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多变量系统传递函数阵零极点的子结构特征分析*

周　军　吕振肃
(兰州大学电子与信息科学系·兰州，730000)

摘要：本文给出多变量系统传递函数阵零极点与其子结构传递函数阵零极点的基于集的最大最小关系.使传递函数阵零极点反映的结构特征更完善和深刻.

关键词：多变量系统；传递函数阵零极点；子结构

1 引　言

传递函数阵零极点是系统输入输出结构的描述[1]. Schrader 和 Sain[2,3]提出系统子结构零点给出系统整体的和局部结构零极点的某些联系和子结构不变零点的反馈可变性.本文讨论子传递函数阵零极点与整体传递函数阵零极点的集的最大最小关系，使传递函数阵零极点在整体和局部结构特性的作用更为明确.结论对多变量系统分析与设计有意义.

2 定义与基本关系

$$G(s) = \{n_{ij}(s)/d_{ij}(s),\quad i=1,2,\cdots,l;\ j=1,2,\cdots,m\}$$

是 $l\times m$ 传递函数阵，$\{n_{ij}(s),d_{ij}(s)\}$是互质多项式. $G(s)\begin{pmatrix} i_1,i_2,\cdots,i_k \\ j_1,j_2,\cdots,j_t\end{pmatrix}$是 $G(s)$第 $i_1,i_2,\cdots,i_k$ 行和第 $j_1,j_2,\cdots,j_t$ 列的 $k\times t$ 子结构传递函数阵. $M(s)$, $M(s)\begin{pmatrix} i_1,i_2,\cdots,i_k \\ j_1,j_2,\cdots,j_t\end{pmatrix}$是 $G(s)$, $G(s)\begin{pmatrix} i_1,i_2,\cdots,i_k \\ j_1,j_2,\cdots,j_t\end{pmatrix}$的 Smith-McMillan 形，即

$$M(s) = \left[\begin{array}{c:c} \mathrm{diag}\{\dfrac{\varepsilon_i(s)}{\varphi_i(s)},\ i=1,2,\cdots,r\} & 0_{r,m-r} \\ \hdashline 0_{l-r,r} & 0_{l-r,m-r}\end{array}\right],$$

$$M(s)\begin{pmatrix} i_1,i_2,\cdots,i_k \\ j_1,j_2,\cdots,j_t\end{pmatrix} = \left[\begin{array}{c:c} \mathrm{diag}\left\{\dfrac{\varepsilon_i(s)\begin{pmatrix} i_1,i_2,\cdots,i_k \\ j_1,j_2,\cdots,j_t\end{pmatrix}}{\varphi_i(s)\begin{pmatrix} i_1,i_2,\cdots,i_k \\ j_1,j_2,\cdots,j_t\end{pmatrix}},\ i=1,2,\cdots,\hat{r}\right\} & 0_{\hat{r},t-\hat{r}} \\ \hdashline 0_{k-\hat{r},\hat{r}} & 0_{k-\hat{r},t-\hat{r}}\end{array}\right].$$

$$r = \mathrm{rank}G(s),\quad \hat{r} = \mathrm{rank}G(s)\begin{pmatrix} i_1,i_2,\cdots,i_k \\ j_1,j_2,\cdots,j_t\end{pmatrix}.$$

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$\{\varepsilon_i(s),\varphi_i(s)\}$和$\left\{\varepsilon_i(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\j_1,j_2,\cdots,j_t\end{pmatrix},\varphi_i(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\j_1,j_2,\cdots,j_t\end{pmatrix}\right\}$是互质多项式对. 定义

$$d(s)=l\cdot c\cdot m\left\{d_{ij}(s),\forall\begin{pmatrix}i\\j\end{pmatrix}\in\begin{pmatrix}1,2,\cdots,l\\1,2,\cdots,m\end{pmatrix}\right\},$$

$$d(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\j_1,j_2,\cdots,j_t\end{pmatrix}=l\cdot c\cdot m\left\{d_{ij}(s),\forall\begin{pmatrix}i\\j\end{pmatrix}\in\begin{pmatrix}i_1,i_2,\cdots,i_k\\j_1,j_2,\cdots,j_t\end{pmatrix}\right\}.$$

多项式 $\hat{d}(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\j_1,j_2,\cdots,j_t\end{pmatrix}$使

$$\hat{d}(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\j_1,j_2,\cdots,j_t\end{pmatrix}d(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\j_1,j_2,\cdots,j_t\end{pmatrix}=d(s). \tag{1}$$

$l\times m,k\times t$ 多项式阵 $N(s),\hat{N}(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\j_1,j_2,\cdots,j_t\end{pmatrix}$使

$$G(s)=N(s)/d(s),$$

$$G(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\j_1,j_2,\cdots,j_t\end{pmatrix}=\hat{N}(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\j_1,j_2,\cdots,j_t\end{pmatrix}/d(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\j_1,j_2,\cdots,j_t\end{pmatrix},$$

显然

$$N(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\j_1,j_2,\cdots,j_t\end{pmatrix}=\hat{d}(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\j_1,j_2,\cdots,j_t\end{pmatrix}\hat{N}(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\j_1,j_2,\cdots,j_t\end{pmatrix}. \tag{2}$$

定义 $G(s),G(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\j_1,j_2,\cdots,j_t\end{pmatrix}$如上,则集

$$Z_i\begin{pmatrix}i_1,i_2,\cdots,i_k\\j_1,j_2,\cdots,j_t\end{pmatrix}=\left\{s\,\middle|\,\varepsilon_i(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\j_1,j_2,\cdots,j_t\end{pmatrix}=0\right\},\quad i=1,2,\cdots,\hat{r},$$

$$P_i\begin{pmatrix}i_1,i_2,\cdots,i_k\\j_1,j_2,\cdots,j_t\end{pmatrix}=\left\{s\,\middle|\,\varphi_i(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\j_1,j_2,\cdots,j_t\end{pmatrix}=0\right\},\quad i=1,2,\cdots,\hat{r},$$

$$Z\begin{pmatrix}i_1,i_2,\cdots,i_k\\j_1,j_2,\cdots,j_t\end{pmatrix}=\left\{s\,\middle|\,\prod_{i=1}^{\hat{r}}\varepsilon_i(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\j_1,j_2,\cdots,j_t\end{pmatrix}=0\right\},$$

$$P\begin{pmatrix}i_1,i_2,\cdots,i_k\\j_1,j_2,\cdots,j_t\end{pmatrix}=\left\{s\,\middle|\,\prod_{i=1}^{\hat{r}}\varphi_i(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\j_1,j_2,\cdots,j_t\end{pmatrix}=0\right\}$$

元分别称 $G(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\j_1,j_2,\cdots,j_t\end{pmatrix}$的第 i 类零点,第 i 类极点,零点和极点. 若

$$G(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\j_1,j_2,\cdots,j_t\end{pmatrix}=G(s),$$

各集记 Z_i,P_i,Z 和 P. 又

$$Z_d\begin{pmatrix}i_1,i_2,\cdots,i_k\\j_1,j_2,\cdots,j_t\end{pmatrix}=P-P\begin{pmatrix}i_1,i_2,\cdots,i_k\\j_1,j_2,\cdots,j_t\end{pmatrix}. \tag{3}$$

其元称 $G(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\j_1,j_2,\cdots,j_t\end{pmatrix}$(相对 $G(s)$)的解耦零点.

引理 $A(s),B(s)$和 $D(s)=A(s)B(s)$是 $l\times m,m\times n$ 和 $l\times n$ 的多项式阵,不变因子记 $a_i(s),b_i(s)$和 $d_i(s)$,若$\forall\ i,a_i(s)=1$,则 $b_i(s)\,|\,d_i(s)$.

引理证明见附录.

3 结论与证明

定理　给定传递函数阵 $G(s)$,则

1) $g\cdot c\cdot d\left\{\varepsilon_i(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}\right\}=\varepsilon_i(s)$,　$l\cdot c\cdot m\left\{\varphi_i(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}\right\}=\varphi_i(s)$,

2) $g\cdot c\cdot d\left\{\prod_{q=1}^{j}\varepsilon_q(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}\right\}=\prod_{q=1}^{j}\varepsilon_q(s)$,　$l\cdot c\cdot m\left\{\prod_{q=1}^{j}\varphi_q(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}\right\}=$

$\prod_{q=1}^{j}\varphi_q(s)$.

其中 $i,j=1,2,\cdots,\min\{k,t\}\leqslant r$, $g\cdot c\cdot d\{\cdot\}$ 和 $l\cdot c\cdot m\{\cdot\}$ 是 $\forall\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}\in\begin{pmatrix}1,2,\cdots,l\\ 1,2,\cdots,m\end{pmatrix}$ 意义的.

证　由(1),(2)

$$\frac{\varepsilon_i(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}}{\varphi_i(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}}=\frac{\hat N(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}\text{的 } i \text{ 阶不变因子 }\hat\lambda_i(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}}{d(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}}$$

$$=\frac{\hat d(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}\hat\lambda_i(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}}{\hat d(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}d(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}}$$

$$=\frac{N(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}\text{的 } i \text{ 阶不变因子 }\lambda_i(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}}{d(s)}.\tag{4}$$

又
$$N(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}=\begin{bmatrix}\vec e_{i_1}\\ \vec e_{i_2}\\ \vdots\\ \vec e_{i_k}\end{bmatrix}N(s)[\vec e_{j_1},\vec e_{j_2},\cdots,\vec e_{j_t}].$$

其中
$$\vec e_i=[0,\cdots,0,\underset{\text{第}i\text{个}}{1},0,\cdots,0]\in\mathrm{R}^l,\quad i=i_1,i_2,\cdots,i_k,$$
$$\vec e_j^{\mathrm T}=[0,\cdots,0,\underset{\text{第}j\text{个}}{1},0,\cdots,0]\in\mathrm{R}^m,\quad j=j_1,j_2,\cdots,j_t.$$

由引理, $N(s)$ 的 i 阶不变因子 $\lambda_i(s)$ 有 $\lambda_i(s)\mid\lambda_i(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}$ 或有多项式 $\tilde\lambda_i(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}$ 使

$$\lambda_i(s)\tilde\lambda_i(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}=\lambda_i(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}.\tag{5}$$

由(5)

$$g\cdot c\cdot d\left\{\prod_{q=1}^{j}\lambda_q(s)\tilde{\lambda}_q(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix},\ \forall\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}\in\begin{pmatrix}1,2,\cdots,l\\ 1,2,\cdots,m\end{pmatrix}\right\}$$

$$=\prod_{q=1}^{j}\lambda_q(s)g\cdot c\cdot d\left\{\prod_{q=1}^{j}\tilde{\lambda}_q(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix},\ \forall\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}\in\begin{pmatrix}1,2,\cdots,l\\ 1,2,\cdots,m\end{pmatrix}\right\}$$

$$=g\cdot c\cdot d\left\{\prod_{q=1}^{j}\lambda_q(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix},\ \forall\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}\in\begin{pmatrix}1,2,\cdots,l\\ 1,2,\cdots,m\end{pmatrix}\right\}$$

$$=g\cdot c\cdot d\left\{N(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}\text{的 } j \text{ 阶行列式因子 } \Delta_j(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix},\right.$$
$$\left.\forall\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}\in\begin{pmatrix}1,2,\cdots,l\\ 1,2,\cdots,m\end{pmatrix}\right\}$$

$=N(s)$ 的 j 阶行列式因子 $\Delta_i(s)$

$$=\prod_{q=1}^{j}\lambda_q(s).$$

于是

$$g\cdot c\cdot d\left\{\prod_{q=1}^{j}\tilde{\lambda}_q(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix},\ \forall\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}\in\begin{pmatrix}1,2,\cdots,l\\ 1,2,\cdots,m\end{pmatrix}\right\}=1,\tag{6}$$

$$g\cdot c\cdot d\left\{\tilde{\lambda}_i(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix},\ \forall\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}\in\begin{pmatrix}1,2,\cdots,l\\ 1,2,\cdots,m\end{pmatrix}\right\}=1.\tag{7}$$

其中 $i=1,2,\cdots,j$.

取多项式 $\tilde{\lambda}_i^{(q)}(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}$, $q=1,2$ 使

$$\tilde{\lambda}_i^{(1)}(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}\tilde{\lambda}_i^{(2)}(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}=\tilde{\lambda}_i(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix},\tag{8}$$

$$\tilde{\lambda}_i^{(2)}(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}=g\cdot c\cdot d\left\{\tilde{\lambda}_i(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix},\varphi_i(s)\right\}.\tag{9}$$

由(6),(7)

$$g\cdot c\cdot d\left\{\prod_{\tau=1}^{j}\tilde{\lambda}_\tau^{(q)}(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix},\ \forall\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}\in\begin{pmatrix}1,2,\cdots,l\\ 1,2,\cdots,m\end{pmatrix}\right\}=1,\tag{10}$$

$$g\cdot c\cdot d\left\{\tilde{\lambda}_i^{(q)}(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix},\ \forall\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}\in\begin{pmatrix}1,2,\cdots,l\\ 1,2,\cdots,m\end{pmatrix}\right\}=1.\tag{11}$$

其中 $q=1,2$. (5),(8),(9)代入(4)

$$\frac{\varepsilon_i(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}}{\varphi_i(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}}=\frac{\varepsilon_i(s)\tilde{\lambda}_i^{(1)}(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}\cdot\tilde{\lambda}_i^{(2)}(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}}{\varphi_i(s)}.\tag{12}$$

由于 $\varepsilon_i(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}$, $\varphi_i(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}$互质, $\varepsilon_i(s)\tilde{\lambda}_i^{(1)}(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}$与 $\varphi_i(s)$互质,(12)表明.

$$\varepsilon_i(s)\begin{pmatrix} i_1,i_2,\cdots,i_k \\ j_1,j_2,\cdots,j_t \end{pmatrix} = \varepsilon_i(s)\tilde{\lambda}_i^{(1)}(s)\begin{pmatrix} i_1,i_2,\cdots,i_k \\ j_1,j_2,\cdots,j_t \end{pmatrix}, \tag{13}$$

$$\varphi_i(s)\begin{pmatrix} i_1,i_2,\cdots,i_k \\ j_1,j_2,\cdots,j_t \end{pmatrix}\tilde{\lambda}_i^{(2)}(s)\begin{pmatrix} i_1,i_2,\cdots,i_k \\ j_1,j_2,\cdots,j_t \end{pmatrix} = \varphi_i(s). \tag{14}$$

由(11),(13),(14)有 1). 又由(13),(14)

$$\prod_{q=1}^{j}\varepsilon_q(s)\begin{pmatrix} i_1,i_2,\cdots,i_k \\ j_1,j_2,\cdots,j_t \end{pmatrix} = \prod_{q=1}^{j}\varepsilon_q(s)\prod_{q=1}^{j}\tilde{\lambda}_q^{(1)}(s)\begin{pmatrix} i_1,i_2,\cdots,i_k \\ j_1,j_2,\cdots,j_t \end{pmatrix}, \tag{15}$$

$$\prod_{q=1}^{j}\varphi_q(s)\begin{pmatrix} i_1,i_2,\cdots,i_k \\ j_1,j_2,\cdots,j_t \end{pmatrix}\prod_{q=1}^{j}\tilde{\lambda}_q^{(2)}(s)\begin{pmatrix} i_1,i_2,\cdots,i_k \\ j_1,j_2,\cdots,j_t \end{pmatrix} = \prod_{q=1}^{j}\varphi_q(s), \tag{16}$$

由(10),(15),(16)有 2). 　　证毕.

关于定理的讨论:

1) 结论 1)表明,Z_i 是 $G(s)$ 所有 $k\times t$ 子结构 $Z_i\begin{pmatrix} i_1,i_2,\cdots,i_k \\ j_1,j_2,\cdots,j_t \end{pmatrix}$ 类零点集均包含的零点的集的最大者;P_i 是包括 $G(s)$ 所有 $k\times t$ 子结构第 i 类极点集 $P_i\begin{pmatrix} i_1,i_2,\cdots,i_k \\ j_1,j_2,\cdots,j_t \end{pmatrix}$ 的最小者.

2) 结论 2)表明 $\sum_{i=1}^{j}Z_i$ 是 $G(s)$ 所有 $k\times t$ 子结构前 j 类零点集 $\sum_{i=1}^{j}Z_i\begin{pmatrix} i_1,i_2,\cdots,i_k \\ j_1,j_2,\cdots,j_t \end{pmatrix}$ 均包含的零点的集的最大者;$\sum_{i=1}^{j}P_i$ 是包含 $G(s)$ 所有 $k\times t$ 子结构前 j 类极点集 $\sum_{i=1}^{j}P_i\begin{pmatrix} i_1,i_2,\cdots,i_k \\ j_1,j_2,\cdots,j_t \end{pmatrix}$ 的最小者.

3)
$$Z_1 = \left\{ s \,\middle|\, g\cdot c\cdot d\left\{ n_{ij}(s), \forall \begin{pmatrix} i \\ j \end{pmatrix} \in \begin{pmatrix} 1,2,\cdots,l \\ 1,2,\cdots,m \end{pmatrix} \right\} = 0 \right\},$$

而且 $P_1 = \left\{ s \,\middle|\, l\cdot c\cdot m\left\{ d_{ij}(s), \forall \begin{pmatrix} i \\ j \end{pmatrix} \in \begin{pmatrix} 1,2,\cdots,l \\ 1,2,\cdots,m \end{pmatrix} \right\} = 0 \right\}$. 这正是[4]的结论.

4) 表 $P(\cdot)$ 是集$(\cdot)$生成的多项式. 如 $P(\{1,2\}) = (s-1)(s-2)$. 当 $j=\min\{k,t\}<r$,则

$$\begin{aligned}
\prod_{i=1}^{j}\varphi_i(s) &= l\cdot c\cdot m\left\{ \prod_{i=1}^{j}\varphi_i(s)\begin{pmatrix} i_1,i_2,\cdots,i_k \\ j_1,j_2,\cdots,j_t \end{pmatrix}, \forall \begin{pmatrix} i_1,i_2,\cdots,i_k \\ j_1,j_2,\cdots,j_t \end{pmatrix} \in \begin{pmatrix} 1,2,\cdots,l \\ 1,2,\cdots,m \end{pmatrix} \right\} \\
&= l\cdot c\cdot m\left\{ P\left(P - Z_d\begin{pmatrix} i_1,i_2,\cdots,i_k \\ j_1,j_2,\cdots,j_t \end{pmatrix} \right), \forall \begin{pmatrix} i_1,i_2,\cdots,i_k \\ j_1,j_2,\cdots,j_t \end{pmatrix} \in \begin{pmatrix} 1,2,\cdots,l \\ 1,2,\cdots,m \end{pmatrix} \right\} \\
&= l\cdot c\cdot m\left\{ \frac{\prod_{i=1}^{r}\varphi_i(s)}{P\left(Z_d\begin{pmatrix} i_1,i_2,\cdots,i_k \\ j_1,j_2,\cdots,j_t \end{pmatrix} \right)}, \forall \begin{pmatrix} i_1,i_2,\cdots,i_k \\ j_1,j_2,\cdots,j_t \end{pmatrix} \in \begin{pmatrix} 1,2,\cdots,l \\ 1,2,\cdots,m \end{pmatrix} \right\} \\
&= \frac{\prod_{i=1}^{r}\varphi_i(s)}{g\cdot c\cdot d\left\{ P\left(Z_d\begin{pmatrix} i_1,i_2,\cdots,i_k \\ j_1,j_2,\cdots,j_t \end{pmatrix} \right), \forall \begin{pmatrix} i_1,i_2,\cdots,i_k \\ j_1,j_2,\cdots,j_t \end{pmatrix} \in \begin{pmatrix} 1,2,\cdots,l \\ 1,2,\cdots,m \end{pmatrix} \right\}},
\end{aligned}$$

即 $$g\circ c\circ d\left\{P\left(Z_d\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}\right),\ \forall\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}\in\begin{pmatrix}1,2,\cdots,l\\ 1,2,\cdots,m\end{pmatrix}\right\}=\prod_{i=j+1}^{r}\varphi_i(s).\tag{17}$$

即 $\sum_{i=j+1}^{r}P_i$ 是满足 $j=\min\{k,t\}$ 的 $k\times t$ 子结构相对 $G(s)$ 的解耦零点；或说 $G(s)$ 既约其子结构相对整体未必既约. 特别当 $\min\{k,t\}\geqslant r$，所有 $k\times t$ 子结构无相同解耦零点.

5) 注意到，由(8)，(12)

$$\left|G(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_k\end{pmatrix}\right|=\alpha\frac{\prod_{i=1}^{k}\varepsilon_i(s)\tilde{\lambda}_i^{(2)}(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_k\end{pmatrix}}{\prod_{i=1}^{k}\varphi_i(s)}.\tag{18}$$

由(10)，(18)，$G(s)$ 任一非零 k 阶子式分子分母同乘适当多项式使分母为 $\prod_{i=1}^{k}\varphi_i(s)$ 时的所有 k 阶子式分子多项式最大公因式是 $\prod_{i=1}^{k}\varepsilon_i(s)$.

又

$$\forall\ k=1,2,\cdots,r,\quad\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_k\end{pmatrix}\in\begin{pmatrix}1,2,\cdots,l\\ 1,2,\cdots,m\end{pmatrix}\prod_{i=1}^{k}\varphi_i(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_k\end{pmatrix}\Bigg|\prod_{i=1}^{k}\varphi_i(s),$$

因此

$$l\cdot c\cdot m\left\{\hat{\varphi}(s)\begin{pmatrix}i_1,i_2,\cdots,i_t\\ j_1,j_2,\cdots,j_t\end{pmatrix},\ \forall\begin{pmatrix}i_1,i_2,\cdots,i_t\\ j_1,j_2,\cdots,j_t\end{pmatrix}\in\begin{pmatrix}1,2,\cdots,l\\ 1,2,\cdots,m\end{pmatrix},\forall\ t=1,2,\cdots,k\right\}\Bigg|\prod_{i=1}^{k}\varphi_i(s).\tag{19}$$

$\hat{\varphi}(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}$ 是 $\left|G(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}\right|$ 既约分式的分母多项式，又 $\varphi_1(s)=d(s)$，且 $\varphi_{i+1}(s)\,|\,\varphi_i(s)$，$i=1,2,\cdots,r-1$. 于是 $\prod_{i=1}^{k}\varphi_i(s)$ 和多项式

$$l\cdot c\cdot m\left\{\hat{\varphi}(s)\begin{pmatrix}i_1,i_2,\cdots,i_t\\ j_1,j_2,\cdots,j_t\end{pmatrix},\ \forall\begin{pmatrix}i_1,i_2,\cdots,i_t\\ j_1,j_2,\cdots,j_t\end{pmatrix}\in\begin{pmatrix}1,2,\cdots,l\\ 1,2,\cdots,m\end{pmatrix},\quad\forall\ t=1,2,\cdots,k\right\}$$

是 $d(s)$ 全部质因子适当幂次的积. 设 $d(s)$ 质因子 $(s+s_0)$ 在 $\prod_{i=1}^{k}\varphi_i(s)$ 幂次为 β，由定理则 $G(s)$ 至少有一个 $k\times k$ 子结构 $G(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_k\end{pmatrix}$ 满足 $k_1\leqslant k$，$\prod_{i=1}^{k_1}\varphi_i(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_k\end{pmatrix}$ 包含 $(s+s_0)^{\beta}$. 且 $\varphi_i(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_k\end{pmatrix}$，$i=1,2,\cdots,k_1$ 均有质因子 $(s+s_0)$. 进而 $G(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_t\end{pmatrix}$ 至少有一个 $k_1\times k_1$ 子结构 $G(s)\begin{bmatrix}t_1,t_2,\cdots,t_{k_1}\\ q_1,q_2,\cdots,q_{k_1}\end{bmatrix}$ 满足 $k_2\leqslant k_1$，$\prod_{i=1}^{k_2}\varphi_i(s)\begin{bmatrix}t_1,t_2,\cdots,t_{k_1}\\ q_1,q_2,\cdots,q_{k_1}\end{bmatrix}$ 包含 $(s+s_0)^{\beta}$，且 $\varphi_i(s)\begin{bmatrix}t_1,t_2,\cdots,t_{k_1}\\ q_1,q_2,\cdots,q_{k_1}\end{bmatrix}$，$i=1,2,\cdots,k_2$ 均有质因子 $(s+s_0)$. 依次进行下去，必有有限正整数 $\hat{k}\leqslant k$ 使 $G(s)$ 至少一个 $\hat{k}\times\hat{k}$ 子结构 $G(s)\begin{pmatrix}\gamma_1,\gamma_2,\cdots,\gamma_{\hat{k}}\\ v_1,v_2,\cdots,v_{\hat{k}}\end{pmatrix}$ 有 $\prod_{i=1}^{\hat{k}}\varphi_i(s)\begin{pmatrix}\gamma_1,\gamma_2,\cdots,\gamma_{\hat{k}}\\ v_1,v_2,\cdots,v_{\hat{k}}\end{pmatrix}$ 包含

$(s+s_0)^\beta$ 且 $\varphi_i(s)\begin{pmatrix}\gamma_1,\gamma_2,\cdots,\gamma_{\hat{k}}\\ v_1,v_2,\cdots,v_{\hat{k}}\end{pmatrix}, i=1,2,\cdots,\hat{k}$ 均有质因子 $(s+s_0)$. 又 $\varepsilon_i(s)\begin{pmatrix}\gamma_1,\gamma_2,\cdots,\gamma_{\hat{k}}\\ v_1,v_2,\cdots,v_{\hat{k}}\end{pmatrix}$ 与 $\varphi_i(s)\begin{pmatrix}\gamma_1,\gamma_2,\cdots,\gamma_{\hat{k}}\\ v_1,v_2,\cdots,v_{\hat{k}}\end{pmatrix}$ 互质，于是 $\left|G(s)\begin{pmatrix}\gamma_1,\gamma_2,\cdots,\gamma_{\hat{k}}\\ v_1,v_2,\cdots,v_{\hat{k}}\end{pmatrix}\right|$ 既约分式分母多项式有因式 $(s+s_0)^\beta$.
结合(19)，则

$$l\cdot c\cdot m\left\{\hat{\varphi}(s)\begin{pmatrix}i_1,i_2,\cdots,i_t\\ j_1,j_2,\cdots,j_t\end{pmatrix},\ \forall\begin{pmatrix}i_1,i_2,\cdots,i_t\\ j_1,j_2,\cdots,j_t\end{pmatrix}\in\begin{pmatrix}1,2,\cdots,l\\ 1,2,\cdots,m\end{pmatrix},\forall\ t=1,2,\cdots,k\right\}$$

$$=\prod_{i=1}^{k}\varphi_i(s).$$

特别当 $k=r$，上述结论正是[5]的结论.

4 结束语

本文讨论表明传递函数阵零极点不仅是系统整体的结构特征描述，也是其子结构部分结构特征的描述. 反映了多变量系统结构的严格与精细.

参 考 文 献

[1] Schrader, C. B. and Sain, M. K.. Research on Systems Zeros: A Survey. Int. J. Control, 1989, 50(4): 1407—1433

[2] Schrader, C. B. and Sain, M. K.. Subzeros of Linear Multivariable Systems. Proceedings of the 1989 American Contorl Conference, 1989, 1:280—285

[3] Schrader, C. B. and Sain, M. K.. Subzeros in Feedback Transimission. Proceedings of the 1989 American Control Conference, 1989, 1:799—804

[4] Ferreira, P. G. and Bhattacharyya, S. P.. On Blocking Zeros. IEEE Trans., Automat. Contr., 1977, AC-22(2):258—259

[5] Postlethwaite, I. and MacFarlane, A. G. J.. 黄琳(译). 线性多变量反馈系统分析的复变方法. 北京:科学出版社, 1986, 27—28

附　录

引理的证明.

单模态阵 $U_i(s), V_i(s), i=1,2$ 使

$$U_1(s)A(s)V_1(s)=\begin{bmatrix}\mathrm{diag}\{a_i(s), i=1,2,\cdots,r_1\} & 0\\ 0 & 0\end{bmatrix},$$

$$U_2(s)B(s)V_2(s)=\begin{bmatrix}\mathrm{diag}\{b_i(s), i=1,2,\cdots,r_2\} & 0\\ 0 & 0\end{bmatrix}.$$

$r_1=\mathrm{rank}A(s)$, $r_2=\mathrm{rank}B(s)$. 于是

$$U_1(s)D(s)V_2(s)=U_1(s)A(s)V_1(s)V_1^{-1}(s)U_2^{-1}(s)U_2(s)B(s)V_2(s)$$

$$=\begin{bmatrix}a_1(s)b_1(s)w_{11}(s) & a_1(s)b_2(s)w_{12}(s) & \cdots & \\ a_2(s)b_1(s)w_{21}(s) & a_2(s)b_2(s)w_{22}(s) & \cdots & 0\\ \vdots & \vdots & & \\ & 0 & & 0\end{bmatrix}$$

$$=\begin{bmatrix}\overline{W}(s) & 0\\ 0 & 0\end{bmatrix}.$$

其中 $\{w_{ij}(s)\}=V_1^{-1}(s)U_2^{-1}(s)$. 由于 $U_1(s)$, $V_2(s)$是单模态的. 因此 $D(s)$与 $\overline{W}(s)$有相同不变因子，又∀ i, $a_s(s)=1$, $b_{j-1}(s)\,|\,b_j(s)$, $j=1,2,\cdots r_2-1$ 于是有多项式 $\beta_{jk}(s)$使

$$b_j(s)=\beta_{jk}(s)b_k(s),\quad k=1,2,\cdots,j\leqslant r_2.$$

由此

$$\left|\overline{W}(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_k\end{pmatrix}\right|$$

$$=b_k(s)\begin{vmatrix}w_{i_1j_1}(s)b_{j_1}(s) & \cdots & w_{i_1j_{k-1}}(s)b_{j_{k-1}}(s) & w_{i_1j_k}(s)\beta_{j_kk}(s)\\ w_{i_2j_1}(s)b_{j_1}(s) & \cdots & w_{i_2j_{k-1}}(s)b_{j_{k-1}}(s) & w_{i_2j_k}(s)\beta_{j_kk}(s)\\ \vdots & & \vdots & \vdots\\ w_{i_kj_1}(s)b_{j_1}(s) & \cdots & w_{i_kj_{k-1}}(s)b_{j_{k-1}}(s) & w_{i_kj_k}(s)\beta_{j_kk}(s)\end{vmatrix}$$

$$=b_k(s)\sum_{i=i_1,\cdots,i_k}(-1)^{i+j_k}w_{ij_k}(s)\beta_{j_kk}(s)\left|\overline{W}(s)\begin{pmatrix}l_1,l_2,\cdots,l_{k-1}\neq i\\ j_1,j_2,\cdots,j_{k-1}\end{pmatrix}\right|$$

$$=b_k(s)\Delta_{k-1}(s)f(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_k\end{pmatrix}.$$

$\Delta_{k-1}(s)$是 $\overline{W}(s)$的 $k-1$ 阶行列式因子，$f(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_k\end{pmatrix}$是适当多项式，从而

$$d_k(s)=\frac{\Delta_k(s)}{\Delta_{k-1}(s)}$$

$$=b_k(s)g\cdot c\cdot d\left\{f(s)\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_k\end{pmatrix},\forall\begin{pmatrix}i_1,i_2,\cdots,i_k\\ j_1,j_2,\cdots,j_k\end{pmatrix}\in\begin{pmatrix}1,2,\cdots,r_1\\ 1,2,\cdots,r_2\end{pmatrix}\right\}.$$

即

$$b_k(s)\,|\,d_k(s),\quad k=1,2,\cdots,r_3(=\mathrm{rank}(D(s))).$$

The Zero-Pole Substructure Analysis of Linear Multivariable Systems

ZHOU Jun and LU Zhensu

(Department of Electronics & Information Science, Lanzhou University • Lanzhou, 730000, PRC)

Abstract: In this paper, the zero-pole relationship among a transfer function matrix and its substructure ones is analysed, and the results give the maximin set relationship of zeros and poles in linear multivariable systems.

Key words: linear multivariable system; transfer function matrix zero and pole; substructure

本文作者简介

周 军 1963年生. 1987年于兰州大学获硕士学位后留校任教，讲师. 目前主要研究领域是多变量系统代数结构理论，系统稳定性理论.

吕振肃 1946年生. 1969年毕业于兰州大学物理系无线电电子学专业，1982年于兰州大学无线电系获硕士学位，现任兰州大学电信系系主任，副教授. 目前主要研究领域是控制理论与应用，数字信号处理，计算机应用.