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A Parametrization of Discrete-Time Low-Order Integral Controllers with Computation Delay

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Abstract: For the output feedback design of discrete-time linear system, we consider the integral controller using prediction type reduced-order observer. A state space representation of the doubly coprime factorization is obtained, and then a converient parametrization of all stabilizing controllers is given by use of YJB parametrization method. In this parmetrization, the requirements on the steady state characteristics and the computation delay can be expressed by simple constraint on the free parameter. Redefining free parameter, we can obtain a modified parametrization where the free parameter can be an arbitrary proper and stable rational matrix.

Key words: ntegral controller; digital control; computation delay; discrete-time system; reduced-order observer; doubly coprime factorization.

1 Introduction

In the control system design, effectiveness of integral controllers has been established. Since computers are applied, it is important to develop design methods for discrete-time integral controllers. An issue inherent in digital control is time delay due to computation time of control law. For example, when the time constant of the plant is short and the dynamic order of the plant becomes high, the time delay due to computation time of the computer cannot be neglected.

Mita[1] has proposed novel state feedback designs of discrete-time integral controllers with computation delay. All the state feedback matrices of the integral controllers are determined by use of the feedback gain matrix of the optimal regulator problem for the original plant. Guo et al have proposed a design[2] based on a prediction type Kalman filter which retaining the advantage of Mita's design. However, he obtained controllers are higher order controllers.

Recently, YJB parametrization [3] of all stabilizing controllers using proper and stable rational matrices has been widely used in control system design[4,5]. To obtain the parametrization, we must find matrices satisfying a doubly coprime factorization. The state space representation found

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by Natt et al[6] has been widely used for this purpose. This representation is derived based on a regulator using a full order observer. For the case of using a reduced-order observer, Hippe[7] has obtained a continuous-time modified doubly coprime factorization based on a regulator. Furthermore, Ishihara et al[8] have developed Hippe's result to discrete-time case which is useful to discrete-time low-order controllers design.

For the output feedback design of discrete-time linear system, to reduce the order and guarantee the steady state characteristics and the computation delay, we consider the integral controller using prediction type reduced-order observer. Based on the controller, we can obtain a imple parametrization of all stabilizing integral controllers with computation delay. The free parameter in the parametrization can be used to enhance the basic integral controller by H_2 or H_∞ optimization techniques.

2 Discrete-Time Low-Order Integral Controller with Computation Delay

Consider a discrete-time linear system described by

$$x(t+1) = Ax(t) + Bu(t), \quad y(t) = Cx(t), \tag{2.1}$$

where $x(t) \in R^n$, $u(t) \in R^m$ and $y(t) \in R^m$. The following assumptions are made.

i) (A, B) is controllable and (C, A) is observable.

ii) The matrix CB is nonsingular.

We shall consider a design of integral controllers wich achieves asymptotic tracking to the step reference input signal under one step computation delay. To guarantee such a design, we assume that the matrix

$$E = \begin{bmatrix} A - I & B \\ C & 0 \end{bmatrix} \tag{2.2}$$

is nonsingular which is equivalent to that the system has no zeros at $z=1$.

2.1 The State Feedback Design[1,2]

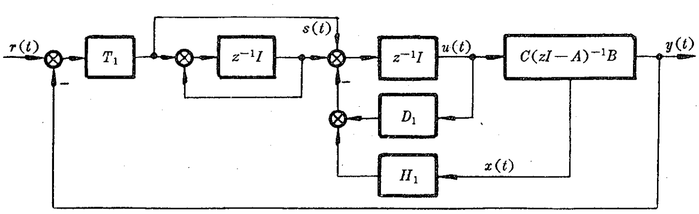

Fig. 1 Structure of the integral controller with computation delay using the state feedback

Consider the integral controller shown in Fig. 1. Let F be an arbitrary matrix such that the matrix $A - BF$ is asymptotically stable. Determine the matrices T_1, H_1 and D_1 so as to satisfy

$$[H_1 + T_1C \quad T_1]E = [FA^2 \quad I + FB + FAB], \quad D_1 = I + FB. \tag{2.3}$$

Then the closed-loop system is internally stable and the transfer function matrix from $r(t)$ to $y(t)$ is given by

$$C_1(z) = z^{-1}C(zI - A + BF)^{-1}B[C(I - A + BF)^{-1}B]^{-1}. \tag{2.4}$$

2.2 The Output Feedback Design

We replace the state required in the state feedback design by an estimate given by the filtering type reduced-order observer

$$\hat{x}(t/t) = Mz(t) + Ny(t), \tag{2.5a}$$

$$z(t+1) = Kz(t) + Hy(t) + TBu(t), \tag{2.5b}$$

where $z(t) \in R^k$ and $n-m \leqslant k \leqslant n$ (see Fig. 2). We choose the matrices M, N, K, H and T such that

$$TA - KT = HC, \quad MT + NC = I, \quad K = TAM, \tag{2.6a}$$

$$\begin{bmatrix} C \\ T \end{bmatrix} [N \quad M] = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}. \tag{2.6b}$$

Then $\hat{x}(t/t) \in R^n$ is a state estimate of the system (2.1).

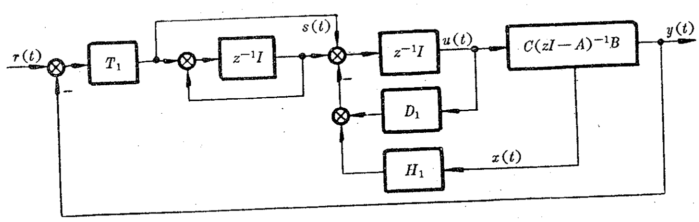

Fig. 2 Structure of the integral controller with computation delay using a filtering type reduced-order observer

For the design of discrete-time low-order integral controller with one step computation delay, we propose to use prediction type reduced-order observer. Combining (2.5) and the state predictor

$$\hat{x}(t+1/t) = A\hat{x}(t/t) + Bu(t), \tag{2.7}$$

we can construct a prediction type reduced-order observer as

$$\hat{x}(t+1/t) = AMz(t) + ANy(t) + Bu(t), \tag{2.8a}$$

$$z(t+1) = Kz(t) + Hy(t) + TBu(t). \tag{2.8b}$$

The structure of the servosystem is shown in Fig. 3.

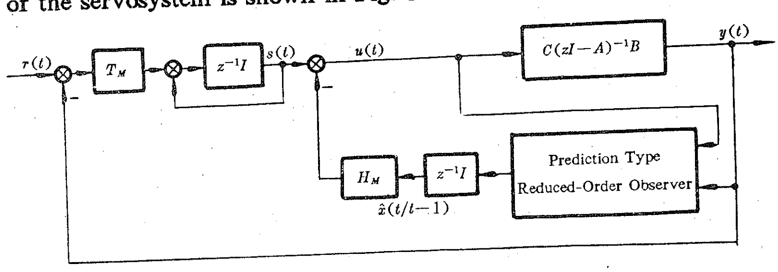

Fig. 3 Structure of the integral controller with computation delay using a prediction type reduced-order observer

The integral controller shown in Fig. 2 can be designed by the state feedback design and the filtering type reduced-order observer (2.5). This controller can be realized by using the prediction type reduced-order observer as shown in Fig. 3 where the matrices H_M and T_M are determined by

$$[H_M \quad T_M]E = [FA \quad I + FB]. \tag{2.9}$$

Because we can easily show that

$$T_1 = T_M, \quad H_1 = H_M A, \quad D_1 = H_M B. \tag{2.10}$$

For the structure shown in Fig. 3, we have

$$u(t+1) = s(t+1) - H_M \hat{x}(t+1/t), \tag{2.11}$$

$$s(t+1) = s(t) + T_M[r(t) - y(t)]. \tag{2.12}$$

2.3 The Controller Transfer Function Matrix

We define the matrices J_M and L_M by the relation

$$E\begin{bmatrix} J_M \\ L_M \end{bmatrix} = \begin{bmatrix} A^2N \\ I + CAN \end{bmatrix}, \tag{2.13}$$

which is clearly dual to (2.9). The solutions of the matrix equations (2.9) and (2.13)are

$$H_M = F + [C(I - A + BF)^{-1}B]^{-1}C(I - A + BF)^{-1}, \tag{2.14a}$$

$$T_M = [C(I - A + BF)^{-1}B]^{-1}, \tag{2.14b}$$

$$J_M = AN + (I - A + ANC)^{-1}B[C(I - A + ANC)^{-1}B]^{-1}, \tag{2.15a}$$

$$L_M = [C(I - A + ANC)^{-1}B]^{-1}. \tag{2.15b}$$

Result 2.1 Consider the integral controller shown in Fig. 3. The controller transfer function matrix from $y(t)$ to $-u(t)$ can be expressed in left factorization form as

$$C_M(z) = \{I + z^{-1}H_M[AM(zI - K)^{-1}T + I]B\}^{-1} \cdot \{(z-1)^{-1}T_M + z^{-1}H_M A[M(zI - K)^{-1}H + N]\} \tag{2.16a}$$

and in right factorization form as

$$C_M(z) = [(z-1)^{-1}L_M + F(zI - A + BF)^{-1}J_M][I + C(zI - A + BF)^{-1}J_M]^{-1}. \tag{2.16b}$$

Outline of proof The expression (2.16a) can easily obtained from (2.1), (2.8), (2.11) and (2.12). It is easily verified that

$$H = TAN, \tag{2.17a}$$

$$A[M(zI - K)^{-1}H + N] = [AM(zI - K)^{-1}T + I]AN, \tag{2.17b}$$

$$AM(zI - K)^{-1}T + I = z(zI - A + ANC)^{-1} \tag{2.17c}$$

from (2.6). Using the above relations and the expressions (2.14) and (2.15), we can derive the expression (2.16b) from (2.16a).

3 Parametrization of Integral Controllers

First, we can obtain the following doubly coprime factorization by using the expressions (2.16).

Result 3.1 Let $G(z)$ be the plant transfer function matrix. The $G(z)$ and $C_M(z)$ can be expressed by coprime factorizations using proper and stable rational function matrices as

$$G(z) = C(zI - A)^{-1}B = ND^{-1} = \overline{D}^{-1}\overline{N}, \tag{3.1a}$$

$$C_M(z) = N_C D_C^{-1} = \overline{D}_C^{-1}\overline{N}_C. \tag{3.1b}$$

Where

$$N = C(zI - A + BF)^{-1}B[C(I - A + BF)^{-1}B]^{-1}, \tag{3.2a}$$

$$D = [I - F(zI - A + BF)^{-1}B][C(I - A + BF)^{-1}]^{-1}, \tag{3.2b}$$
$$\bar{D} = z^{-1}[C(I - A + ANC)^{-1}B]^{-1}C[zI - A - AM(zI - K))^{-1}TA]N, \tag{3.2c}$$
$$\bar{N} = z^{-1}[C(I - A + ANC)^{-1}B]^{-1}C[AM(zI - K)^{-1}T + I]B, \tag{3.2d}$$
$$N_C = z^{-1}[I + (z-1)F(zI - A + BF)^{-1}(I + ANC)(I - A + ANC)^{-1}B], \tag{3.3a}$$
$$D_C = z^{-1}(z-1)[C(I - A + ANC)^{-1}B + C(zI - A + BF)^{-1}(I + ANC)(I - A + ANC)^{-1}B], \tag{3.3b}$$
$$\bar{D}_C = z^{-1}(z-1)\{C(I - A + BF)^{-1}B + z^{-1}C(I - A + BF)^{-1}(I + BF)[AM(zI - K)^{-1}T + I]B\}, \tag{3.3c}$$
$$\bar{N}_C = z^{-1}\{I + z^{-1}(z-1)C(I - A + BF)^{-1}(I + BF)A[M(zI - K)^{-1}H + N]\}. \tag{3.3d}$$

In addition, there exist the following relation of doubly coprime factorization.

$$\begin{bmatrix} \bar{D}_C & \bar{N}_C \\ -\bar{N} & \bar{D} \end{bmatrix} \begin{bmatrix} D & -N_C \\ N & D_C \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}. \tag{3.4}$$

Outline of proof The expressions (3. 1a) and (3. 2) are easily proved[2,5]. The expressions (3. 1b) and (3. 3) can be obtained from (2. 16). The relation (3. 4) is decomposed into

$$\bar{D}_C D + \bar{N}_C N = I, \tag{3.4a}$$
$$\bar{N} D = \bar{D} N, \tag{3.4b}$$
$$\bar{D}_C N_C = \bar{N}_C D_C, \tag{3.4c}$$
$$\bar{N} N_C + \bar{D} D_C = I. \tag{3.4d}$$

It is clear from (3. 1) that identities (3. 4b) and (3. 4c) hold. The identity (3. 4d) is dual to the identity (3. 4a) completely. Using the expressions (3. 3c), (3. 2b), (3. 3d), (3. 2a), (2. 16b) and (2. 17) and the following relation

$$ANC - BF = (zI - A + ANC) - (zI - A + BF), \tag{3.5}$$

we can directly prove that the identity (3. 4a) hold.

From the YJB parametrization, the above result can be used to parametrize all the stabilizing controllers as

$$C(z) = (\bar{D}_C - Q\bar{N})^{-1}(\bar{N}_C + Q\bar{D}) \tag{3.6a}$$
$$= (N_C + DQ)(D_C - NQ)^{-1}, \tag{3.6b}$$

where free parameter Q is proper and stable rational matrix. For the controller described by (3. 6), the sensitivity matrix at the input of the plant can be expressed as

$$\Sigma(z) = D(\bar{D}_C - Q\bar{N}). \tag{3.7}$$

The following result provides a simple parametrization of the stabilizing integral controllers with computation delay.

Result 3. 2 In the parametrization (3. 6), define a class of the free parameter as

$$Q = z^{-2}(z-1)Q_1 \tag{3.8}$$

where Q_1 is arbitrary proper and stable rational matrix. Then the corresponding controllers ensure the integral action and admit one step computation delay.

Proof Note that the integral action is guaranteed if and only if the sensitivity matrix

(3. 7) satisfies $\Sigma(1)=0$. It follows from (3. 3c) that $\bar{D}_C(1)=0$. Since we have assumed that the plant has no zero at $z=1$, the matrix $\bar{N}$ defined (3. 2d) has no zero at $z=1$. It is possible but not always that $D(1)=0$. If the free parameter Q satisfies $Q(1)=0$, the sensitivity matrix (3. 7) always satisfies $\Sigma(1)=0$. To admit one step computation delay, $C(z)$ should be strictly proper, i. e., $C(\infty)=0$. It can be verified that $C(\infty)=D(\infty)Q(\infty)D_{\bar{C}}^{-1}(\infty)$. It follows easily from (3. 2b) and (3. 3b) that $D(\infty)$ and $D_{\bar{C}}^{-1}(\infty)$ are nonsingular. Consequently, $C(\infty)=0$ is equivalent to $Q(\infty)=0$. These requirements for Q are clearly satisfied if Q is expressible as in (3. 8).

The above result shows that the requirements on the steady state characteristics and the computation delay can be expressed by simple constraints on the free parameter in the parametrization.

4 An Example

We consider the following case

$$A=\begin{bmatrix}0&1&0\\0&0&1\\1&2&3\end{bmatrix},\quad B=\begin{bmatrix}0\\0\\1\end{bmatrix},\quad C=[0\quad 1\quad 0] \tag{4. 1}$$

for the system described by (2. 1). For the prediction type reduced-order observer described by (2. 8), we can choose the matrices M, N, K, H and T as

$$M=\begin{bmatrix}1&0\\0&0\\0&1\end{bmatrix},\quad N=\begin{bmatrix}0\\1\\3\end{bmatrix},\quad K=\begin{bmatrix}0&0\\1&0\end{bmatrix},\quad H=\begin{bmatrix}1\\2\end{bmatrix},\quad T=\begin{bmatrix}1&0&0\\0&3&1\end{bmatrix} \tag{4. 2}$$

which satisfy the relation (2. 6). Choosing the state feedback gain matrix F for the regulator problem as $F=[1\ 2\ 3]$, we can obtain that

$$N=\bar{N}=1/z^2,\quad D=\bar{D}=(z^3-3z^2-2z-1)/z^3, \tag{4. 3}$$

$$N_C=\bar{N}_C=(55z^3-20z^2-19z-15)/z^4,\ D_C=\bar{D}_C=(z^3+3z^2+11z-15)/z^3. \tag{4. 4}$$

which satisfy the expression (3. 1) and the relation (3. 4). Using the parametrization (3. 6) and the constraint condition (3. 8) on the free parameter Q, we obtain all the stabilizing controllers as

$$C(z)=\frac{z(55z^3-20z^2-19z-15)+q_1(z^4-4z^3+z^2+z+1)}{z[z(z^3+3z^2+11z-15)-(z-1)q_1]}. \tag{4. 5}$$

which can ensure the integral action and admit one step computation delay, where q_1 is proper and stable rational polynomial.

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离散时间低阶积分控制器考虑运算延迟的参数化形式

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摘要：对于离散时间线性系统的输出反馈设计，我们考虑使用预测型降维观测器的积分控制器. 在找到了一个二重互质分解的状态空间表达式后，通过使用YJB参数化方法，给出了所有稳定化控制器的一个方便的参数化形式. 在这个参数化表达式中，关于稳态特性和运算延迟的要求被描述为对自由参数的约束. 重新定义这个自由参数，我们可以获得一个改进的参数化形式，而其中的自由参数可以是一个任意真的和稳定的有理函数矩阵.

关键词：积分控制器；数字控制；运算延迟；离散时间系统；降维观测器；二重互质分解

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