foundation of China (19782003).
Manuscript received Sep.15,1998, revised Jul.26,1999.

holonomic constraint of system, the objective function of control system is built by dynamics equations of spacecraft with solar arrays. The rule of control input is approximated by wavelet expansion, the construction of chromosome and fitness function of optimal control is determined, and genetic operators are designed. Finally, the optimal control input rule and the optimal path of stretching process are obtained. Numerical simulations show that the control problem of spacecraft attitude during solar arrays stretching process can be efficiently resolved by the proposed method.

2 Orthonormal wavelet and multi-resolution analysis

The discrete orthonormal wavelet is characterized by the translation and dilation of a single function $\psi(t)$[6]:

$$\psi_{m,n}(t) = 2^{\frac{m}{2}}\psi(2^m t - n), \quad m,n \in \mathbb{Z}, \quad (1)$$

where $\psi_{m,n}(t)$ constitutes a basis for the space of square integral function $L^2(R)$, and any $f(x) \in L^2(R)$ can be expressed as:

$$f(x) = \sum_m \sum_n d_{m,n}\psi_{m,n}(x), \quad m,n \in \mathbb{Z}, \quad (2)$$

where coefficient $d_{m,n}$ is defined as follows:

$$d_{m,n} = \int_{-\infty}^{+\infty} f(x)\psi_{m,n}(x)\mathrm{d}x, \quad m,n \in \mathbb{Z}, \quad (3)$$

orthonormal wavelet $\psi(t)$ are derived from scaling function $\varphi(t)$ and have the two scale relationships:

$$\varphi(t) = \sum_{n=-\infty}^{\infty} h(n)\varphi(2t - n), \quad (4)$$

$$\psi(t) = \sum_{n=-\infty}^{\infty} g(n)\varphi(2t - n). \quad (5)$$

For any $f(x) \in L^2(R)$ whose approximate solution at the m-order resolution can be described as:

$$P_m f(x) = \sum_n C_{m,n}\varphi_{m,n}(x), \quad n \in \mathbb{Z}, \quad (6)$$

where

$$C_{m,n} = \int_{-\infty}^{+\infty} f(x)\varphi_{m,n}(x)\mathrm{d}x, \quad m,n \in \mathbb{Z}, \quad (7)$$

$P_m f(x)$ represents the projection of $f(x)$ onto the space of m-order resolution scale,

$$\varphi_{m,n}(t) = 2^{\frac{m}{2}}\varphi(2^m t - n), \quad m,n \in \mathbb{Z}, \quad (8)$$

which constitutes a basis for the function space of m-order resolution scale, the $(m+1)$-order resolution can be written as

$$P_{m+1}f(x) = P_m f(x) + \sum_n d_{m,n}\psi_{m,n}(x), \quad n \in \mathbb{Z}, \quad (9)$$

which shows that the solution at $(m+1)$-order resolution can be obtained from the solution at m-order resolution superimposed by projection of function $f(x)$ onto m-order wavelet space. As m_0 is the lowest resolution degree, any function $f(x) \in L^2(R)$ can be expressed as the expansion of wavelet series

$$f(x) = \sum_n C_{m_0,n}\varphi_{m_0,n}(x) + \sum_{m=-\infty}^{m_0}\sum_n d_{m,n}\psi_{m,n}(x), \quad n \in \mathbb{Z}, \quad (10)$$

where $\psi_{m,n}(t)$ and $\varphi_{m,n}(t)$ are orthonormal and satisfy:

$$\begin{cases} \int_{-\infty}^{+\infty} \varphi_{m,n}(x)\varphi_{m,n'}(x)\mathrm{d}x = \delta_{n,n'}, \\ \int_{-\infty}^{+\infty} \psi_{m,n}(x)\psi_{m',n'}(x)\mathrm{d}x = \delta_{m,m'}\delta_{n,n'}, \\ \int_{-\infty}^{+\infty} \varphi_{m,n}(x)\psi_{m',n'}(x)\mathrm{d}x = 0, \quad m' \geqslant m. \end{cases} \quad (11)$$

In practice, we may obtain an initial solution at resolution m_0, and then successively refine the solution by superimposing "detail signal", i. e. the second item in Eq.(10). Because of the orthonormality of wavelet and scaling function, a little computation is needed to refine the solution, especially when both $\varphi(t)$ and $\psi(t)$ are compactly supported.

3 Optimal control based on wavelet approximation

3.1 Dynamics equations of stretching process of solar arrays and optimal control

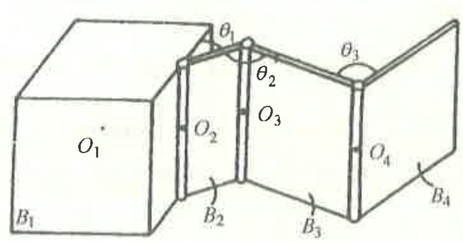


Fig. 1 Spacecraft with solar arrays

Consider a spacecraft with solar arrays consisting of four rigid bodies, connected by cylinder hinge joint in single side (Fig. 1), where B_1 is the main body of spacecraft, B_2 is the connecting array, B_3 and B_4 are the internal and external solar arrays respectively. The dynamic equation of system is written as[2]

$$I_z\dot{\varphi}_1 + [-(I_{2z} + I_{3z} + I_{4z})\dot{\theta}_1 + (I_{3z} + I_{4z})\dot{\theta}_2 - I_{4z}\dot{\theta}_3] = 0, \tag{12}$$

where I_z and $I_{iz}(i = 2,3,4)$ are respectively the equivalent moment of inertia of the system and its parts. φ_1 is the attitude angle of the spacecraft main body, $\theta_i(i = 1,2,3)$ is the relative angle between the adjacent solar arrays. Equation (12) has the form of nonholomonic behavior, which shows that any relative motion of the components in the system can disturb the attitude of the main body. The relative angular velocity between the adjacent solar arrays $\dot{\theta}_i(i = 1,2,3)$ is regarded as input variables, denoted as $u = (\dot{\theta}_1 \quad \dot{\theta}_2 \quad \dot{\theta}_3)^{\mathrm{T}}$. The state variables of system are defined as $x = [\theta_1 \quad \theta_2 \quad \theta_3 \quad \varphi_1]^{\mathrm{T}}$, therefore the state equation of the system is as follows:

$$\dot{x} = B(x)u. \tag{13}$$

where

$$B(x) = \begin{bmatrix} b_1 & b_2 & b_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$b_1 = (I_{2z} + I_{3z} + I_{4z})/I_z,$$
$$b_2 = -(I_{3z} + I_{4z})/I_z,$$
$$b_3 = I_{4z}/I_z.$$

Equation (13) contains four state variables and three control inputs. In the stretching process of arrays, the optimal control inputs to decide the attitude of the main body is obtained by minimizing cost. Choose the dissipative energy at the hinge joint between two solar arrays as the cost of optimal control, and then objective function becomes

$$J(u) = \int_0^T \langle u, u \rangle \mathrm{d}t, \tag{14}$$

where $u(t)$ is measurable vector function in Hilbert space $L^2(R)$, expanding the components of $u(t)$ according to Eq.(10), and taking finite terms, $u(t)$ can be expressed as a linear combination of wavelet basis vectors

$$u = \sum_{i=1}^{N} \alpha_i a_i = \Phi\alpha, \tag{15}$$

where $\alpha_i(i = 1,2,\cdots,N)$ is the projection of the function $u(t)$ onto the basis vector $\{a_i\}_{i=1}^N$, Φ is a $n \times N$ dimension matrix. By regarding α as a new control variable and then taking into account the term as the orthonormality of Eq.(11) and the constrained term of the system terminal of spacecraft, the objective function $J(u)$ of Eq.(14) can be written as:

$$J(\alpha,\lambda) = \sum_{i=1}^{N} \alpha_i^2 + \lambda \| x(T) - x_f \|^2, \tag{16}$$

where λ is a penalty coefficient, $x(T)$ is the state of system at $t = T$. Obviously, $x(T)$ is a function about α, and denoted by $f(\alpha)$. When N and λ are given, Eq.(14) becomes

$$J(\alpha) = \langle \alpha, \alpha \rangle + \lambda \| f(\alpha) - x_f \|^2. \tag{17}$$

Therefore, the problem of minimizing the value of Equation (14) by searching control input $u(t)$ is changed into that of minimizing the value of Equation (17) by searching α.

3.2 Genetic algorithm of optimal control

In general, binary strings are used to encode the optimal parameter space in GA. When the dimensions of parameter become high and the range of parameter value becomes great, the converge speed of the algorithm will become slowly. Because the real-coded GA[7] has the advantages of higher precise and convenience to search in large space, it is utilized in the paper. Considering the optimal control problem in the solar arrays stretching process, the GA is designed to obtain the optimal control input rule and the optimal path of stretching process of system. GA is designed as follows:

1) Chromosomes representation: utilizing the parallel searching mechanics of GA, α, the projections of function $u(t)$ on wavelet basis, are encoded to an N-dimensions vector composed by $\alpha_i(i = 1,2,\cdots,N)$,

$$\alpha = [\alpha_1 \quad \alpha_2 \quad \cdots \quad \alpha_N]^{\mathrm{T}}. \tag{18}$$

2) Initialization of the population of parent: the P parents, whose components are normal Gauss random variables, are generated randomly.

3) The selection of fitness function: we define the fitness score function as:

$$g(\alpha) = 1/J(\alpha), \tag{19}$$

where $J(\alpha)$ is the objective function defined in Eq.(17), α is the chromosomes, and the α with small objective function value has high fitness score.

4) Selection: the fitness score $g(\alpha_i)(i = 1,2,\cdots, P)$ of each chromosome is calculated according to Eq.(19), the survival probability of ith chromosome is

$$P_i = g(\alpha_i)/\sum_{i=1}^{P} g(\alpha_i). \quad (20)$$

This process is accomplished by using roulette wheel selection.

5) Crossover: a simple one-point crossover is employed. According to the crossover probability P_c, a splice point is determined uniformly at random. The genetic codes following the splice point are interchanged, and offspring are generated as shown in Fig.2

$$\alpha_1^k\alpha_2^k\cdots\alpha_l^k|\alpha_{l+1}^k\cdots\alpha_N^k \rightarrow \alpha_1^k\alpha_1^k\cdots\alpha_l^k|\alpha_{l+1}^s\cdots\alpha_N^s$$
$$\alpha_1^s\alpha_2^s\cdots\alpha_l^s|\alpha_{l+1}^s\cdots\alpha_N^s \rightarrow \alpha_1^s\alpha_2^s\cdots\alpha_l^s|\alpha_{l+1}^k\cdots\alpha_N^k$$

Fig.2 Crossover chart

6) Mutation: some components $\alpha_{ij}(i = 1,2,\cdots,P, j = 1,2,\cdots,N)$ of the chromosome α_i are chosen to mute according to the mutation probability P_m, and a Gaussian random variable on it:

$$\alpha_{ij} = \alpha_{ij} + \delta_j, \quad (21)$$

where δ_j is Gaussian random variable. The Steps (4) ~ (6) is repeated, the fitness scores of population become higher and the optimal solution can be obtained in the end.

4 Simulation examples

The mass and geometry parameters of the spacecraft with stretchable solar arrays are: main body $m_1 = 200\text{kg}, J_1 = 32.2\text{kgm}^2, l_1 = 0.5\text{m}$, the solar arrary $m_2 = 5\text{kg}, l/2 \times l = 0.5 \times 1\text{m}^2; m_3 = m_4 = 10\text{kg}, l \times l = 1 \times 1\text{m}^2$. The stretching time is $t = [0,5]$. The control parameters of GA are selected as: the dimension of the population of parents is $P = 48$, the length of chromosomes is $N = 15$, the probability of crossover is $P_c = 0.9$, the probability of mutation is $P_m = 0.1$. The number of evolution generations approximated by scale function is $R = 2000$, and the number of superimposing wavelet basis function is 200.

Choose Daubechies[3] wavelet with $L = 4$. According to Eq.(10), take the period characterstic of $u(t)$ into account, the scaling function and superimposing wavelet basis function are respectively utilized to numerical calculation. Taking $m_0 = 0$, approximately substitute by the translation of scaling function, the period is 5. Φ are constituted by 15 discrete orthonormal basis function. $\{a_i\}_{i=1}^5$ (basis for first competent) are shown as follows:

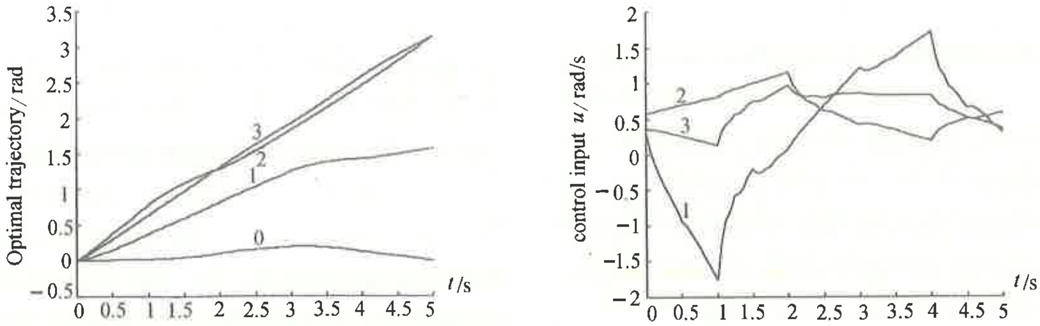
$$\begin{Bmatrix}\varphi(t)\\0\\0\end{Bmatrix}\begin{Bmatrix}\varphi(t-1)\\0\\0\end{Bmatrix}\begin{Bmatrix}\varphi(t-2)\\0\\0\end{Bmatrix}$$
$$\begin{Bmatrix}\varphi(t-3)+\varphi(t+2)\\0\\0\end{Bmatrix}\begin{Bmatrix}\varphi(t-4)+\varphi(t+1)\\0\\0\end{Bmatrix},$$

the remaining terms $\{a_i\}_{i=6}^{10}$, $\{a_i\}_{i=11}^{15}$ are obtained by permuting the rows of the above element. Set $m = 1$, function $u(t)$ is a linear combination of scaling function $\varphi_{0,n}(t)$ and wavelet function $\psi_{0,n}(t)$. Eq.(15) combined 15 scaling basis function and 15 wavelet basis functions, the $\{a_i\}_{i=1}^{10}$ as

$$\begin{Bmatrix}\varphi(t)\\0\\0\end{Bmatrix}\begin{Bmatrix}\varphi(t-1)\\0\\0\end{Bmatrix}\begin{Bmatrix}\varphi(t-2)\\0\\0\end{Bmatrix}$$
$$\begin{Bmatrix}\varphi(t-3)+\varphi(t+2)\\0\\0\end{Bmatrix}\begin{Bmatrix}\varphi(t-4)+\varphi(t+1)\\0\\0\end{Bmatrix}$$
$$\begin{Bmatrix}\psi(t)\\0\\0\end{Bmatrix}\begin{Bmatrix}\psi(t-1)\\0\\0\end{Bmatrix}\begin{Bmatrix}\psi(t-2)\\0\\0\end{Bmatrix}$$
$$\begin{Bmatrix}\psi(t-3)+\psi(t+2)\\0\\0\end{Bmatrix}\begin{Bmatrix}\psi(t-4)+\psi(t+1)\\0\\0\end{Bmatrix},$$

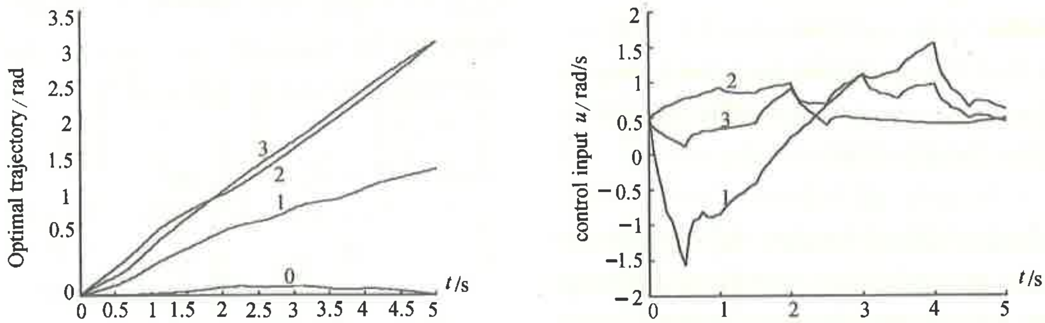
the remaining terms $\{a_i\}_{i=11}^{20}$, $\{a_i\}_{i=21}^{30}$ are obtained by permuting the rows of the above element.

Example 1 The designed target is that the solar arrays completely stretches from the folded state, while the attitude of the spacecraft main body remains unchanged throughout. The initial and final configurations are $x_0 = [0 \quad 0 \quad 0 \quad 0]^T$ and $x_f = [0 \quad \pi/2 \quad \pi \quad \pi]^T$ respectively. The result of simulation is shown in Fig.3 and Fig.4, where Fig.3(a) and Fig.4(a) are the attitude of the main body and the optimal trajectory for the relative angle between two solar arrays (0,1,2,3 corresponding $\varphi_1, \theta_1, \theta_2, \theta_3$), and Fig.3(b) and Fig.4(b) are the optimal control input for solar arrays (1,2,3 corresponding u_1, u_2, u_3). $J(\alpha) = 4.624865$ for scaling function approximating after 2000 iteration, and $J(\alpha) = 4.589580$ for superimposing wavelet function approximating after 200 iteration.



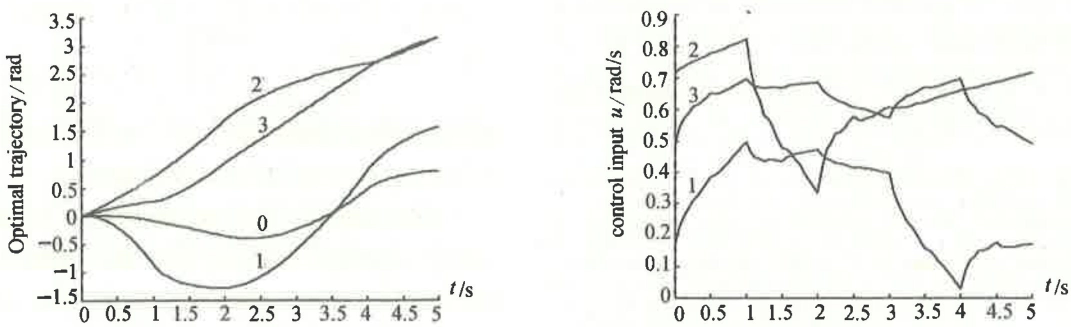
(a) Optimal trajectory (b) Optimal control input

Fig. 3 Approximated by scaling function



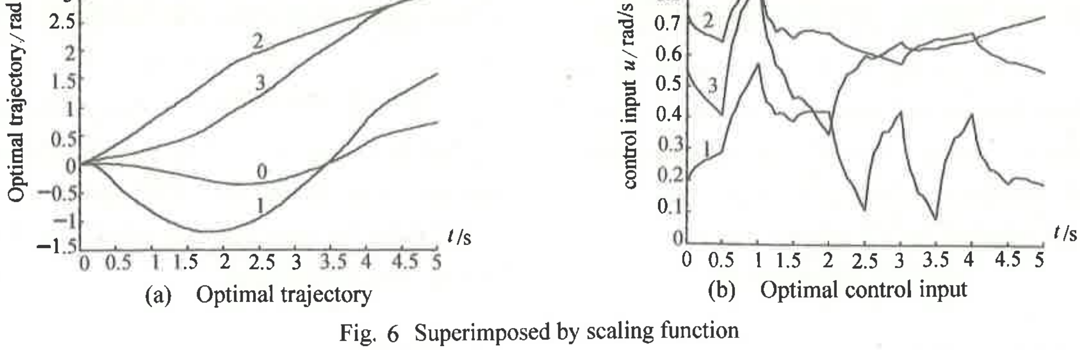
(a) Optimal trajectory (b) Optimal control input

Fig. 4 Superimposed by scaling function



(a) Optimal trajectory (b) Optimal control input

Fig. 5 Approximated by scaling function



(a) Optimal trajectory (b) Optimal control input

Fig. 6 Superimposed by scaling function

Example 2 The designed target is that the solar arrays completely stretches from the folded state, while the attitude of the spacecraft main body rotates $\pi/4$. The initial and final configurations are $x_0 = [0 \quad 0 \quad 0 \quad 0]^T$ and $x_f = [\pi/4 \quad \pi/2 \quad \pi \quad \pi]^T$ respectively. The result of simulation is shown in Fig.5 and Fig.6, where Fig.5(a) and Fig.6(a) are the attitude of the main body and the optimal trajectory for the relative angle between two solar arrays (0,1,2,3 corresponding $\varphi_1, \theta_1, \theta_2, \theta_3$), and Fig.5(b) and Fig.6(b) are the optimal control input for solar arrays (1,2,3 corresponding u_1, u_2, u_3). $J(\alpha) = 9.213321$ for scaling function approximating after 2000 iteration, and $J(\alpha) = 8.636225$ for superimposing wavelet function approximating after 200 iteration.

The algorithm is implemented using Turbo C 2.0 on Pentium 133-MHz PC. The computation time is 4320 sec after 2000 "scale" iteration and 613 sec after 200 "superimposing wavelet" iteration. Two examples are shown, "rough" solution can be obtained by scaling function approximation, and a little computation is needed to refine the solution by the superimposing wavelet function.

5 Conclusions

1) It is a new and useful attempt that the wavelet function is used in multi-resolution approximation, and GA is introduced into optimal control of nonlinear systems. The simulation calculations show that GA is an effective method to solve the optimal control problem of the spacecraft system with stretchable solar arrays.

2) In the application of the above method, we may obtain a scaling function approximation, and then successively refine the solution by superimposing "detail signal". Because of the orthonormality of wavelet and scaling function, little computation is needed to refine the solution. This solving process of hierarchy agrees with the human cognitive rule.

3) Theoretically, binary string is more reasonable than real representations. But for real-valued numerical optimization problem considered in this paper, real representations are a best selection, because they are more comprehensible, more precise, and conducive to faster execution.

4) Differential and continuity condition is not required in GA. Compared with gradient method, it can be applied to a larger range, and is a global optimal algorithm. An optimal control algorithm for the stretching process of spacecraft arrays is proposed in this paper, and it can also be used to solve other optimal control problems. It is worth noticing that GA is not so efficient as the gradient method and considering the present technical level, it is difficult for us to make a real-time online application of the method.

References

1 Liu Yanzhu. The Spacecraft Attitude Dynamics. Beijing: The National Industry Press, 1995 (in Chinese)

2 Ge Xinsheng and Liu Yanzhu. The optimal control of stretching process of spacecraft solar arrays. Space Science Jouranl, 1997, 17(4): 360 – 366(in Chinese)

3 Daubechies I. Orthonormal bases of compactly supported wavelets. Comm. Pure and Appl. Math. 1988, 41(4): 909 – 996

4 Mallat S. Multifrequency channels decomposition of images and wavelet models. IEEE Trans. ASSP, 1989, 37(12): 2091 – 2109

5 Cheng Guoliang. Genetic Algorithm and Its Application. Beijing: The Public Post Press, 1996(in Chinese)

6 Zhao Songnian and Xong Xiaoyun. Wavelet Transform and Wavelet Analysis. Beijing: Electrical Industry Press, 1997(in Chinese)

7 Goldberg D E. Real-coded genetic algorithm, virtual alphabets and bloking. Complex Systems, 1991, 5(2): 139 – 167

本文作者简介

张奇志 1963 年生.副教授.分别于 1986 年和 1989 年在吉林大学获得学士和硕士学位,1996 年在东北大学获博士学位.目前研究兴趣为机器人智能控制和噪声智能控制.

戈新生 1957 年生.教授.目前研究兴趣为多体动力学系统理论及其控制.

刘延柱 1936 年生.教授.博士生导师.目前研究兴趣为多体动力学系统理论及其控制.