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Fault-Tolerant Pole Assignment for Multivariable System Using a Fixed State Feedback*

YE Yinzhong
(Research Institute of Automatic Control, East China University of Chemical Technology·Shanghai, 200237, PRC)

Abstract: The problem of fault-tolerant pole assignment for multivariable system against the actuator failure is investigated. A design procedure is proposed based on the n-linear characteristic coefficient system and the parameter space design method. With such a procedure a state feedback law can be achieved to locate all the closed-loop poles in a prescribed region for a given plant under various actuator failure modes.

Key words: fault-tolerant pole assignment; multivariable system; n-linear characteristic coefficient system; parameter space design

1 Introduction

In this paper, a new design procedure for multivariable state feedback system is proposed by which the fault-tolerant control against the actuator failures is achieved. This stems from locating the closed-loop poles in a prescribed region on the s-plane (i. e. the pole region assignment) using a dyadic feedback structure. The design problem is formulated below.

Consider the system in Fig. 1, where the plant equation is

$$\dot{x} = Ax + Bu, \tag{1}$$

$x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$. Assume $\{A, B\}$ is a controllable pair. The state feedback is applied to the system

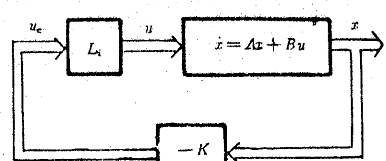

Fig. 1 Multivariable feedback system with actuator failure

$$u_c = -Kx, \tag{2}$$

with

$$u = L_i u_c, \tag{3}$$

where

$$L_i = \mathrm{diag}(l_1, l_2, \cdots, l_m) \tag{4}$$

is the actuator failure matrix, and

$$l_j = \begin{cases} 1, & \text{actuator } j \text{ 'normal'}, \\ 0, & \text{actuator } j \text{ 'failure'}. \end{cases}$$

In practise, L_i may be viewed as an element of the set $\mathscr{L}$, which includes all possible actuator failure modes of interest. Thus

$$L_i \in \mathscr{L} = \{L_0, L_1, \cdots, L_N\}.$$

Let an n-vector Λ denote all closed-loop poles. It is known that the system will operate satis-

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factorily if Λ lie in a region Γ_s' on the s-plane, as shown in Fig. 2. For convenience, a closed region Γ_s' in Fig. 3 is substituted for Γ_s'. In effect, as Γ_s' is on the left half of s-plane with at least v_1 far from the imaginary axis, the system with $\Lambda \in \Gamma_s$ is stable and possesses certain stability margin. Besides, the design requirement on dynamic response will be guaranteed by the boundary of the hyperbola.

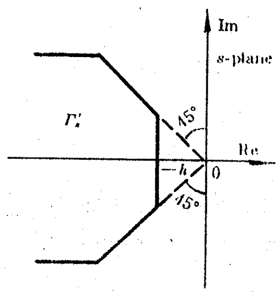

Fig. 2 A feasible region Γ_s' of closed-loop poles on s-plane

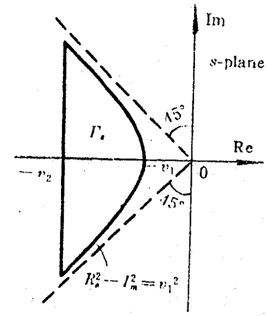

Fig. 3 The alternative to Γ_s' in fault-tolerant pole assignment problem

The traditional pole assignment problem, i. e. for $L_i = I$ in Fig. 1, finding a feedback matrix K to locate Λ in a given point within Γ_s', was nearly closed due to wide investigation. Recently, the requirement on control system reliability has provided incentives for taking account of the actuator failure in system Fig. 1, e. g. see a study from LQR theory by E. Shimemura and M. Fujita (1985).

This paper will give a new design procedure for such problem. Some preliminary development is introduced below.

2 Development

Definition 1 Consider a sysetm

$$\begin{cases} \dot{x} = Ax + Bu, \\ y = Cx, \end{cases} \tag{5}$$

it is called n-linear characteristic coefficient system of K, if the coefficients in its closed-loop characteristic polynomial under an output feedback $u = -Ky$ are all the linear functions of K.

The n-linear characteristic coefficient system was first studied by M. Tarokh (1980). An error in his study was corrected by G. K. G. Kolka with the lemma below.

Lemma 1 (G. K. G. Kolka, 1985) The system (5) is n-linear characteristic coefficient system of K, if and only if

$$\text{rank } C\text{adj}(sI - A)B \leqslant 1. \tag{6}$$

Lemma 2 If expressing $\text{adj}(sI - A)$ as

$$\text{adj}(sI - A) = H_{n-1}s^{n-1} + H_{n-2}s^{n-2} + \cdots + H_0, \quad H_{n-1} = I, \tag{7}$$

then

$$A^i = \text{span}\{H_{n-1-i}, H_{n-i}, \cdots, H_{n-1}\}, \quad i = 0, 1, \cdots, n-1, \tag{8}$$

with the unity spanning coefficient of H_{n-1-i}.

Proof Let

$$\det(sI - A) = s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0.$$

It is easy to prove that $\mathrm{adj}(sI-A)$ can be equivalently expressed as

$$\mathrm{adj}(sI - A) = A^{n-1} + (s + a_{n-1})A^{n-2} + \cdots + (s^{n-1} + a_{n-1}s^{n-2} + \cdots + a_1)A^0. \tag{9}$$

Simply comparing (7) with (9) gives

$$\begin{bmatrix} 1 & a_{n-1} & a_{n-2} & \cdots & a_1 \\ & 1 & a_{n-1} & \cdots & a_2 \\ & & 1 & \cdots & a_3 \\ & & & \ddots & \vdots \\ & & & & 1 \end{bmatrix} \begin{bmatrix} A^{n-1} \\ A^{n-2} \\ A^{n-3} \\ \vdots \\ A^0 \end{bmatrix} = \begin{bmatrix} H_0 \\ H_1 \\ H_2 \\ \vdots \\ H_{n-1} \end{bmatrix}.$$

Thus lemma 2 results immediately.

Generally, each coefficient of the characteristic polynomial of system (1) under state feedback (2,3) is non-linear on K, containning $n\times m$ parameters to be determined. This leads to a formidable-treating task in fault-tolerant pole assignment. However, this may be simplified using n-linear characteristic coefficient system. Consider a state feedback of dyadic structure

$$K = -fk^{\mathrm{T}}, \tag{10}$$

$f\in \mathrm{R}^m$ is a give vector, $k\in \mathrm{R}^n$ is the feedback gain to be determined. Then system (1,2,3) becomes

$$\dot{x} = (A - BL_i fk^{\mathrm{T}})x. \tag{11}$$

Theorem 1 For any given f, system (11) is n-linear characteristic coefficient system of k.

Proof System (11) is equivelent to the following single-input system in the characteristic polynomial

$$\begin{cases} \dot{x} = Ax + BL_i fu', \\ y = x, \\ u' = -k^{\mathrm{T}}y. \end{cases} \tag{12}$$

Comparing (12) with (5), this theorem holds obviously from lemma 1.

For $L_i\in \mathscr{L}$, denote

$$l_{f_i} = L_i f \tag{13}$$

and

$$T_i = [H_0Bl_{f_i}, H_1Bl_{f_i}, \cdots, H_{n-1}Bl_{f_i}]^{\mathrm{T}}. \tag{14}$$

Theorem 2 Let $\{A, Bl_{f_i}\}$ be controllable, then $\mathrm{rank}T_i=n$.

Proof Consider

$$\Phi_c = [A^0Bl_{f_i}, A^1Bl_{f_i}, \cdots, A^{n-1}Bl_{f_i}],$$

by lemma 2

$$\Phi_c = [\mathrm{span}(H_{n-1})Bl_{f_i}, \mathrm{span}(H_{n-2}, H_{n-1})Bl_{f_i}, \cdots, \mathrm{span}(H_0, \cdots, H_{n-1})Bl_{f_i}].$$

Clearly $\mathrm{rank}\Phi_c=\mathrm{rank}T_i$.

Theorem 3 For $L_i\in \mathscr{L}$, let

$$\det(sI - A + Bl_{f_i}k^{\mathrm{T}}) = s^n + p_{n-1}(k)s^{n-1} + \cdots + p_0(k)s^0$$

$$=[p_0(k),p_1(k),\cdots,p_{n-1}(k)]\begin{bmatrix}s^0\\ s_1\\ s^{n-1}\end{bmatrix}+s^n=p(k)^{\mathrm{T}}\mathscr{S}+s^n \tag{15}$$

and

$$\det(sI-A)=s^n+a_{n-1}s^{n-1}+\cdots+a_0s^0$$

$$=[a_0,a_1,\cdots,a_{n-1}]\begin{bmatrix}s^0\\ s_1\\ s^{n-1}\end{bmatrix}+s^n=a^{\mathrm{T}}\mathscr{S}+s^n, \tag{16}$$

then

$$p(k)=a+T_ik.$$

Proof

$$\det(sI-A+Bl_{f_i}k^{\mathrm{T}})=\det(sI-A)+k^{\mathrm{T}}\mathrm{adj}(sI-A)Bl_{f_i}$$

$$=\det(sI-A)+k^{\mathrm{T}}(H_{n-1}s^{n-1}+H_{n-2}s^{n-2}+\cdots+H_0s^0)Bl_{f_i}. \quad \text{(by (7))}$$

Thus

$$p_j(k)=a_j+k^{\mathrm{T}}H_jBl_{f_i},\quad j=0,1,\cdots,n-1.$$

Note that $k^{\mathrm{T}}H_jBl_{f_i}=[H_jBl_{f_i}]^{\mathrm{T}}k$, (16) results.

3 Fault-Tolerant Pole Assignment

3.1 Feasible Region Γ_p in $\mathscr{P}$-Space

Consider (15), where $p(k)\in R^n\triangleq\mathscr{P}$. $\mathscr{P}$ is the parameter space of $p(k)$.

Definition 2 A region $\Gamma_p\in\mathscr{P}$ is called the feasible region of p, if and only if for any $p\in\Gamma_p$, the zeros of (15) lie in Γ_s, i.e.

$$\Lambda\in\Gamma_s\Leftrightarrow p\in\Gamma_p\subset\mathscr{P}. \tag{17}$$

By (17) it is possible to carry out the fault-tolerant pole assignment in $\mathscr{P}$-space rather than on s-plane. Such designing idea is named parameter space method. Using this design method, J. Ackermann (1980, 1984) studied the robust controller design problem against sensor failures. Y. Z. Ye et al (1987) developed a designing scheme for MIMO stability fault-tolerant controller in the case of both actuator and sensor failures. A detailed investigation on the property and the construction of the feasible region Γ_p in $\mathscr{P}$-space is beyond the scope of this paper. There were extensive researches on this respect in[4,7]. Actually, a graphical description for Γ_p is available when $n\leqslant 3$. Fig. 4 shows the Γ_p for $n=2$, which is based on $v_1=1$ and $v_2=5$.

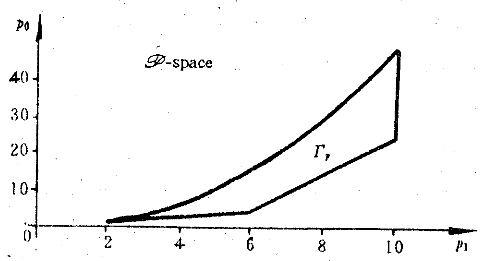

Fig. 4 The parameter space Γ_p for the second-order system

3.2 Designing in $\mathscr{K}$-Space

So far the fault-tolerant pole assignment has been reduced to finding a feedback gain k in (10) for a given f, such that $p\in\Gamma_p$ for any $L_i\in\mathscr{L}$. Note that when $\{A,Bl_{f_i}\}$ is controllable, (16) defines a one-to-one mapping relationship between $\mathscr{P}$-space and $\mathscr{K}$-space. Denote the corresponding feasible region of k for L_i in $\mathscr{K}$-space by Γ^i_k, that is

$$p(k) \in \Gamma_p \subset \mathscr{P} \Leftrightarrow k \in \Gamma_k^i \subset \mathscr{K}, \quad \text{for } L_i, \tag{18}$$

then Γ_k^i can easily be constructed from Γ_p and (16). In fact these two are of the same dimension. It is obvious from (17) and (18) that

Theorem 4 For all $L_i \in \mathscr{L}, \Lambda \in \Gamma_s$ if and only if

$$k \in \bigcap_{i=0}^{N} \Gamma_k^i. \tag{19}$$

3.3 Design Procedure

The following design procedure for fault-tolerant pole assignment is proposed based on the preceding development.

Step 1 For a given Γ_s on s-place, define the feasible region Γ_p in $\mathscr{P}$-space. It is possible to constructe graphically Γ_p when $n \leqslant 3$.

Step 2 Determine f in (10) such that $\{A, BL_i f\}$ is controllable for all $L_i \in \mathscr{L}$. Work out T_i for $i=0,1,\cdots, N$ from (7) and (14).

Step 3 Construct the corresponding feasible region Γ_k^i $(i=0,1,\cdots,N)$ in $\mathscr{K}$-space from (16).

Step 4 Finally, any k within the intersection in (19) will ensure the availability of fault-tolerant pole assignment.

Remark 1 For a system with $n>3$, only analytical description on Γ_p is possible. Thus a suitable CAD program is needed to carry out the design procedure above.

Remark 2 In step 2, a fixed f needs to be specified such that $\{A, BL_i f\}$ is controllable for all $L_i \in \mathscr{L}$. When $\{A, BL_i\}$ is controllable, there does exist an m-vector f such that for a given L_i, $\{A, BL_i f\}$ is controllable according to W. M. Wonham (1967). However, the condition for the existence of such a common f for all $L_i \in \mathscr{L}$ is still open up to this point. As such problem has much to do with the fault-tolerability condition, here it has not been involved for the time being.

Remark 3 In certain cases, an empty intersection in (19) may result. Again this is related to the problem of fault-tolerability mentioned above. When this happens, a relaxed Γ_s is needed to lead to a wider feasible region Γ_p in $\mathscr{P}$-space and so Γ_k^i in $\mathscr{K}$-space from the viewpoint of designing.

Remark 4 If zero element is concerned in $\mathscr{L}$, which means that the full failure of m actuators is under consideration, $T_i=0$ for certain L_i and by (16) $p(k)$ is independent on k. In this case it is necessary for the open-loop poles of system (1) to lie in Γ_s in order to achieve fault-tolerant pole assignment.

4 Illustrative Example

In system Fig. 1, consider

$$A = \begin{bmatrix} -5 & 4 \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & -5 \\ 1 & 1 \end{bmatrix}.$$

and

$$\mathscr{L}=\left\{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix},\begin{bmatrix}0 & 0\\0 & 1\end{bmatrix},\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}\right\}.$$

A Γ_s like that in Fig. 3 is taken with $v_1=1$, $v_2=20$. The corresponding Γ_p is similar to that in Fig. 4. It is easy to prove that $\{A, Bl_{f_i}\}$ is controllable for any $L_i\in\mathscr{L}$ when $f=(1,1)^T$, Therefore Γ_k^i $(i=0,1,2)$ can be constructed from (16). A local inspection on them is given in Fig. 5, which shows also the common intersection of these three feasible regions in $\mathscr{K}$-space. As a result, it suffices to chose k within the hatched region. For instance,

$$k=\begin{bmatrix}2\\10\end{bmatrix}$$

and so
$$K=fk^T=\begin{bmatrix}2 & 10\\2 & 10\end{bmatrix}$$

is one of the feasible state feedback laws. In fact, it is easy to prove that with such scheme being adopted, Λ will be:

$(-6, -17)^T$ for L_0,

$(-4, -1)^T$ for L_1,

$(-5.315, -17.685)^T$ for L_2.

All of them lie in Γ.

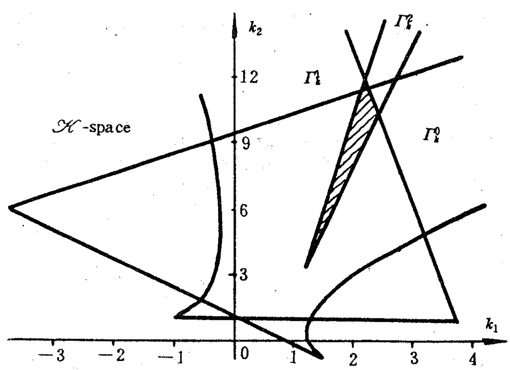

Fig. 5 The feasible regions in $\mathscr{K}$-space

5 Conclusions

The problem of fault-tolerant pole assignment for multivariable system is investigated. A design procedure is proposed, by which the traditional pole assignment technique and fault-tolerant control are connected with each other so that a system can operate stably and satisfactorily under various actuator failure modes by a fixed state feedback. According to n-linear characteristic coefficient system theory, a dyadic state feedback is used, which greatly simplifies the designing for multi-input system. When $n\leqslant 3$, the proposed design procedure can be carried out graphically. Computer graphics will make it easier to construct Γ_p in $\mathscr{P}$-space and Γ_k^i in $\mathscr{K}$-space. If $n>3$, a suitable CAD program is required to carry out the proposed procedure.

The major open research problems are those related to fault-tolerability conditons mentioned in Remark 2 and 3, which will be a theme of further research.

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多变量系统固定状态反馈下的容错极点配置

叶银忠

(华东化工学院自动化研究所·上海,200237)

摘要: 本文研究了多变量系统对执行器故障的容错极点配置问题. 基于 n-线性特征系数系统理论及参数空间设计方法,提出了一种容错极点配置的方法. 借助这一方法,对于给定的被控对象,可以设计出一个固定的状态反馈控制律,在执行器的各种故障模式下都将系统的闭环极点设置在预定的区域内.

关键词: 容错极点配置; 多变量系统; n-线性特征系数系统; 参数空间设计

本文作者简介

叶银忠 1964 年生. 分别于 1982 年、1985 年和 1989 年在华东化工学院工业自动化专业获得工学学士、硕士和博士学位. 自 1985 年始在华东化工学院自动化研究所工作. 主要研究兴趣是容错控制技术及过程系统的故障检测与诊断方法.