

# Multi-Input, Multi-Output Control System Analysis and Synthesis by Singular Value Decomposition\*

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**Abstract:** A systematic procedure has been developed for the synthesis of robust regulatory control systems comprising multi-input multi-output controllers. The technique is based on singular value decomposition and results in a simple matrix-proportional-plus-integral controller. An interactive computer-aided control system package has also been developed with the systematic technique incorporated. Two design examples are given to demonstrate the systematic nature of the design approach as well as the utility of its computer software.

**Key words:** multivariable control system design; robustness; singular values; frequency domain techniques

## 1 Introduction

Control system design for a chemical process is a very challenging task which confronts control engineers. The activities involved include identifying the model of the chemical plant, analyzing the model, formulating a design problem, determining the specifications and determining the appropriate control law which satisfies the specifications. The above is a highly complex task. The aim of these activities is to obtain a controlled plant which can perform as specified by the process specifications.

In this paper, a novel approach to analyze and design robust process control is given. We will present a unified framework for the analysis of multivariable control systems. The analysis is based on the eigenvalue and singular value decomposition of matrices. An indicator called primary indicators<sup>[1]</sup> is defined which can provide a designer with the information on stability, performance as well as the robustness of a system. The primary indicators of a system are similar to the Generalized Bode Diagrams except that it contains the crucial information on robustness.

We also present a new synthesis method called simple design technique for the design of robust process control systems. The simple design technique is a frequency-domain design method and like many other frequency-domain techniques, it makes intensive use of interactive graphics. As discussed in Arkun<sup>[2]</sup>, one disadvantage of most existing design techniques such as Nyquist Array<sup>[3]</sup>, Characteristic Locus<sup>[4]</sup>, Multivariable Root Locus<sup>[5]</sup>, Method of Inequalities<sup>[6]</sup>, Fractional Representation Approach<sup>[7]</sup>, etc. is that they do not provide adequate assessment on the robust-

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ness aspect. The simple design technique, however, can handle the robustness aspect of a design problem in addition to the aspects of stability and performance. The controller synthesis of this technique is based on the singular value decomposition. Similar attempts on the design and analysis of control systems based on the singular value decomposition were carried out by Bequette and Edgar<sup>[8]</sup>, and Lau et al.<sup>[9]</sup>. In this paper, the systematic nature of the simple design technique is shown and the design procedures are intuitively appealing.

## 2 Robustness in Controller Design

The problem of robustness in control system design has received the attention of many researchers<sup>[10,11,12,2]</sup>. By robustness, we mean the ability of a system to remain stable in the face of model inaccuracies and parameter variations of the system. It should be reminded that no mathematical model of a physical system is exact. However, these models are used for control system analysis and controller synthesis. Hence, the issue of robustness is crucial in the analysis and synthesis procedures for a successful design. During the design of a process control system, the three key aspects of feedback system behaviour are stability, performance and robustness. The assessment of closed-loop stability can be obtained from the Generalized Nyquist Stability Criterion<sup>[13,14]</sup>. Very often, this assessment is based on the nominal model of a system. A robust design would minimize the amount of perturbations in the loci of eigenvalues especially around the critical point. Regarding the performance of a control system, it can be shown that closed-loop performance in terms of input tracking, disturbance rejection and noise rejection depends on the singular values of the return difference and inverse return difference operators. These in turn are related to the singular values of the open-loop gain matrix. Therefore, the maximum and minimum principal gains of the open-loop gain matrix can be used as indicators of the closed-loop performance<sup>[15]</sup>. To obtain a robust system is very important because the prime reason for using feedback control is to combat uncertainties. From a performance point of view, we are concerned with the effect of perturbations on singular values, and from a stability point of view their effect on characteristic values.

It is well-known that the eigenvalues of a normal matrix are relatively insensitive to perturbations<sup>[16]</sup>. Therefore, we aim to obtain a compensated system whose open-loop transfer function matrix is as normal as possible over the frequency range of interest. It has been found that a matrix is normal when its singular values equal the magnitude of its eigenvalues. Therefore, the non-normality of a matrix is associated with the divergence between the singular values and the magnitude of the eigenvalues.

Although normal matrices have nice spectral properties, they constitute only a relatively small set among general matrices. Also, it would be difficult to obtain a compensated system which is normal over the entire frequency range of interest. Therefore, we have to introduce a measure of departure from normality for a general matrix<sup>[17,18,19]</sup>.

**Definition**  $MS(G)$ —A Normality Indicator

Let  $G \in C^{m \times m}$  have a Schur triangular decomposition

$$G = S(D + T)S^*$$

We define

$$MS(G) = \frac{\|T\|_F}{\|G\|_F}, \quad (1)$$

where  $\|\cdot\|_F$  denotes the Frobenius norm of a matrix.

Note that

$$\|G\|_F = \|S(D + T)S^*\|_F = \|D + T\|_F, \quad (2)$$

since the Frobenius norm is invariant under a unitary transformation.

Hence,

$$\|G\|_F^2 = \|D\|_F^2 + \|T\|_F^2 \quad (3)$$

and

$$\|T\|_F^2 = \|G\|_F^2 - \sum_{i=1}^m |g_i|^2. \quad (4)$$

Where  $g_i$  are the eigenvalues of  $G$  ( $i=1, \dots, m$ ). Therefore, although a Schur triangular decomposition is not unique,  $\|T\|_F$  is unique because it is independent of the particular schur triangular decomposition taken.

$MS(G)$  has been defined as a quantitative measure of departure from normality. A matrix is called skew if it is not normal. Hence,  $MS(G)$  is a measure of skewness of a matrix. When the matrix is very skew,  $\|T\|_F$  tends to  $\|G\|_F$  and  $MS(G)$  tends to 1. In the limiting case,  $MS(G) = 1$ . On the other hand,  $MS(G) = 0$  when  $G$  is normal. Therefore, the normality indicator  $MS(G)$  has a range from 0 to 1. As a heuristic, a value of  $MS(G)$  below 0.3 is considered to be very good and a value above 0.7 is considered to be high.

In fact, it can be shown<sup>[1]</sup> that the normality indicator  $MS(G)$  for a matrix  $G \in C^{m \times m}$  can be expressed as

$$MS(G) = \left[ \frac{\sum_{i=1}^m (\sigma_i^2 - |g_i|^2)}{\sum_{i=1}^m \sigma_i^2} \right]^{1/2}, \quad i = 1, \dots, m, \quad (5)$$

where  $G$  have eigenvalues  $g_i$  and singular values  $\sigma_i$  ( $i=1, \dots, m$ ) and they are arranged in descending order of their magnitude.

Therefore, it can be seen that the amount of divergence between the magnitude of the corresponding pairs of eigenvalues and singular values is neatly related to the normality indicator  $MS(G)$ . That is, the skewness of a matrix may be expressed as a summation of divergences of magnitude of the corresponding pairs of eigenvalues and singular values.

### 3 Control System Analysis

Indicators are any computer-generated graphics that are presented to the user during a control system design and analysis session. The well-known Nyquist and Bode diagrams are examples of indicators in the classical frequency-response approach. Such graphs are extremely effective means of representing and conveying information about relationships which can help us better understand the control system being considered. Hence, interactive graphics is a very powerful tool for discovery, understanding, communication and teaching.

In the frequency-response analysis of a multi-input-multi-output (MIMO) feedback control system, it is important to obtain a complete set of indicators which can characterize accurately all the relevant aspects of closed-loop system behaviour. Different indicators of the MIMO system represent different aspects of the behaviour of the system. Essentially, the large set of indicators arises because of the complexity of the multivariable feedback problem. With a large number of these indicators, a user has the problem of observing changes in them at each stage of the design, which can make the user difficult to proceed in a design process. Therefore, it is useful to obtain a set of primary indicators which would specify only the major aspects of the behaviour of the system.

For the analysis of multivariable feedback control system, the primary indicators are defined as the characteristic gains and phases (obtained from eigenvalues) as well as with the principal gains (from singular values) plotted together on the same drawing in Bode form. Fig. 1, 2 and 3 show the primary indicators of a system at different design stage. These indicators are 'primary' in the sense that they contain the most crucial information about the system i. e. stability, performance and robustness. The interpretation of the primary indicators are as follows:

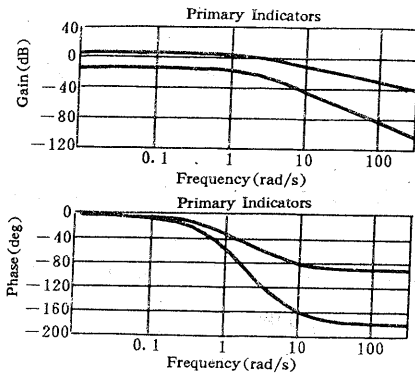


Fig. 1 The primary indicators of the uncompensated system HUT

1) The characteristic gains and phases give an indication of the closed-loop stability of the system. This is accessed by the loci which must satisfy the Generalized Nyquist Stability Criterion.

2) The principal gains give an indication of the performance of the system. The maximum and minimum principal gains of the open-loop gain matrix can be used as indicators of the closed-loop performance.

3) The divergences between the characteristic and principal gains give an indication of the robustness of the system. It has been shown that the divergences between the corresponding pairs of principal gain and characteristic gain loci of the open-

loop transfer function gives information on the normality of the system, and hence information

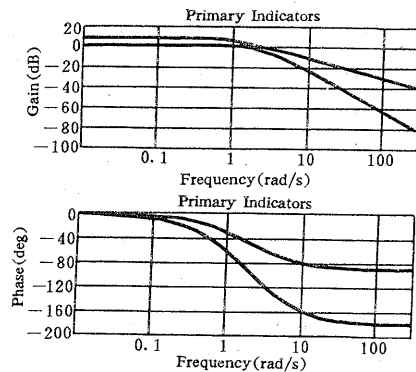


Fig. 2 The primary indicators of HUT after high frequency design stage

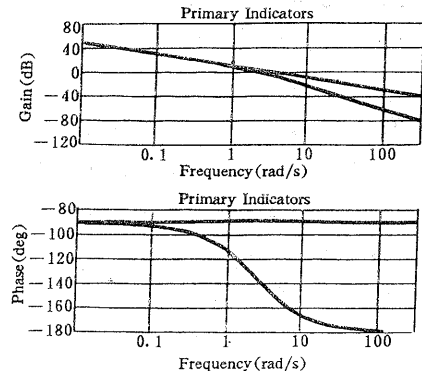


Fig. 3 The primary indicators of HUT after low frequency design stage

on robustness. In the manipulation of the primary indicators, one strives to minimize the degree of divergence between the characteristic and principal gains.

With the definition of the primary indicators, all other indicators are called secondary indicators. Examples are graphs that give us information such as misalignment angles, normality, robust stability, condition numbers etc. on the system. Other secondary indicators include the generalized Nyquist diagram, generalized root locus diagram, Nyquist array and closed-loop step responses of the system. The secondary indicators allow the user to obtain more precise values for other measurements that characterize a control system.

The standard multivariable feedback arrangement is shown in Fig. 4. In the design of multivariable feedback control system, the user is concerned with a suitable compromise between the design objectives: stability, performance and robustness, which may be conflicting in nature. It can be shown that these important aspects may be discussed in terms of the primary indicators of the system. Therefore, the design techniques presented in the next section is based on an appropriate manipulation of the primary indicators in such a way as to give them a required set of properties. The desired properties of the primary indicators are summarized as follows.

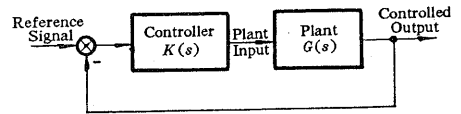


Fig. 4 The standard multivariable feedback arrangement

### 3.1 Closed-Loop Stability

The primary objective of controller design is to obtain a closed-loop feedback system which is stable. Closed-loop stability can be assessed by the generalized Nyquist diagram of the open-loop transfer function. The characteristic loci must satisfy the Generalized Nyquist Stability Criterion. In the intermediate frequency region, the rate at which the deployed gain can be rolled off without violating the Nyquist Stability Criterion determines the phase shift involved as well as the gain bandwidth. We also strive for reasonable stability margin in the intermediate frequency region. Therefore, appropriate gain-phase trade-offs have to be made for an acceptable compromise between stability and performance.

### 3.2 Closed-Loop Performance

The closed-loop system performance in terms of input tracking, disturbance rejection and noise rejection depends on the singular values of the return difference and inverse return difference operators. These can be related to the singular values of the open-loop gain matrix. Therefore, the maximum and minimum principal gains of the open-loop gain matrix can be used as indicators of the closed-loop performance. In the low and intermediate frequency region, the minimum principal gain of the open-loop transfer function has to be suitably large for good tracking and disturbance rejection. This can be achieved by introducing high gains or an integral action. An integral action will also bring about zero steady-state error in closed-loop step responses. In general, high gain will give 'good' transient response, as determined by overshoot and settling time. In the high frequency region, the maximum principal gain should be small for sensor noise

rejection as well as for stability reason. To reduce interaction, one aims to balance up the principal gains at high frequencies and obtain high gains at low frequencies.

### 3.3 Robustness

A robust design should be insensitive to changes in the parameters of the controller as well as the plant nominal model. It has been shown that the robustness of the system can be assessed by the divergences between the corresponding pairs of principal gain and characteristic gain loci of the open-loop transfer function. In the manipulation of gains and phases at both ends of the frequency range, one strives to minimize the degree of divergence between the characteristic and principal gains in the intermediate frequency region.

## 4 A Systematic Approach to Control System Synthesis

### 4.1 Simple Design Technique (SDT)

The Simple Design Technique results in a matrix proportional-plus-integral (PI) controller for a system and the controller synthesis is based on the singular value decomposition. The design technique is based on the manipulation of the primary indicators of the open-loop transfer function at the high and low frequency regions. The two main stages of this technique is summarized below:

1) In the first stage, a simple-structured controller called high-frequency sub-controller (HFS) is constructed. The objective of this sub-controller is to balance up the principal gains (singular values), as well as to align the input and output gain frames of the system at high frequencies.

2) During the second stage, another sub-controller called low-frequency sub-controller (LFS) is formed and it has to be cascaded to the high-frequency sub-controller (Fig. 5). Its

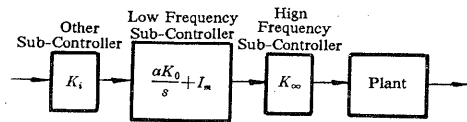


Fig. 5 The structure of the controller using SDT

objective is to balance the principal gains and align the gain frames of the cascaded system at low frequencies. In addition, it attempts to inject gains into the system by incorporating an integral action.

Consider a multivariable system described by the transfer function matrix  $G(s)$ . Let  $G(s) \in \mathbb{C}^{m \times m}$  have an singular value decomposition (SVD) at one specific frequency  $s_a$

$$G(s_a) = Y_a \Sigma_a U_a^* \tag{6}$$

where  $Y_a$  and  $U_a$  are unitary, and  $\Sigma_a = \text{diag}\{\sigma_i^a\}$ ,  $i = 1, \dots, m$ .

Asymptotically, as  $|s| \rightarrow \infty$ ,  $G(s)$  takes the form

$$G(s_\infty) = Y_\infty \Sigma_\infty^a U_\infty^T \tag{7}$$

where  $Y_\infty$  and  $U_\infty$  are real orthogonal matrices, and

$$\Sigma_\infty^a = \text{diag}\left\{\frac{\sigma_i^\infty}{s^{\tau_i}}\right\} \tag{8}$$

where  $\sigma_i^\infty$  are real and  $\tau_i$  are the orders of infinite zeros of  $G(s)$ .

A high frequency sub-controller (HFS) for  $G(s)$  is defined as

$$K_{\infty} = U_{\infty} \Sigma_{\infty}^k Y_{\infty}^T, \quad (9)$$

where  $\Sigma_{\infty}^k = \text{diag}\{k_i^{\infty}\}$ , with  $k_i^{\infty}$  real constants.

When this sub-controller is cascaded with the system, and at high frequencies,

$$G(s_{\infty})K_{\infty} = Y_{\infty} \Sigma_{\infty}^a U_{\infty}^T U_{\infty} \Sigma_{\infty}^k Y_{\infty}^T = Y_{\infty} \Sigma_{\infty}^a \Sigma_{\infty}^k Y_{\infty}^T. \quad (10)$$

The product  $G(s_{\infty})K_{\infty}$  has the following properties:

1) The input and output gain frames are aligned and thus the system is normal at high frequencies.

2) Since the system is normal, the singular values and the magnitude of eigenvalues tend to coincide in a neighbourhood of  $s = \infty$ .

3) Since the system is normal as  $|s| \rightarrow \infty$ , it is robust in the high frequency region.

4) Since the eigenloci behave asymptotically as

$$\frac{\sigma_i^{\infty} k_i^{\infty}}{s^i},$$

the phases of the eigenloci will approach  $\pm r_i \pi / 2$ .

5) The loci of singular values can be balanced up by a suitable choice of  $k_i^{\infty}$  if the orders of infinite zeros,  $r_i$ , are the same.

For obtaining a low frequency sub-controller (LFS), we let  $G_H(s)$  be the transfer function matrix of the system cascaded with the high frequency sub-controller. Let  $G_H(s)$  be real at  $s=0$  and take the form

$$G_H(0) = Y_0 \Sigma_0 U_0^T, \quad (11)$$

where  $Y_0$  and  $U_0$  are orthogonal matrices.

Let

$$K_0 = U_0 \Sigma_0^k Y_0^T \quad (12)$$

and one basic type of low frequency sub-controller (LFS) for  $G(s)$  is defined as

$$K_L(s) = \frac{\alpha K_0}{s} + I_m, \quad (13)$$

where  $\Sigma_0^k = \text{diag}\{k_i^0\}$ ,  $i=1, \dots, m$ , with  $k_i^0$  and  $\alpha$  real constants.

At high frequencies,  $G_H(s)K_L(s)$  tends to  $G_H(s)$  and therefore  $K_L(s)$  will not affect the system in the high frequency region.

As  $|s| \rightarrow 0$ ,

$$G_H(s)K_L(s) \rightarrow G_H(0) \left( \frac{\alpha K_0}{s} + I_m \right), \quad (14)$$

which approximates

$$G_H(0) \frac{\alpha K_0}{s}$$

and has the following properties:

1) Since

$$G_H(0) \frac{\alpha K_0}{s} = Y_0 \Sigma_0 U_0^T U_0 \Sigma_0^k Y_0^T \frac{\alpha}{s} = Y_0 \Sigma_0 \text{diag} \left\{ \frac{\alpha k_i^0}{s} \right\} Y_0^T, \quad (15)$$

the input and output gain frames are aligned and the system is normal at very low frequencies.

2) Since the system is normal, the loci of singular values and magnitude of eigenvalues tend to coincide in a neighbourhood of  $s=0$ .

3) Since the system is normal as  $|s| \rightarrow 0$ , it is robust in the low frequency region.

4) The loci of singular values can be balanced up by a suitable choice of the gain parameters  $k_i^0$ .

5) The phases of the eigenloci of  $G_{\pi}(s) aK_0/s$  will approach  $-\pi/2$  because of the effect of integrator  $1/s$ .

6) The effect of the integral action can be controlled by a suitable choice of the parameter  $a$ .

7) Steady-state error and low frequency interaction are eliminated because of the presence of the integrator term.

The Simple Design Technique is based on a cascaded combination of the high and low frequency sub-controllers, plus some basic types of sub-controller which may be suitable (Fig. 6). The design is broken down into high and low frequency region design. Let  $K_{\infty}$  be the HFS in the high frequency region design and the sub-controllers for the low frequency region design be

$$\left( \frac{aK_0}{s} + I_m \right) K_i$$

$K_i$  is a basic type of sub-controller which may be needed for further improvement of the design results (e. g. a constant gain sub-controller). The final controller  $K(s)$  is the product

$$K(s) = K_{\infty} \left( \frac{aK_0}{s} + I_m \right) K_i \quad (16)$$

The effects of the high and low frequency sub-controllers are combined and each comes into proper operation in the appropriate frequency region. The complete controller becomes a matrix-proportional-plus-integral controller.

## 5 Example

In this section, we demonstrate the above design approach which is based on the singular value decomposition. Two examples from the literature are used and the systematic nature of the simple design technique is illustrated.

### 5.1 Example One

The following example is taken from Hutchison<sup>[20]</sup>. The state space representation of the system is described below and it is referred to as HUT.

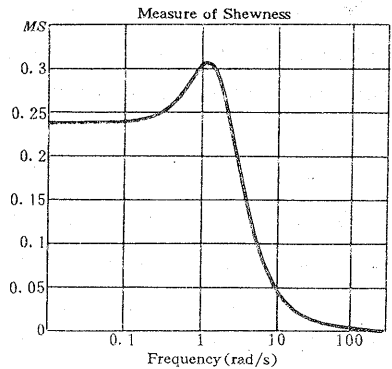


Fig. 6 The normality indicator  $MS(G)$  of HUT after high frequency design stage



$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}. \quad (17)$$

The system is minimum phase with poles at  $-1$ ,  $-2$  and  $-3$ . The suggested bandwidth is around  $\omega=1$ . The primary indicators of the uncompensated system and its measure of skewness are shown in Fig. 1 and 7 respectively. We observe that the system has different roll-off rate at high frequencies. Hence, we can only balance the principal loci at an intermediate frequency. This is usually taken to be around the bandwidth frequency. Therefore, in the first stage of design, a high frequency sub-controller  $K_\infty$  is obtained to balance up the gains at around  $\omega=1$ . The primary indicators of the system after high frequency design is shown in Fig. 2.

$$K_\infty = \begin{bmatrix} 4.2095 & -2.9446 \\ -6.8379 & 7.4703 \end{bmatrix}. \quad (18)$$

The measure of skewness of the system after the first stage of design is shown in Fig. 6. We observe that  $MS(G)$  tends to zero and the system is normal at high frequencies, which is what we expect after stage one. One of eigenloci is rolling off at  $-20\text{db/dec}$  which gives rise to the  $-90$  phase shift. The other eigenloci rolls off at  $-40\text{db/dec}$  and gives rise to the  $-180$  phase shift. Also, we can observe that the corresponding loci the eigenvalues and singular values almost coincide over the entire frequency region.

In the low frequency design stage, a low frequency sub-controller  $K_L(s)$  is used to align the input and output gain frames of the system, balance up the gain and add integral action (Fig. 3).

$$K_L(s) = \left( \frac{K_0}{s} + I_2 \right), \quad (19)$$

where

$$K_0 = \begin{bmatrix} 1.0138 & 0.5394 \\ -0.0704 & 2.2908 \end{bmatrix}. \quad (20)$$

The primary indicators of the final compensated system has shown that the gain divergences is small over the entire frequency range. Hence, we expect the system to be robust. The secondary indicator  $MS(G)$  (Fig. 8) shows that the the measure of skewness is indeed small especially at both ends of the frequency range. Even the peak value at around  $\omega=1$  is around 0.19. The compensated system is found to have fast closed-loop step responses and the interaction is around 20% (Fig. 9 and 10). The frequency response of the closed-loop system is shown in Fig. 11.

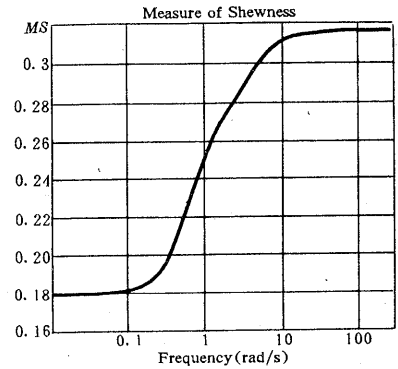


Fig. 7 The normality indicator  $MS(G)$  of the uncompensated system HUT

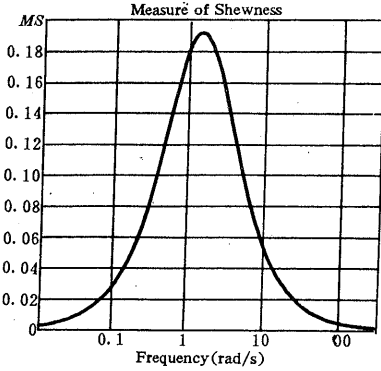


Fig. 8 The normality indicator  $MS(G)$  of HUT after low frequency design stage

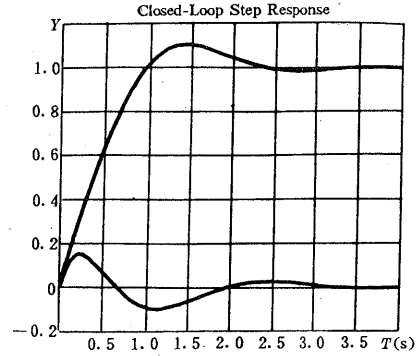


Fig. 9 Closed-loop step response of HUT after SDT (step at input 1)

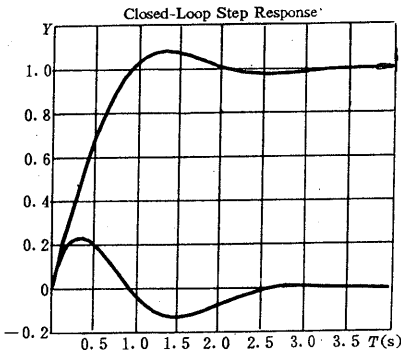


Fig. 10 Closed-loop step response of HUT after SDT (step at input 2)

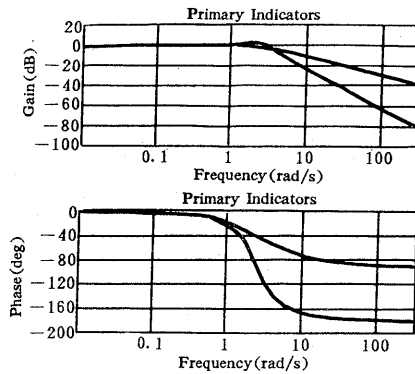


Fig. 11 Frequency response of the closed-loop HUT system

### 5. 2 Example Two

The second example is a binary distillation column<sup>[21]</sup> referred to as Ray in this paper. The concentrations of the heavier component in the top and in a side stream are the control variables. The manipulated variables are the drawoff rates of those streams. The nominal transfer function model of this system is given by

$$G(s) = \begin{bmatrix} \frac{0.7}{1 + 9s} & 0 \\ \frac{2}{1 + 8s} & \frac{0.4}{1 + 6s} \end{bmatrix} \quad (21)$$

The design of the above system was attempted by Arkun et al. <sup>[2]</sup> using an approach called Model Reference Scheme. Here, the simple design technique is used and the design procedures are straightforward and systematic.

#### 5. 2. 1 High Frequency Region Design

The primary indicators of the uncompensated system and its measure of skewness are shown in Fig. 12 and 13 respectively. The uncompensated system is very skew as shown by the gain divergences (Fig. 12) as well as the measure of skewness in Fig. 13. We observe that the system has the same roll-off rate at high frequencies. Therefore, we can balance up the principal loci at

high frequencies. A high frequency sub-controller  $K_\infty$  is obtained to balance up the gains at high frequencies and align the gain frames. The primary indicators of the system after high frequency design is shown in Fig. 14.

$$K_\infty = \begin{bmatrix} 3.1342 & 0 \\ -10.4474 & 3.6566 \end{bmatrix}. \tag{22}$$

The gain loci all coincide at high frequencies and the system is robust at that region. Now, we are ready to move to the next stage.

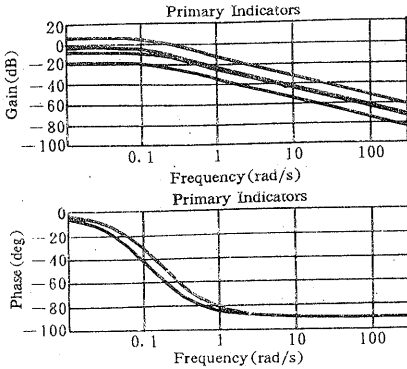


Fig. 12 The primary indicators of the uncompensated system RAY

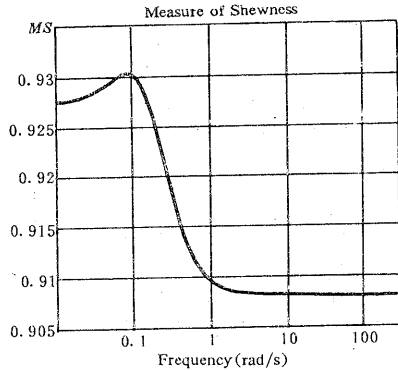


Fig. 13 The normality indicator  $MS(G)$  of the uncompensated system RAY

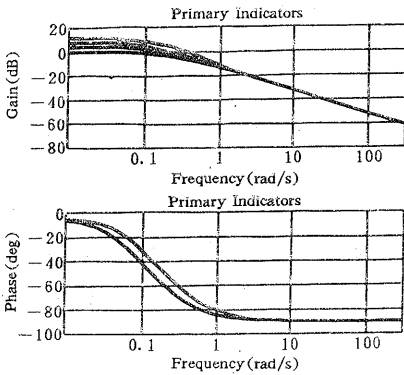


Fig. 14 The primary indicators of RAY after high frequency design stage

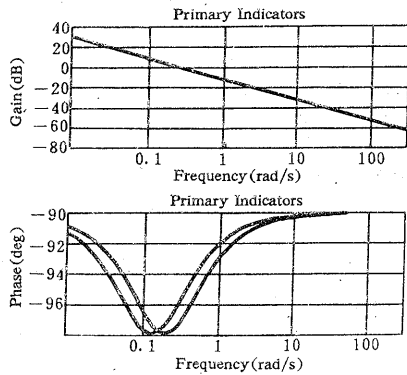


Fig. 15 The primary indicators of RAY after low frequency design stage

### 5. 2. 2 Low Frequency Region Design

In the low frequency design stage, a low frequency sub-controller  $K_L(s)$  is used to align the input and output gain frames of the system, balance up the gain and add an integral action (Fig. 15).

$$K_L(s) = \left( \frac{0.1K_0}{s} + I_2 \right), \tag{23}$$

where

$$K_0 = \begin{bmatrix} 1.4643 & 0 \\ -2.0919 & 2.1965 \end{bmatrix}. \tag{24}$$

The primary indicators of the compensated system has shown that all the four gain loci have coin-

side over the entire frequency range. Hence, we expect the system to be robust. The secondary indicator  $MS(G)$  (Fig. 16) shows that the measure of skewness is very small. Finally, we apply a gain of 35 to both channels and the primary indicators of the final compensated system are shown in Fig. 17. The compensated system is found to have fast closed-loop step responses with no interaction (Fig. 18 and 19).

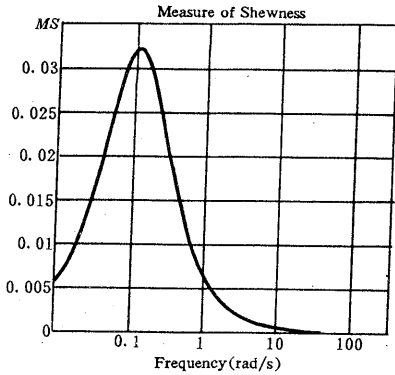


Fig. 16 The normality indicator  $MS(G)$  of RAY after low frequency design stage

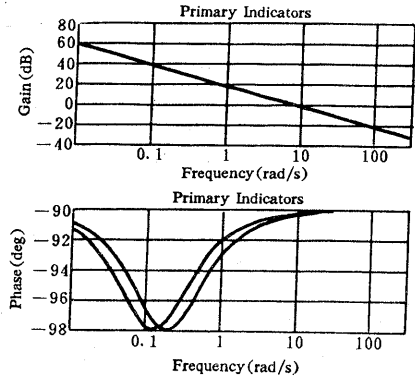


Fig. 17 The primary indicators of the final compensated system

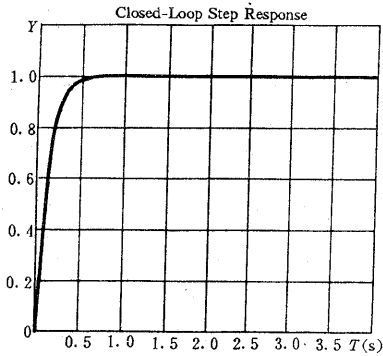


Fig. 18 Closed-loop step response of RAY after SDT (step at input 1)

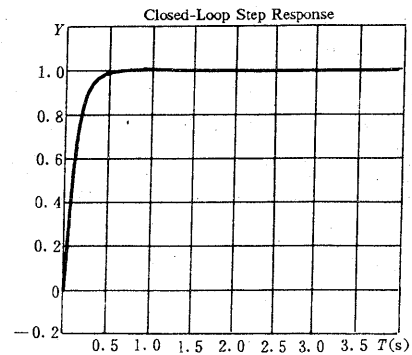


Fig. 19 Closed-loop step response of RAY after SDT (step at input 2)

## 6 Conclusion

The simple design technique is a systematic approach for the design of multivariable feedback control systems in the frequency domain. The technique has been based on the singular value and eigenvalue decompositions and consideration regarding the robustness of the system can be handled in the design framework. The design is carried out in two stages; high frequency design and low frequency design. The alignment of gain frames and balancing up of singular values will ensure that the multivariable system is in the correct orientation at both the high and low frequency regions. Particular attention is paid to the primary indicators at the intermediate frequency region. It is found that the technique can handle a large class of multivariable systems satisfactorily. For systems which require more phase compensation or gain adjustment in an intermediate frequency region than can be provided by the simple design technique, more elaborate steps are required to

provide a satisfactory design. The interested should refer to Pang and MacFarlane<sup>[1]</sup> for more details.

The simple design technique has already been implemented in a computer-aided control system design package called SFPACK<sup>[22]</sup> at the University of Waterloo. The package is interactive and command-driven, and it has the same command syntax as the well-known MATLAB program. The synthesis of the controllers are programmed with a high level command language using the facility of user-defined functions in SFPACK. All the graphs in this paper are generated by the design package. The development of SFPACK has made the design technique more accessible for research and teaching purposes.

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## 采用奇异值分解研究 多输入多输出控制系统的分析及综合

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**摘要:** 本文对于包含多输入多输出控制器的鲁棒控制系统作了系统化的研究. 根据奇异分解法, 文中得到了一个简单的矩阵-比例-积分控制器. 采用上述系统化的技术, 我们研制出一个交互式的控制系统计算机辅助设计软件包. 文中例举了两个例子以说明本设计方案的系统性质及本计算机软件包的有效性.

**关键词:** 多变量控制系统设计; 鲁棒性; 奇异值; 频域法

### 本文作者简介

Professor Grantham K. H. Pang obtained his Ph. D. degree from the University of Cambridge in 1986 for research in multivariable control system design and expert systems. Since then, he has been with the Department of Electrical and Computer Engineering, University of Waterloo. Since 1988, he published more than 50 technical papers and has authored or coauthored three books. Dr. Pang is the co-author of a research monograph, written with Professor A. G. J. MacFarlane, entitled 'An Expert Systems Approach to Computer-Aided Design of Multivariable Systems'. His research interests include expert systems for control system design, intelligent control, neural networks, control theory and computer-aided design. In 1989, he was awarded the ICI Prize for authorship of the best paper on the application of the theory of control published in the *Transaction of Institute of Measurement and Control*.

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