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Sufficient Condition for Stability of Interval Matrices

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Abstract: In this paper, some sufficient conditions are presented for the stability of a real matrix whose elements are not known precisely. These conditions have improved the results of symmetric interval matrices.

Key words: interval matrix; Hurwitz stability; Schur stability; strictly diagonally dominant matrix

1 Introduction

Suppose the linear dynamical system described by the equation

$$x'(t) = Ax(t), \quad x(t_0) = x_0. \tag{1}$$

Where A is a real matrix with dimensions $n\times n$. Because of rounding error, we only know the ends of the interval within which the elements of the matrix are confined. Let

$$A = \{a_{ij}\}, \quad P = \{p_{ij}\}, \quad Q = \{q_{ij}\}, \quad (i,j = 1,2,3,\cdots,n)$$

are real matrices with dimensions $n\times n$. Denote $N[P, Q]=\{A=\{a_{ij}\}\in \mathrm{R}^{n\times n}: P\leqslant A\leqslant Q,\ p_{ij}\leqslant a_{ij}\leqslant q_{ij}, i,j=1,2,\cdots,n\}$. Let $\lambda_1, \lambda_2, \cdots, \lambda_n$ denote the eigenvalues of matrix $A\in N[P, Q]$. For the continuous time-invariant case, the set $N[P, Q]$ is said to be Hurwitz if and only if $\mathrm{Re}\lambda_i<0$ $(i=1,2,\cdots,n)$. For the time-invariant discrete time system, the set $N[P, Q]$ is said to be Schur if and only if $|\lambda_i|<1$. $(i=1,2,\cdots,n)$.

Bialas[1] has shown that the family of interval matrices is Hurwitz if and only if all its vertex matrices are Hurwitz. But Barmish and Hollot[2] have shown that the result of Bialas[1] is incorrect; Xu, D. Y.[7] has obtained the simple criteria for Hurwitz of interval matrices; Mori and Kokame[9,11] have shown that interval matrices and interval polynomials are Hurwitz in some certain conditions; Petkovski[3] has attempted to improve the bounds of interval matrices to keep it Hurwitz, however his result has also been shown to be incorrect by Buslowiz[5]; Necessary and sufficient condition for Hurwitz and Schur have been obtained by Soh[6], but these results are based on symmetric matrices.

In this paper, we present several sufficient conditions for the stability of real interval matrix.

2 Hurwitz Stability

Mori and Kokame[9] and Soh[6] have obtained the result of symmetric interval matrices.

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Now we consider the case of general interval matrices.

Suppose $A=\{a_{ij}\}$ is a $n\times n$ real matrix. We have $A=(A+A^T)/2+(A-A^T)/2$. Obviously $(A+A^T)/2$ is a symmetric matrix. Denote $B=(A+A^T)/2$. Let $\max(\lambda(B))=\beta$, $\min(\lambda(B))=\alpha$, then $\alpha\leqslant\mathrm{Re}(\lambda(A))\leqslant\beta$ (see [8]).

Lemma 1 Let A be a real matrix with dimension $n\times n$. Denote $B=(A+A^T)/2$. Then A is Hurwitz if B is Hurwitz.

Lemma 2 Let A_1 and A_2 be two real matrices with dimensions $n\times n$. Denote $B_1=(A_1+A_1^T)/2$, $B_2=(A_2+A_2^T)/2$. Then $A=(1-r)A_1+r\,A_2$ is Hurwitz for all $r\in[0,1]$, if B_1 and B_2 is Hurwitz.

Proof We have the fact that (Lancaster and Tismenetsky[10]) a symmetric Hurwitz matrix is negative-definite and vice versa. Because B_1, B_2 and $(A+A^T)/2$ are symmetric matrices, it will suffice to show $(A+A^T)/2$ is negative-definite for all $r\in[0,1]$. We have

$$(A+A^T)/2=(1-r)(A_1+A_1^T)/2+r(A_2+A_2^T)/2=(1-r)B_1+rB_2$$

Here B_1 and B_2 are Hurwitz. So $x^T((A+A^T)/2)x=(1-r)x^TB_1x+rx^TB_2x<0$ for all $0\neq x\in R^{n\times 1}$. $(A+A^T)/2$ is Hurwitz, then A is Hurwitz.

For the following polytope of $n\times n$ real matrices

$$R=\sum_{i=1}^{m}r_iA_i,\quad \sum_{i=1}^{m}r_i=1,\quad r_i\geqslant 0,\quad m=m(n). \tag{2}$$

Theorem 1 The polytope of matrices R is Hurwitz if matrices B_i are Hurwitz. $(i=1,2,\cdots,m)$, where $B_i=(A_i+A_i^T)/2$, $(i=1,2,\cdots,m)$.

Proof From the condition:

$$R_*=(R+R^T)/2=\sum_{i=1}^{m}r_i(A_i+A_i^T)/2=\sum_{i=1}^{m}r_iB_i. \tag{3}$$

Soh[6] has shown that the polytope of symmetric matrices R_* is Hurwitz if and only if tne vertex matrices B_i are Hurwitz. From Lemma 1, R is Hurwitz.

Suppose $V^*[P,Q]=\{T=\{t_{ij}\}\in R^{n\times n}: t_{ij}=(p_{ij}+p_{ji})/2$ or $(q_{ij}+q_{ji})/2$ for any $i,j=1,2,\cdots,n\}$. There are 2^{n^2-1} numbers of T in $V^*[P,Q]$.

Theorem 2 If $V^*[P,Q]$ is Hurwitz, then $N[P,Q]$ is Hurwitz.

Proof Because $P\leqslant A\leqslant Q$ for all $A\in N[P,Q]$

$$(P+P^T)/2\leqslant(A+A^T)/2\leqslant(Q+Q^T)/2.$$

Also[6] there exist constant numbers $r_i\geqslant 0$, $\sum_{i=1}^{m}r_i=1$, such that

$$(A+A^T)/2=\sum_{i=1}^{m}r_iT_i,\quad T_i\in V^*[P,Q],\ m=2^{n^2-1}.$$

It followed from Theorem 1:

$(A+A^T)/2$ is Hurwitz, A is Hurwitz. So $N[P,Q]$ is Hurwitz.

3 Hurwitz Stability of Interval Diagonally Dominant Matrices

Let $A=\{a_{ij}\}$ be a real matrix with dimension $n\times n$. If $|a_{ii}|>\sum_{\substack{j=1\\j\neq i}}^{n}|a_{ij}|$ for each $1\leqslant i\leqslant n$, we say that A is a strictly diagonally dominant matrix, denoted by $A \in D$.

Lemma 3[4] Let $A \in D$, $a_{ii} > 0 (i = 1, 2, \cdots n)$, then $\det A \neq 0, \operatorname{Re}\lambda_i A > 0, (i = 1, \cdots, n)$.

Theorem 3 Let $t_{ii} < 0$, $T \in D$ for all $T \in V^*[P, Q]$, then $N[P, Q]$ is Hurwitz.

Proof $T \in V^*[P, Q]$, $T \in D$, therefore $V^*[P, Q]$ is Hurwitz. From Theorem 2, $N[P, Q]$ is Hurwitz.

Remark 1 This theorem has improved the corresponding results of Xu, D. Y. [7].

Remark 2 Soh[6] has shown that a family of symmetric interval matrices is Schur if and only if the vertex matrices are Schur. For the case of general interval matrices. We have result: the polytope of matrices R is Schur for all $r_i \in [0, 1]$, if each B_i is Schur. Where

$$B_i = (A_i + A_i^T)/2, \quad (i = 1, 2, \cdots, n)$$

4 Example

In this section we have a simple example.

Let

$$P = \begin{bmatrix} -2.1 & 1.4 \\ 0.6 & -4.0 \end{bmatrix}, \quad Q = \begin{bmatrix} -1.5 & 2.0 \\ 0.8 & -3.5 \end{bmatrix},$$

$$N[P, Q] = \{A \mid A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, -2.1 \leqslant a_{11} \leqslant -1.5,$$

$$1.4 \leqslant a_{12} \leqslant 2.0,\ 0.6 \leqslant a_{21} \leqslant 0.8,\ -4.0 \leqslant a_{22} \leqslant -3.5\},$$

$$(P + P^T)/2 = \begin{bmatrix} -2.1 & 1.0 \\ 1.0 & -4.0 \end{bmatrix}, \quad (Q + Q^T)/2 = \begin{bmatrix} -1.5 & 1.4 \\ 1.4 & -3.5 \end{bmatrix},$$

$$V^*[P, Q] = \{T_i : i = 1, 2, \cdots, 8\}, \quad \text{where } m = 2^{n^2-1} = 2^{2^2-1} = 8.$$

$$T_1 = \begin{bmatrix} -2.1 & 1.0 \\ 1.0 & -4.0 \end{bmatrix}, \quad T_2 = \begin{bmatrix} -1.5 & 1.0 \\ 1.0 & -4.0 \end{bmatrix}, \quad T_3 = \begin{bmatrix} -2.1 & 1.4 \\ 1.4 & -4.0 \end{bmatrix},$$

$$T_4 = \begin{bmatrix} -2.1 & 1.0 \\ 1.0 & -3.5 \end{bmatrix}, \quad T_5 = \begin{bmatrix} -2.1 & 1.4 \\ 1.4 & -3.5 \end{bmatrix}, \quad T_6 = \begin{bmatrix} -1.5 & 1.4 \\ 1.4 & -4.0 \end{bmatrix},$$

$$T_7 = \begin{bmatrix} -1.5 & 1.0 \\ 1.0 & -3.5 \end{bmatrix}, \quad T_8 = \begin{bmatrix} -1.5 & 1.4 \\ 1.4 & -3.5 \end{bmatrix}.$$

1) $\forall A \in N[P, Q]$, we have

$$(A + A^T)/2 = \sum_{i=1}^{m} r_i T_i, \quad \sum_{i=1}^{m} r_i = 1, \quad r_i \geqslant 0, \quad (i = 1, 2, \cdots, m).$$

For example

$$A = \begin{bmatrix} -1.8 & 1.7 \\ 0.7 & -3.6 \end{bmatrix} \in N[P, Q], \quad (A + A^T)/2 = \begin{bmatrix} -1.8 & 1.2 \\ 1.2 & -3.6 \end{bmatrix} = \sum_{i=1}^{8} r_i T_i,$$

Hence

$$r_1 = 0.1, \quad r_2 = 0.1, \quad r_3 = 0, \quad r_4 = 0.3,$$

$$r_5 = 0.1, \quad r_6 = 0, \quad r_7 = 0, \quad r_8 = 0.4,$$

or

$$r_1 = 0, \quad r_2 = 0.2, \quad r_3 = 0, \quad r_4 = 0.3,$$

$$r_5 = 0.2, \quad r_6 = 0, \quad r_7 = 0, \quad r_8 = 0.3,$$

or another.

Because

$$\begin{bmatrix} -2.1 & -1.5 & -2.1 & -2.1 & -2.1 & -1.5 & -1.5 & -1.5 \\ 1.0 & 1.0 & 1.4 & 1.0 & 1.4 & 1.4 & 1.0 & 1.4 \\ -4.0 & -4.0 & -4.0 & -3.5 & -3.5 & -4.0 & -3.5 & -3.5 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_8 \end{bmatrix} = \begin{bmatrix} -1.8 \\ 1.2 \\ -3.6 \\ 1 \end{bmatrix}.$$

2) $V^*[P, Q]$ is Hurwitz, Because T_1 is Hurwitz $(i = 1, 2, \cdots, 8)$, so $N[P, Q]$ is Hurwitz.

For example

$$A = \begin{bmatrix} -1.8 & 1.7 \\ 0.7 & -3.6 \end{bmatrix}, \quad \det(\lambda I - A) = |\lambda I - A| = 0, \quad \text{i.e. } \lambda^2 + 5.4\lambda + 5.29 = 0,$$

$\mathrm{Re}\lambda(A) < 0$, and A is Hurwitz.

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区间矩阵稳定的充分条件

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摘要:本文讨论了几类元素不确定区间矩阵的稳定性问题,得到了若干充分判据,改进了对称区间矩阵稳定性的结果.

关键词:区间矩阵;Hurwitz 稳定;Schur 稳定;严格对角占优矩阵

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