Analysis and Improvement of the Nonlinear Tracking-Differentiator

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Abstract: This paper proposes an extensive analysis of the nonlinear tracking-differentiator on the working process and parameters tuning, based on which the source of the overshoot and the cause that coupling to tune are cleared. Thus, an improved switching function is put forward to the developed nonlinear tracking-differentiator, which exhibits an agreeable performance and owns a flexible, uncouple tuning technique. This amelioration renders this unit more applicable to practice than ever.

Key words: nonlinear system; tracking-differentiator; switching function

非线性跟踪-微分器的分析与改进

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摘要: 本文对非线性跟踪-微分器的工作机理和参数意义进行了深入分析,发现了超调产生的根源及参数相互制约的原因,通过对其开关平面函数的改进,改善了信号的品质,得到了灵活解耦的参数整定方法,使非线性跟踪—微分器更具有工程实用意义.

关键词: 非线性系统; 跟踪-微分器; 开关平面函数

1 Introduction

Successive signals and their differentials are commonly used in control projects. Their quality, especially that of the differentials is greatly influenced the property of the whole control system. Therefore, a structure of nonlinear tracking-differentiator is proposed in [1]. It generates tracking signal and its differential by integral method which is very useful to those noncontinual or non-differential signals, for instance, to disturbed signals. It is frequently used in nonlinear control systems and results in good performance[2~5]. But its mechanism is not clear and its tuning is difficult, especially the condition R/δ = constant that causes a new conflict between fast tracking and chatter suppressing. So its application is severely restricted. The analysis and simulation of the unit are made in this paper and a developed structure is introduced, which is proved to be little overshoot, flexible and uncouple tuning.

2 Nonlinear tracking-differentiator and analysis

2.1 Ideal nonlinear tracking-differentiator

Nonlinear tracking-differentiator is such a structure: to an input signal $v(t)$, $x_1(t)$ and $x_2(t)$ yielded, herein $x_1(t)$ tracks the input signal $v(t)$ and $x_2(t) = \dot{x}_1(t)$, in other words that $x_2(t)$ is the extended differential of $v(t)$.

An ideal second order nonlinear tracking-differentiator can be expressed as

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -R \cdot \mathrm{sgn}\left(x_1 - v(t) + \dfrac{|x_2| \cdot x_2}{2R}\right), \end{cases} \tag{1}$$

where R is a real number greater than zero.

Suppose $v(t)$ is a step signal that $v(t) = c, (t > 0)$, the property of the nonlinear tracking-differentiator can be analyzed as follows approximatively:

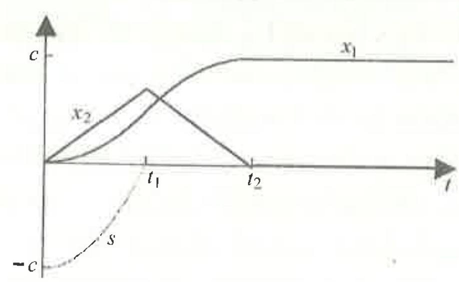

Fig. 1 Property of ideal tracking-differentiator

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Response of $v(t)$ is shown in Fig.1. Defining a switching function of the unit as

$$s = x_1 - c + \frac{|x_2| \cdot x_2}{2R}. \tag{2}$$

Assuming the first time $s = 0$ at time t_1, during $t \in [0, t_1]$ the unit satisfies

$$\begin{cases} x_1(t) = R \cdot t^2/2, \\ x_2(t) = R \cdot t, \quad t \leqslant t_1, \\ s = R \cdot t^2 - c, \end{cases} \tag{3}$$

where

$$t_1 = \sqrt{c/R}, \quad x_1(t_1) = \frac{c}{2}, \quad x_2(t_1) = \sqrt{R \cdot c}.$$

Assuming the unit arrives at steady state $x_1(t_2) = c$ at time t_2, during $t_1 < t \leqslant t_2$ the unit satisfies

$$\begin{cases} s = 0, \\ x_1(t) = \frac{c}{2} + \int_{t_1}^{t_2} (\sqrt{R \cdot c} - R \cdot (t - t_1)) \mathrm{d}t, \\ x_2(t) = \sqrt{R \cdot c} - R \cdot (t - t_1), \\ \quad t_1 < t \leqslant t_2, \end{cases} \tag{4}$$

where

$$t_2 = 2\sqrt{c/R}, \quad [x_1(t_2), x_2(t_2), s]^{\mathrm{T}} = [c, 0, 0]^{\mathrm{T}}.$$

After $t > t_2$, the unit will end the tracking process and be kept in the steady state.

2.2 Nonlinear tracking-differentiator with linear area

To diminish the chatter in the steady state of the unit, the function sgn (s) is substituted with a linear saturation function sat (s, δ) in [1] and Equation (1) turned to be

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -R \cdot \mathrm{sat}\left(x_1 - v(t) + \frac{|x_2| \cdot x_2}{2R}, \delta\right), \end{cases} \tag{5}$$

where

$$\mathrm{sat}(s, \delta) = \begin{cases} \mathrm{sgn}(s), & |s| \geqslant \delta, \\ s/\delta, & |s| < \delta. \end{cases}$$

2.2.1 Tracking property

In Fig.2, the solid line represents the property of an ideal nonlinear tracking-differentiator and dashed line represents the property of those with a linear area. Assuming it is t_0 when the unit satisfies $s = -\delta$, we obtain

$$t_0 = \sqrt{\frac{c - \delta}{R}}, \quad x_{10} = \frac{c - \delta}{2},$$

$$x_{20} = \sqrt{R \cdot (c - \delta)};$$

Differentiate Equation (2) on both sides ($x_2 > 0$) and we obtain that

$$\dot{s} = x_2 + \frac{x_2}{R} \cdot \dot{x}_2. \tag{6}$$

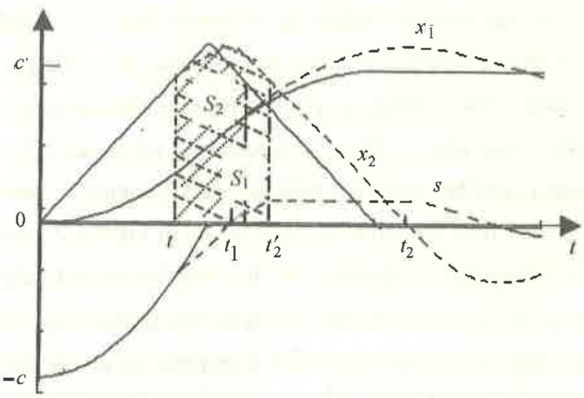

Fig. 2 Property of nonlinear tracking-differentiators

From (5) and (6), we can obtain that

$$\dot{s} = x_2 \cdot \left(1 - \frac{s}{\delta}\right). \tag{7}$$

Assuming Δx_2 is the increment of x_2 during t_0 to t_1, $\dot{s}$ must vary in $(2x_{20}, x_{20} + \Delta x_2)$. Because δ and $\Delta t = t_1 - t_0$ are both very small, Δx_2 must be much less than x_{20}. Considering $\dot{s}$ as a constant and $\Delta x_2 \approx 0$, thus

$$\dot{s} \approx (2x_{20} + x_{20} + \Delta x_2)/2 = 1.5 x_{20}. \tag{8}$$

At time t_1

$$s_{t1} \approx s_{t0} + \dot{s} \cdot \Delta t = -\delta + \dot{s} \cdot \Delta t = 0. \tag{9}$$

So

$$\Delta t = 1.5\delta / x_{20}. \tag{10}$$

Thus, the equations below can be obtained:

$$\begin{cases} x_{2t_1} = x_{20} + \int_{t_0}^{t_1} \dot{x}_2 \mathrm{d}t \approx x_{20} + R \cdot \Delta t/2, \\ \left(\int_{t_0}^{t_1} (-s)\mathrm{d}t \approx \delta/2\right), \\ x_{1t_1} = x_{10} + \int_{t_0}^{t_1} x_2 \mathrm{d}t \approx x_{10} + x_{20} \cdot \Delta t + R \cdot \Delta t^2/4. \end{cases} \tag{11}$$

Simplify (11), then

$$\begin{cases} x_{2t_1} = \sqrt{R \cdot (c - \delta)} + \frac{\delta}{3}\sqrt{\frac{R}{c - \delta}}, \\ x_{1t_1} = \frac{c}{2} + \frac{\delta^2}{16(c - \delta)}. \end{cases} \tag{12}$$

In the same way, when $s = +\delta$ at time t_2', we can obtain that

$$\begin{cases} x_{2t'_2} = \sqrt{R \cdot (c - \delta)}, \\ x_{1t'_2} = \frac{c - \delta}{2} + \frac{4}{3}\delta - \frac{\delta^2}{8(c - \delta)}. \end{cases} \tag{13}$$

After $t > t_2'$, it is easy to find that the switching function will track along with the inside of $s = +\delta$ until it enters the steady state. The curve is similar to that of ideal nonlinear tracking-differentiator during (t_1, t_2).

In the analysis above, it is found that x_2 varied more slowly when it is closer to zero because of the linear area. This feature is conducive to diminishing the chatter, and also shifting the decreasing curve to right. When x_2 reaches zero, overshoot yields because x_1 must be greater than the value of $v(t) = c$. In Fig.2, S_1 and S_2 are respectively defined as the area between x_2 and time axis, t_0 and t_2' of the unit with linear area and the ideal unit, which are shadowed with right ramp line and left ramp line. The overshoot is equal to the difference between S_1 and S_2. Because S_1 can be expressed as

$$S_1 = \frac{4}{3}\delta - \frac{\delta^2}{8(c-\delta)}, \tag{14}$$

and S_2 can be expressed as

$$S_2 = +\delta, \tag{15}$$

the overshoot is nearly equal to

$$\Delta S = S_1 - S_2 = \frac{\delta}{3} - \frac{\delta^2}{8(c-\delta)}. \tag{16}$$

The results of the simulation shows that the real overshoot is 1.5 to 3 times of (16) because the linearization above is conservative and in fact x_2 has delayed to decrease since s tracks inside the boarder of $+\delta$ after $t > t_2$.

2.2.2 Steady property

The steady state of the nonlinear tracking-differentiator with linear area is indicated by x_2 decreasing to zero. In the state x_1 fluctuates nearby $x_1 = c$. Assuming at time a it satisfies $\Delta x_{1a} = x_{1a} - c, x_{2a} = 0$ and $\Delta x_1 = x_1 - c$. Simplifying the analysis with considering $|x_2|$ as a constant and substitute (b) with (a) in the following equations

$$\begin{cases} \Delta\dot{x}_1 = x_2, & (17a) \\ \dot{x}_2 = -\dfrac{R}{\delta}\left(\Delta x_1 + \dfrac{|x_2|\cdot x_2}{2R}\right). & (17b) \end{cases}$$

Transfer it with a Laplace operator p:

$$p(p\Delta x_1(p) - \Delta x_{1a}) = -\frac{R}{\delta}\left(\Delta x_1(p) + \frac{p\Delta x_1(p) - \Delta x_{1a}}{2a}\cdot|x_2|\right). \tag{18}$$

Defining $T = \delta/R$, (18) can be described as

$$\Delta x_1(p) = \frac{\dfrac{\Delta x_{1a}\cdot|x_2|}{2\delta} + p\Delta x_{1a}}{\left(p + \dfrac{|x_2|}{4\delta}\right)^2 + \dfrac{1}{T} - \dfrac{|x_2|^2}{16\delta^2}}. \tag{19}$$

Because

$$|x_2| \ll \sqrt{R\cdot\delta},$$

(19) can be further simplified to

$$\Delta x_1(p) = \frac{\dfrac{\Delta x_{1a}\cdot|x_2|}{2\delta} + p\Delta x_{1a}}{\left(p + \dfrac{|x_2|}{4\delta}\right)^2 + \dfrac{1}{T}}. \tag{20}$$

Inversely transfer (20) with a Laplace operator, thus

$$\Delta x_1(t) = a_1\cdot e^{-\beta(t)\cdot t}\cdot\sin\omega t + \Delta x_{1a}\cdot e^{-\beta(t)\cdot t}\cdot\cos\omega t, \tag{21}$$

where

$$a_1 = \frac{\Delta x_{1a}\cdot|x_2|}{2\delta}\cdot\sqrt{T} = \Delta x_{1a}\cdot\frac{|x_2|}{2\sqrt{R\cdot\delta}} \ll \Delta x_{1a},$$

$$\beta(t) = \frac{|x_2|}{4\delta}, \quad \omega = \sqrt{\frac{1}{T}} = \sqrt{\frac{R}{\delta}}.$$

Ignoring the first term of right side of (21), then

$$\Delta x_1(t) \approx \Delta x_{1a}\cdot e^{-\beta(t)\cdot t}\cdot\cos\omega t. \tag{22}$$

It means that if any disturbance Δx_{1a} to x_1 in the steady state happens, the error of tracking signal will fluctuate at frequency $\omega = \sqrt{R/\delta}$, initial scope Δx_{1a} and decreasing rate $\beta(t) = \dfrac{|x_2|}{4\delta}$.

3 Developed nonlinear tracking-differentiator

When s approaches zero at the first time in which process $|x_2|$ has a large value, the overshoot of x_1 yields for the delayed decrease of x_2 by the action of linear area. To weaken the effect of the linear area in that process but not influence the property of the steady state, a penalty function $e^{|x_2|}$ is multiplied to the switching function s and the advanced nonlinear tracking-differentiator should be expressed as

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -\dfrac{R}{\delta}\cdot\mathrm{sat}(s,\delta), \\ s = e^{|x_2|}\cdot\left(x_1 - v(t) + \dfrac{|x_2|\cdot x_2}{2R}\right). \end{cases} \tag{23}$$

It is easy to understand that the new nonlinear tracking-differentiator has both the tracking property of an ideal unit and the steady property of the one with linear area. In all the processes, the tracking signal and differential signal have an admirable quality.

4 Parameters tuning

In the developed nonlinear tracking-differentiator, it becomes smooth to uncouple tuning of R and δ for the weak relation with each other and clear meanings. Define $T_{td} = 2\sqrt{c/R}$ as the time inertia of the nonlinear tracking-differentiator, which is the time needed for the output tracking signal reaching the value of the step input. If T_{td} can be confirmed at first, R can be tuned as $R = 4c/T_{td}^2$, where c is the amplification of the input. If the input can be expressed as a standard sinuous $v(t) = c \cdot \sin\left(\frac{2\pi \cdot t}{T_0}\right)$, T_{td} should satisfy $T_{td} < \frac{T_0}{4}$ for a good tracking property and R should be tuned as $R > \frac{64c}{T_0^2}$, where T_0 is the period of the sinuous input. If the input is a non-sinuous signal, FFT should be needed to calculate T_0 that should be the period of the highest order harmonic. A great value of R is beneficial to a fast tracking to the variation of the input but it also increases the sensitivity of x_2 to the disturbance. So R should not be too huge if it is great enough to satisfy the tracking property.

Though the linear erea is useful to get rid of the chatter, the frequency of the decreasing fluctuation $\omega = \sqrt{R/\delta}$ declines with the augment of δ. This feature is nuisance to the steady property obviously. So δ should be tuned to a relatively small value if it is great enough to reject the disturbance.

5 Digital simulation

Three examples are given to make the simulation in step $h = 0.001$s of the 4th order Lunge-kutta method. In Fig.3 to Fig.5, the solid line represents the result of simulation of a developed nonlinear tracking-differentiator, dashed line represents that of a unit with a linear area and dotted line represents the input (overlapped with the solid line sometimes).

Example 1 $v(t) = 5(t \geqslant 0), R = 100, \delta = 0.5$, disturbance of $+5\%$ is attached to the input $v(t)$ at $t = 1$s. The result of simulation is shown in Fig.3.

Example 2 $v(t) = 3\sin 12.56t, R = 300, \delta = 0.5$, disturbance of $+5\%$ was attached to the input $v(t)$ at $t = 0.8$s. The result of simulation is shown in Fig.4.

Example 3 $v(t) = 3\sin 9.42t + \sin(12.56t + 0.1), R = 300, \delta = 0.5$, disturbance of $+5\%$ was attached to the input $v(t)$ at $t = 0.8s$. The result of simulation is shown in Fig.5.

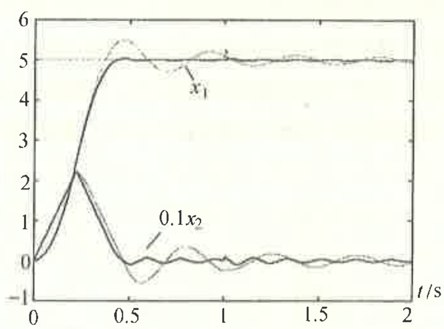

Fig. 3 Result of Example 1

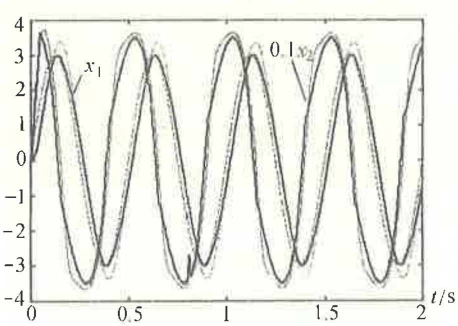

Fig. 4 Result of Example 2

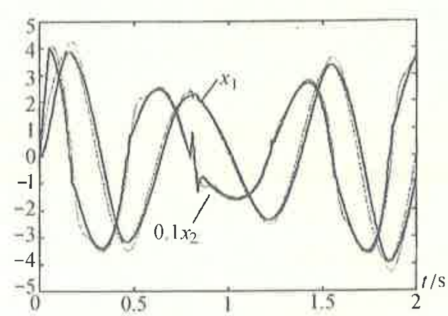

Fig. 5 Result of Example 3

6 Conclusion

Nonlinear tracking-differentiator is a significant unit in the control field, but it is far from being widely used because of the difficulty of tuning. This paper has proposed an improved switching function and got a new structure of the unit whose parameters can be uncoupled to tune. The digital simulation has proved that the new unit has not only an agreeable tracking property but also a good property of rejecting disturbance and chatter.

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