Tracking Flexible Objects by Multiple Robot Systems

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Abstract: Control issues of multiple robot systems in tracking flexible objects are studied in this paper. To simplify the exposition of this very complicated matter, a flexible long beam is considered instead of flexible bodies in general form. After some mechanical properties of the object are established, a hierarchical control law is suggested.

Key words: Robot control; flexible object tracking; multiple-robot systems; hierarchical control strategy

1 Introduction

Multiple robot systems, especially two-arm robot are studied by many researchers[1~21], where a lot of interesting results have been reported. Obviously, multiple robot systems are undoubtedly needed in order to accomplish complicated tasks which is beyond the ability of one single robot. Such cases may be pointed out: case where the control activity is limited[8] owing to the finite power source, case when the holding ability of the end-effectors of the robots is limited owing to the finite strength of material[9], case in tracking a mechanical system consisting of several parts moving relatively each to the other as an assembly set[23] and many others. A new case where the object is flexible is studied in this paper. For simplicity of exposition we consider here only slender flexible beams and more complex flexible objects can also be treatd in similar way.

The objective of our work is to find suitable control laws which track the flexible object to follow a desired trajectory while keeping its form nearly undeformed, i. e. keeping always straight in moving. About the form of the object we make:

Assumption 1 Suppose the coordinate (x_i, y_i, z_i) attached to each of the end-effectors of the n robots grasping firmly the beam are chosen such that x_i are all along the longitudinal axis of the beam. The beam is called formally undeformed if all x_i lay on the same straight line, while y_i and z_i remain parallel to that just as in the natural undeformed form. We assume such a formally undeformed beam is undeformed.

Assumption 2 All the deformations, deflections and torsions, are small.

Assumption 3 In the tracking process only the static deformation under static and dynamic loads on the beam are considered.

Assumption 3 ignores all the elastic vibrations (elastic modes) of the beam while moving as

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a body with deformations caused by the static and dynamic loads.

A series of mechanical properties of the flexible-plus-robots systems are formulated which serve a foundation for solving our control problem.

The control strategy is established to fulfill the tracking task formulated above in a hierarchy with three layers:

1) Local control, the first layer: control on the robots to track themselves;

2) Global control, the second layer: we call it the whole motion control which is shared by the robots to track the object considered as rigid;

3) Coordination control, the third layer: we call it fine motion control which is contributed by the robots to coordinate the motion of the end-effectors such that the flexible object keeps a undeformed form.

The paper is organized as below. In section 2, preliminaries about definitions and notations are given. A series of mechanical properties are stated in section 3. The hierarchical control strategy is suggested in section 4 and the control law is formulated then in section 5. Finally a short conclusion is drawn in the last section.

2 Preliminaries: definitions and notations

2.1 Matrix Representation of Force Systems with Respect to Moving Frame

Any force system (system of forces) may be simplified to some arbitrarily chosen point O to a force f and a torque m_o and then be representated by a 6×1 matrix as

$$F = [f_x,\ f_y,\ f_z,\ m_{ox},\ m_{oy},\ m_{oz}]^{\mathrm{T}}, \tag{2.1}$$

where f_x, f_y, f_z, m_{ox}, m_{oy}, m_{oz} are projections of f and m_o on arbtitrary coordinate system (x, y, z) with origin at O. In our study (x, y, z) is always chosen fixedly attached to some relevant moving body. Such force systems are, e. g. , force system acting on a body, force system exerted by the end-effector on the body, reactive force system from other body or constraint, etc. Besides, another form representating the force system of the generalized control on a manipulator is: $\tau = [\tau_1, \tau_2, \cdots, \tau_6]^{\mathrm{T}}$.

2.2 Matrix Representation of Motions with Respect to Moving Frame

Position/orientation of a rigid body is representated by

$$P_0 = [x_0,\ y_0,\ z_0,\ \varphi,\ \theta,\ \psi]^{\mathrm{T}}, \tag{2.2}$$

where (x_0, y_0, z_0) is the coordinates of a point on the body O and (φ, θ, ψ) the Euler's angles all with respect to the base coordinate frame.

But for the velocity of body we have two different representations. The first is

$$V_o = [V_{ox},\ V_{oy},\ V_{oz},\ \omega_x,\ \omega_y,\ \omega_z]^{\mathrm{T}}, \tag{2.3}$$

where V_{ox}, V_{oy}, V_{oz} is the components of velocity of point O on moving axes (x, y, z), and ω_x, ω_y, ω_z the angular velocity components of the body on the same moving axes. The second is

$$\dot{P}_0 = [V_{ox},\ V_{oy},\ V_{oz},\ \dot{\varphi},\ \dot{\theta},\ \dot{\psi}]^{\mathrm{T}}. \tag{2.4}$$

We will obviate the often used representations where V_{ox}, V_{oy}, V_{oz}, ω_x, ω_y, ω_z are components on axes of the base frame.

The drawback of this representation is that the time derivative of P_o has no direct meaning in sense of "velocity". As is well-known, the Euler's kinematic equations

$$\omega_x = \dot{\varphi}\sin\theta\sin\psi + \dot{\theta}\cos\psi, \quad \omega_y = \dot{\varphi}\sin\theta\cos\psi - \dot{\theta}\sin\psi, \quad \omega_z = \dot{\varphi}\cos\theta + \dot{\psi} \tag{2.5}$$

give the relations between Euler's angles and the angular velocities about x, y, z axes and relation (2.5) may also represented as

$$\omega = W\dot{\varepsilon}, \quad \omega = [\omega_x, \omega_y, \omega_z]^T, \quad \varepsilon = [\varphi, \theta, \psi]^T, \tag{2.6}$$

where

$$W = \begin{bmatrix} \sin\theta\sin\psi & \cos\psi & 0 \\ \sin\theta\cos\psi & -\sin\psi & 0 \\ \cos\theta & 0 & 1 \end{bmatrix}. \tag{2.7}$$

These two representations are related by

$$V_o = X\dot{P}_o, \quad X = \begin{bmatrix} U & 0 \\ 0 & W \end{bmatrix}, \quad U = R_B^O,$$

where B refers to the base coordinates, O to that fixed with the body at O and R_B^O is the orientation matrix of B relative to O.

Besides, other often met representation of position and velocity is that of a manipulator by generalized coordinates

$$q = [q_1, \cdots, q_6]^T, \quad \dot{q} = [\dot{q}_1, \cdots, \dot{q}_6]^T.$$

2.3 Dynamic Equations of Object-Robots Systems

Dynamic equations of robots may be compactly described by Lagrange equation

$$M_i(q_i)\ddot{q}_i + N_i(q_i, \dot{q}_i) = \tau_i + G_i f_{ei}, \quad (i = 1, 2, \cdots, m), \tag{2.8}$$

where the subscript i denotes the number of the robot supposing they are totally m. As usual, q_i is the joint vector, $M_i(q_i)$ the inertia matrix, $N_i(q_i, \dot{q}_i)$ the vector including Coriolis, centrifugal and gravity forces. τ_i the control vector and f_{ei} the force acting on the end-effector of the ith robot. Matrix G_i will be defined later.

As to object is temporarilly considered as rigid, it is plausible to employ a coordinate system $Oxyz$ with O at the mass center and xyz fixed with the body along principal axes of inertia. Then the equations of motion of the body are given by the Newton's law of motion and the Euler's momentum equation[10]:

$$\begin{aligned} \dot{V}_x &= L_x + m^{-1}f_x, & \dot{\omega}_x &= H_x(\omega) + J_x^{-1}m_x, \\ \dot{V}_y &= L_y + m^{-1}f_y, & \dot{\omega}_y &= H_y(\omega) + J_y^{-1}m_y, \\ \dot{V}_z &= L_z + m^{-1}f_z, & \dot{\omega}_z &= H_z(\omega) + J_z^{-1}m_z, \end{aligned} \tag{2.9}$$

where m is the mass of the moving body, f_x, f_y, f_z, and m_x, m_y, m_z are the components of the principal vector and the principal moment of the system of all external forces acting on the body including reactive forces from the end-effectors of the manipulators, gravity and reactions from the environments.

It is to be noted that $Oxyz$ is a moving coordinate system, so in the equations of motion such terms L_x, L_y, L_z appear. The reason why we choose a moving coordinate system which induces these complex nonlinear terms is that otherwise J_x, J_y, J_z will not remain constant during the motion and in that case the equatons of motion become nonstationary which is much more complicated.

In equation (2.9):

$$L_x = \omega_z V_y - \omega_y V_z, \quad L_y = \omega_x V_z - \omega_z V_x, \quad L_z = \omega_y V_x - \omega_x V_y, \tag{2.10}$$

$$H_x = J_x^{-1}(J_y - J_z)\omega_y\omega_z, \quad H_y = J_y^{-1}(J_z - J_x)\omega_z\omega_x, \quad H_z = J_z^{-1}(J_x - J_y)\omega_x\omega_y. \tag{2.11}$$

Now we are ready to put the equations of motion of the moving body presented by (2.9) to (2.11) into a compact form

$$\frac{\mathrm{d}}{\mathrm{d}t}V = K + F, \tag{2.12}$$

$$K = [L_x, L_y, L_z, H_x, H_y, H_z]^{\mathrm{T}}, \tag{2.13}$$

$$F = Ef, \tag{2.14}$$

$$E = \mathrm{diag}[m^{-1}, m^{-1}, m^{-1}, J_x^{-1}, J_y^{-1}, J_z^{-1}]^{\mathrm{T}}. \tag{2.15}$$

$$V = [V_x, V_y, V_z, \omega_x, \omega_y, \omega_z]^{\mathrm{T}}, \quad f = [f_x, f_y, f_z, m_x, m_y, m_z]^{\mathrm{T}},$$

where the subscript o is omitted.

3 Some Basic Mechanical Properties of the Object-Robots Systems

The following properties of the object-robots systems are basic to our study.

Property 1 Force transition chain. The representations of force systems acting on various parts of the object-robots system relative to different local moving frames are connected by a typical force transition chain established in [10]. For the system shown in Fig. 1 we have

$$\tau_{i+1} = G_{i+1}^{\mathrm{T}} F_{Bi+1}, \quad F_{Bi+1} = S_{i+1} F_i, \quad F_i = S_i^{-1} F_{Bi}, \quad F_{Bi} = G_i^{-\mathrm{T}} \tau_i, \tag{3.1}$$

where F_{Bi}, F_i, F_{Bi+1} are statically equivalent force systems referred to different points of application, whereas τ_i causes the same effect on the ith robot as F_{Bi}, and the same to τ_{i+1} and $-F_{Bi+1}$, but they are not equivalent pairs.

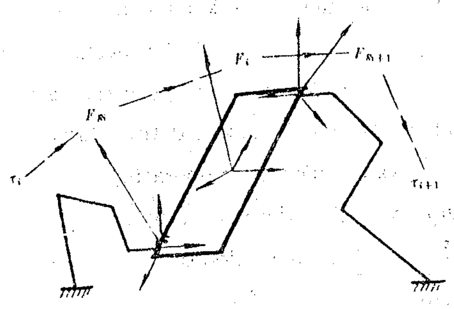

Fig. 1 Force transition chain

Force transition chain may be constructed similarily for any other cases.

Property 2 Velocity transition chain. The representation of velocities of various parts of the object-robots system relative to different moving frames are connected by typical velocity transition chains[10]. For case shown in Fig. 2, we have (3.2).

$$\dot q_{i+1} = G_{i+1}^{-T} V_{Bi+1}, \quad V_{Bi+1} = Y_{i+1} V_i, \quad V_i = Y_i^{-1} V_{Bi}, \quad V_{Bi} = G_i^{\mathrm{T}} \dot q_i, \tag{3.2}$$

where V_{Bi+1}, V_i, V_{Bi} are different representations of the velocity of the same body referred to either the mass center or the end-effectors.

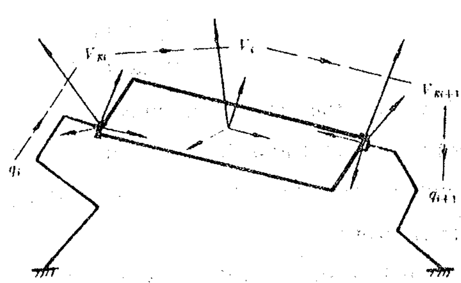

Fig. 2 Velocity transition chain

Any other case can be treated by using these equations too.

Property 3 Independency of motion and force contorl. For mechanical systems the motion of the system and part of the force system acting on the system, whether they are external active forces, interactive forces between parts of the system or reactive forces from constraints, can be controlled separately. That is, we can always split the control vector into two parts: $u=u_m+u_f$, such that u_m is devoted to track the motion of the system to follow the desired motion while u_f to track the designated part of force to follow the desired forces.

Proof Consider a mechanical system with q as its generalized coordinate vector $q=[q_1, q_2, \cdots, q_m]^T$, where m is equal to its degree of freedom.

The differential equation of motion of the mechanics system is described by Lagrange equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}}\right)-\frac{\partial L}{\partial q}=Q, \tag{3.3}$$

where L is the Lagrangian $L=T(q, \dot{q})-\Pi(q)$, T the kinetic energy, Π the potential energy, Q the generalized force

$$Q=\sum_{j=1}^{N}\left[\frac{\partial r_j}{\partial q}\right]^T F_j. \tag{3.4}$$

In equation (3.4), $F_j(j=1,\cdots,N)$ is the actual acting force system and r_j denotes the radius vector of the point of application of f_j with respect to the base coordinate frame. Obviously r_j are function of q.

Equation (3.4) shows that Q with its components and in consequence the dynamic equation of motion (3.3) are linear with respect to forces F_j. The force system $\{F_j\}$ includes: control vector u, reaction force from the constraints which may be forces from the end-effectors of relevant manipulators, and all other forces applied on the machanical system: $F_i(i=1, \cdots, N)$.

In its developed form, (3.3) may be represented as

$$M(q)\ddot{q}+N(q,\dot{q})=G_o u+\sum_{i=1}^{N}G_i^T F_i, \tag{3.5}$$

where G_o, G_i are all nonsingular transformation matrices.

Suppose the task of control is to track the motion $q(t)$ and $f_i(t)$ to follow desired ones $q^d(t)$ and $f_i^d(t)$. On accouting of the linearity of (3.5) in u, we can split it into two parts: $u=u_m+u_f$, where u_m and u_f are designated to track the motion and the force separately. For the motion control part we have

$$M(q)\ddot{q}+N(q,\dot{q})=u_m \tag{3.6}$$

So as to the force control part it can be realized in someways, for example, two schemes are suggested in [12]: the programed force control and the dynamic feedback control. In case of the so-called programmed force control, we have

$$u_f=\sum_{i=1}^{N}G_i^T f_i^d(t), \tag{3.7}$$

but in the dynamic feedback control scheme

$$u_f = \sum_{i=1}^{N} G_i^{\mathrm{T}}[f_i^d(t) + K_i\int(f_i^d(t) - f_i)\mathrm{d}t], \tag{3.8}$$

where K_i are feedback matrices. This property is valid for any control schemes.

Property 4 Deformation relation. Suppose the beam is supported at points O and O' by two manipulators as shown in Fig. 3. The deformed position and orientation of $O'x'y'z'$ with respect to $Oxyz$ is denoted by $p=[\delta_x, \delta_y, \delta_z, \theta_x, \theta_y, \theta_z]^{\mathrm{T}}$, where $\delta_x, \delta_y, \delta_z$ are the deflections of O' along the x, y, z directions, and $\theta_x, \theta_y, \theta_z$ rotational angles around ox, oy, oz.

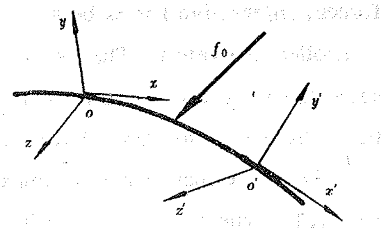

Fig. 3 Deformation relation

Denoting the force system applied at O' by $f'=[f_x, f_y, f_z, m_x, m_y, m_z]^{\mathrm{T}}$, which is the force system exerted by the end-effector of the manipulator at O'.

Now we have a deformation relation

$$p = Sf' + l_o, \tag{3.9}$$

where l_o is a vector depending on all the known forces applied on the beam section OO', and

$$S = \begin{bmatrix} \Delta\overline{S} & 0 \\ D\overline{S} & H\overline{S} \end{bmatrix}, \quad \overline{S} = R_O^{O'}, \tag{3.10}$$

$$\Delta = \begin{bmatrix} \Delta_1 & 0 & 0 \\ 0 & \Delta_2 & 0 \\ 0 & 0 & \Delta_3 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & d_3 \\ 0 & d_2 & 0 \end{bmatrix}, \quad H = \begin{bmatrix} \theta_1 & 0 & 0 \\ 0 & \theta_2 & 0 \\ 0 & 0 & \theta_3 \end{bmatrix}.$$

In (3.10), $\Delta_1, \Delta_2, \Delta_3, d_2, d_3, \theta_1, \theta_2, \theta_3$ are constants depending on the dimensions and strength coefficients of the beam, and $\overline{S}$ is equal to the orientation matrix of $O'x'y'z'$ relative to $Oxyz$.

When the axial strength is very strong such that the axial deformation along x is negligible, then $\Delta_1=0$, then the rank of S is 5.

Now we define the form of a section of the beam considered in Property 4 by a couple of vectors $\Phi=\{p_o, p_{o'}\}$. In conformity with our simplifying assumptions, so if OO' is a undeformed section we have $\Phi=\{p_o, 0\}$.

Property 5 Equation of the beam. Relation between the form of the beam and forces acting on the beam is determined by (3.9) and the Dalamber's principal, i.e. the equilibrium equations of all forces acting on the beam including the dynamic loads:

$$F = \{f_{Eo}, f_{Eo'}, f^s, f^d\}^{\mathrm{T}}, \tag{3.11}$$

where F_{Eo}, $f_{Eo'}$ are the reactions of end-effectors. f^s the known static load, f^d the dynamic load depending on the motion of the beam. They are

$$\begin{aligned} \Sigma X = 0, \quad \Sigma Y = 0, \quad \Sigma Z = 0, \\ \Sigma M_x = 0, \quad \Sigma M_y = 0, \quad \Sigma M_z = 0, \end{aligned} \tag{3.12}$$

where X, Y, Z are projections of all forces (3.11) on axes x, y, z whereas M_x, M_y, M_z projections of all moments.

4 The Hierarchical Control Strategy

The tracking task may be considered in two phases:

1) The whole motion tracking, i. e. the flexible object is temporarily taken as solidified;

2) The fine motion tracking, where the form of the flexible beam is adjusted to keep it in a formally underformed status (defined in Assumption 1).

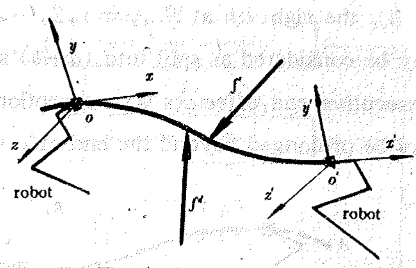

Fig. 4 Dynamic equations

First the desired whole motion is given in form $p_o^d(t)$ or $v_o^d(t)$ as the position vector or the velocity vector of the solidified beam with o at its mass center.

By the velocity transition chain and through computations the desired motion of each manipulator may be obtained

$$q_i^d(t), \quad \dot{q}_i^d(t), \quad \ddot{q}_i^d(t), \quad (i = 1,2,\cdots,m). \tag{4.1}$$

On base of property 3, the control on each robot is decomposed into two parts: $u_i = u_{il} + u_{ig}$, $(i=1,2,\cdots,m)$. The local control u_{il} tracks the ith robot to following its desired motion $q_i^d(t)$ when the end-effector grasps nothing. We have a zero payload tracking case:

$$M_i(q_i)\ddot{q}_i + N_i(q_i,\dot{q}_i) = \tau_i = u_{il}. \tag{4.2}$$

Here any of the well-established control techniques can be used, for example, the computed torque technique, the variable structure control technique, etc. By the first we have

$$u_{il} = N_i(q_i,\dot{q}_i) + M_i(q_i)(\ddot{q}_i^d + K_v\dot{e}_i + K_pe_i), \tag{4.3}$$

$$e_i = q_i^d(t) - q_i(t).$$

The global control u_{ig} is just that part of control activity which contributes to the whole motion tracking u_{ig}^w and fine motion adjusting control u_{ig}^f.

The asymptotic tracking : $e_i \to 0$ in this control layer is just the well-known "Computed Torque Method" as exposed in [10], or its original version [14].

$$u_{ig} = u_{ig}^w + u_{ig}^f. \tag{4.4}$$

The additiveness in (4.4) is justified by reasoning the fine motion adjusting control as a force control which is postulated in Assumption 3 and illustrated in Property 5. Now we have $u_i = u_{il} + u_{ig}^w + u_{ig}^f$.

This control law is in a hierarchy with three layers: first layer for local control, second one for fine motion adjusting control and the high layer for whole motion control. At this stage it is reasonable to have

Assumption 4 we assume that the maximun possible control activity of the ith robot is stronger than the activity: u_{il}, but may be weaker than $u_i = u_{il} + u_{ig}^f + u_{ig}^w$. So the limits on the control activity must be taken into account and determine u_{ig} so that u_i is admissible. In any case, it is assumed that the multi-robot system can achieve the proposed tracking task.

5 The Control Law Formulation

We are now going on to find the global control.

Suppose beam AB is held by $(1+a+b)$ robots: the 0th one at the mass center by end-effector E_o, the right ith at $E_{+i}(i=1,2,\cdots,a)$, the left jth at $E_{-j}(j=1,2,\cdots,b)$. Such the beam may be considered as split into $(a+b)$ segments, each of which is the part of beam between two consecutive end-effectors with exceptions possibly of the two end segments, where the segment may be prolonged beyond the end-effector. Both cases are shown in Fig. 5.

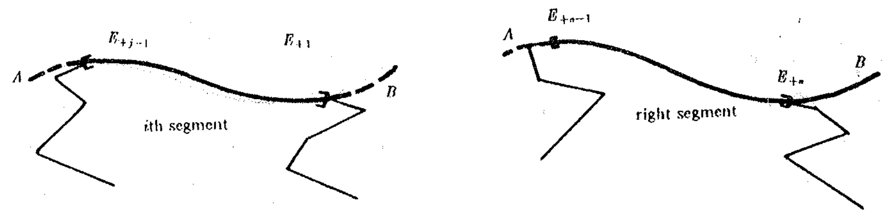

Fig. 5 Segment of beam

Denote the internal forces at the right side cross sections of E_i and left side cross sections of E_{i+1} by

$$\begin{aligned} &f^l_0,\ f^l_{E+i},\quad (i=1,2,\cdots,a),\quad f^l_{E-i},\quad (i=1,2,\cdots,b),\\ &f^r_{E+i},\quad (i=1,2,\cdots,a),\quad f^r_{E-i},\quad (i=1,2,\cdots,b), \end{aligned} \tag{5.1}$$

and the reaction forces at the end-effectors by

$$f_{E0},\ f_{E+i},\quad (i=1,2,\cdots,a),\quad f_{E-i},\quad (i=1,2,\cdots,b). \tag{5.2}$$

We now have the following equations for solving the reaction forces on the end-effectors of the $(1+a+b)$ robots.

For segment $E_{+i-1}E_{+i}$, deformation equation (3.9) and (3.12) give

$$p^l_{E+i} = S_i f^l_{E+i} + l_{oi}, \tag{5.3}$$

where subscript i is added, and

$$\begin{aligned} &\Sigma X(f^r_{Ei-1},\ f^l_{Ei},\ f^g_i,\ f^d_i) = 0,\quad \Sigma Y(f^r_{Ei-1},\ f^l_{Ei},\ f^g_i,\ f^d_i) = 0,\\ &\Sigma Z(f^r_{Ei-1},\ f^l_{Ei},\ f^g_i,\ f^d_i) = 0,\quad \Sigma M_x(f^r_{Ei-1},\ f^l_{Ei},\ f^g_i,\ f^d_i) = 0,\\ &\Sigma M_y(f^r_{Ei-1},\ f^l_{Ei},\ f^g_i,\ f^d_i) = 0,\quad \Sigma M_z(f^r_{Ei-1},\ f^l_{Ei},\ f^g_i,\ f^d_i) = 0, \end{aligned} \tag{5.4}$$

where $\Sigma X(\cdot)$, $\Sigma Y(\cdot)$, $\Sigma Z(\cdot)$ denote the sums of projections of all forces indicated in the brocket on x, y, z axes and $\Sigma M_x(\cdot)$, $\Sigma M_y(\cdot)$, $\Sigma M_z(\cdot)$ denote the sums of moments of all forces included in the brocket around x, y, z axes.

We obtain 12 equations (5.3) and (5.4), which can be solved for two six-dimensional force vector f^r_{Ei-1} and f^l_{Ei}. They are all functions of p^l_{Ei}, which is the six-dimensional deformation vector. If give p^l_{Ei}, i.e. the ith segment of the beam any assumed form, we get at the same time, the forces f^r_{Ei-} and f^l_{Ei}.

Now we can immediately determine the reaction force on the ith end-effector $f_{Ei}=f^l_{Ei}+f^r_{Ei}$, which in turn determines the fine motion adjusting contorl by the force transition chain (3.1)

$$u^f_{ig} = G_i f_{Ei},\quad (i=+1,\ \cdots,\ +a,\ -1,\ \cdots,\ -b). \tag{5.5}$$

In order to find the whole motion control u^w_{ig}, we should distribute the dynamic load of the beam which is postulated staying in a formally undeformed status. For this aim the simple and plausible way is to build a Master-helper self-organizing control strategy seggested in [8,9].

The dynamic load in performing the whole motion, which follows from the differential equations of the object treated as a rigid body, may be derived from (2. 12)

$$f^d = E^{-1}\dot{V} - E^{-1}K. \qquad (5.6)$$

This force system is to be distributed among the $(1+a+b)$ robots. If the activity of individual robot is strong enough, then by any of the distribution techniques[6,7], one obtains the global control for the whole motion tracking u_{ig}^{w} and in consequence the control strategy $u_i = u_{il} + u_{ig}^{f} + u_{ig}^{w}$, where u_{il} and u_{ig}^{f} are already obtained as (2. 4) and (5. 5). The block diagram of a three layer hierarchical control scheme is shown in Fig. 6.

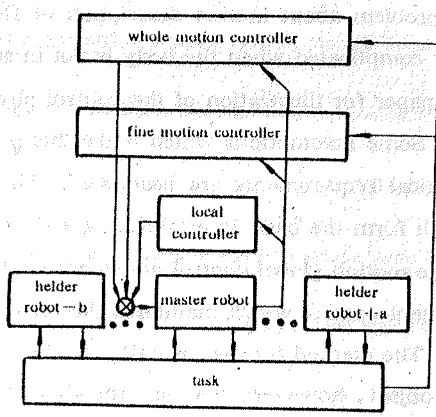

Fig. 6 Control hierarchy

Consider now the case when the contorl activity of individual robot is finite owing to the limited power source or other limitation [9], here

$$|u_i| \leqslant u_i^*, \quad (i = 0, +1, \cdots, +a, -1, \cdots, -b). \qquad (5.7)$$

We use the Master-helper control strategy [10] to realize such a distribution. Based on the procedure suggested in [10] the control law for the $(1+a+b)$ robots is established one by one. Suppose E_0 corresponds to the Master robot, we have

$$u_0 = u_{0l} + u_{0g}^f + u_{0g}^w, \quad u_{0g}^f = 0,$$

$$u_{0l} = N_0(q_0, \dot{q}_0) + M_0(q_0)[\ddot{q}_0^d(t) - K_{0v}\dot{e}_0 - K_{0p}e_0],$$

$$u_{0g}^w = u_{0l} + u_{0g}^f + f^d - \mathrm{tr}(u_{0l} + u_{0g}^f + S_0 f^d, u_0^*),$$

where function tr(•) is a truncation function defined by

$$\mathrm{tr}(x, x^*) = \begin{cases} x - x^*, & \text{when } x \geqslant x^*, \\ 0, & \text{when } |x| < x^*, \\ x + x^*, & \text{when } -x \geqslant x^* \end{cases} \qquad (5.8)$$

and x^* is the bound on x such that $|x| < x^*$. Then for E_{+1} which corresponds to the first helper robot, we have

$$u_{+1} = u_{+1l} + u_{+1g}^f + u_{+1g}^w, \qquad (5.9)$$

$$u_{+1l} = N_{+1}(q_{+1}, \dot{q}_{+1}) + M_{+1}(q_{+1})[\ddot{q}_{+1}^d(t) - K_{+1v}\dot{e}_{+1} - K_{+1p}e_{+1}],$$

$$u_{+1g}^f = G_{+1}^{\mathrm{T}} f_{E+1}, \quad u_{+1g}^w = \bar{u}_{+1g}^w - \mathrm{tr}(\bar{u}_{+1g}^w, u_{+1}^*),$$

$$\bar{u}_{+1g}^w = u_{+1l} + u_{+1g}^f + S_{+1}S_0^{-1}\mathrm{tr}(u_{0l} + u_{0g}^f + S_0 f^d, u_0^*).$$

Going on in this way we find the control strategy for all robots, the master robot and the $(a+b)$ helper robots, $u_0, u_{+1}, u_{-1}, \cdots, u_{+a}, u_{-b}$. We stress here that the order in the selection of helper robots is irrelevent to the tracking task, even different control law is resulted for different order.

6 Conclusion

Tracking problem of flexible object consists of the motion tracking and the form maintenance of the object. So in solving such a tracking control problem, besides the control issue, a mechanical problem about motion description of flexible bodies is involved. This problem may become very complicated when the body is not in simple regular form. Only a slender beam is treated in this paper for illustration of the control phylosophy developed in this paper.

Some assumptions which make this problem easy to attack and at the same time fulfill the practical requirements are postulated. Then a series of basic mechanical properties are drawn which form the basis in establishing a three-layer hierarchical control; local for the robot itself, whole motion global control which accomplishes the motion tracking task and the fine motion adjustment control which maintains the form of the flexible objects.

The method for the formulation of the control strategy is general and applicable for any flexible object, however, the mechanical issue inevitably becomes fairly complex.

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用多机器人系统完成柔性物体的跟踪控制

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摘要: 本文研究了用多机器人系统完成柔性物体的跟踪的控制问题,为了简化对这一十分复杂问题的表述,用一个柔性长杆代替了一般形状的柔性物体,在对象的一些力学性质被确立之后,提出了一个递阶控制策略.

关键词: 机器人控制; 柔性物体跟踪; 多机器人系统; 递阶控制策略

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