

An Iterative Learning PD Longitudinal Controller for Unmanned Land Vehicles

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Abstract: For the velocity-following problem of a unmanned land vehicle, the iterative learning PD controller is shown to be adaptive with respect to highly nonlinear dynamics by properly choosing the control gains and training factor. The choices for the gains hardly depend on the nonlinear dynamics. This paper presents a learning control scheme that provides the ability for machines to utilize their past experiences. Experiment results are presented to validate the conclusions.

Key words: learning control; land vehicles; longitudinal control

1 Introduction

In recent year, much attention has been focused on individual-vehicles automated ground transport, as one anticipated future transportation problems. A facet which is common to this classes of automted ground transportation is the velocity control of individual vehicles. A number of efforts have been devoted. The velocity controllers of the land vehicles were initiated in the late 1950's in a joint venture by the Radio Corporation of America, General Motors Corporation (GMC), and involved vehicle control at speeds up to approximately 13.4m/s. subsequent the feasibility of automated roadway systems was a topic of much research in the 1960's, 1970's and 1980's (Caudill al, 1977, Garrard et al. 1988; D. H. McManon et. al. 1990). Longitudinal control strategies of land vehicles are necessary to regulate the velocity of each vehicles. The longitudinal controller algorithm must insure good performance over a variety of operating points and external conditions.

On the other hand, the PD control has been widely used in industry for decades. It has been shown in practice that a PD control law can be effective not only for linear plants, but for complicated nonlinear system as well. However, the classical PD control law can not make the tracking error arbitrarily small. Now most of the research in the control of complicated nonlinear system has used more complicated control law. At the same time, practicing engineers are reluctant to use more complicated control laws, because these control law are not easy to implement and generally require more information about the controlled system.

In this article, the velocity following problem of a land vehicle is considered using adaptive PD control. It is shown the adaptive PD controller is applicable to the highly nonlinear vehicular

velocity-tracking problem, and the tracking error can be made arbitrarily small. Experimental and theoretical results are presented to validate the conclusions above.

2 Problem Formulation

Consider a class of nonlinear dynamic systems which can be described as follows:

$$\dot{X}(t) = f(X(t), t) + g(X(t), t)u(t), \tag{1}$$

$$y(t) = C^T X(t). \tag{2}$$

Where $X(t) \in R^{n \times 1}$, $u(t) \in R$ and $y(t) \in R$ are state vector, input and output, respectively. $f(X(t), t)$, $g(X(t), t) \in R^{n \times 1}$. Nonlinear function vectors are assumed to be Lipschitz continuous as follows:

$$\| f(X_1, t) - f(X_2, t) \| \leq f_0 \| X_1 - X_2 \| \tag{3}$$

and
$$\| g(X_1, t) - g(X_2, t) \| \leq g_0 \| X_1 - X_2 \| . \tag{4}$$

Where f_0, g_0 are positive constants. $\| \cdot \|$ denotes the vector norm $\| \cdot \|_\infty$. $C^T \in R^{1 \times n}$, $C^T = [c_1 \text{sgn}(\tilde{X}_1), \dots, c_n \text{sgn}(\tilde{X}_n)]$, $\forall c_i > 0$.

Now, when the desired trajectory $y_d(t)$ is specified as a reference input for system (1) (2), the fundamental control problem is to find a control input $u(t)$ with which the system output $y(t)$ follows the reference input $y_d(t)$ for all $t \in [0, t_f]$ as close as possible. Within the framework of learning control, this objective can be stated as follows:

Problem Statement:

Suppose that $y_d(t)$ ($t \in [0, t_f]$), the control problem is to find a sequence of piecewise continuous control input $u^j(t) \in R$, ($t \in [0, t_f]$), for the uncertain system (1) (2) with which the system output $y^j(t)$ follows $y_d(t)$ with a give accuracy ϵ as follows

$$|y_d(t) - y^j(t)| < \epsilon, \text{ for all } t \in [0, t_f].$$

where j denotes the j th iteration. The resulting learning control system is schematically shown in Fig. 1.

Where $\text{sgn}(\cdot)$ is defined as follows:

$$\text{sgn}(\tilde{x}_i(t)) = \begin{cases} 1, & \tilde{x}_i(t) > 0, \\ 0, & \tilde{x}_i(t) = 0, \\ -1, & \tilde{x}_i(t) < 0, \end{cases} \tag{5}$$

$$e(t) = y_d(t) - y(t).$$

Therefore, we obtain a continuous function,

$$e(t) = \sum_{i=1}^n c_i |\tilde{x}_i(t)|, \quad \forall i, c_i \geq 1. \tag{6}$$

In Fig. 1, "memory" describes a unit, which memorizes the past inputs.

3 Stabilization of the Dynamic System

The tracking capability of the iterative learning process hinges upon the stability of the closed-loop system at each iteration. Therefore we firstly design a PD controller which, when applied to target system (1) (2), forms a stable closed-loop system. However, the tracking error

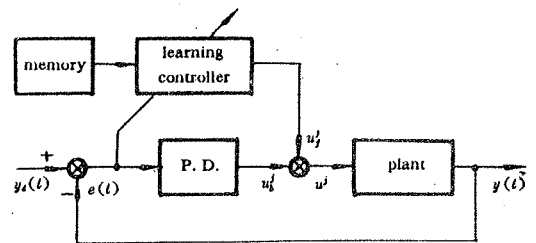


Fig. 1 Schematic diagram of the learning controller system

can not be made arbitrarily small, then we introduce a learning controller in the feedforward loop, which provides tracking of the entire profile of $y_d(t)$ over a sequence of iterative operations. At the initial stage of learning the appropriate feedback gains are selected that ensures system stability with an initial error bound of $e(0)$ much greater than the prespecified target tolerance ϵ . Then the feedforward learning controller reduces the system output error, every iteration and finally makes it smaller than ϵ . The following theorem formalizes the learning operation by means of feedforward input correction strategy.

Theorem Assume that $u_d(t)$, $|C^T g(\cdot)|^{-1}$ and $|\frac{d}{dt}(C^T g(\cdot))^{-1}|$ are bounded as follows:

$$\begin{aligned} \sup_{t \in [0, t_f]} |u_d(t)| &\leq u_0 < \infty, \\ \sup_{t \in [0, t_f]} |(C^T g(\cdot))^{-1}| &\leq r_0 < \infty, \\ \sup_{t \in [0, t_f]} \left| \frac{d}{dt}(C^T g(\cdot))^{-1} \right| &\leq r_1 < \infty. \end{aligned} \tag{7}$$

Furthermore, assume the vector $C^T = [C_1 \text{sgn}(\tilde{X}_1), \dots, C_n \text{sgn}(\tilde{X}_n)]$, $\forall c_i \geq 1$, where $\tilde{X}^j(t) = X_d(t) - X^j(t) = ((\tilde{x}_i^j(t))_{n \times 1})$. The feedback gains k_p, k_v satisfies the following inequalities:

$$\begin{aligned} k_v &> r_0, \\ 2k_p &> \beta + r_1 + 2r_0 \|c\|_\infty (f_0 + g_0 u_0), \\ 1 &> \beta > 0. \end{aligned}$$

Then, with the iterative learning algorithm:

$$\begin{aligned} u^j(t) &= u_d^j(t) + u_f^j(t), \\ u_f^j(t) &= u_f^{j-1}(t) + \beta e^j(t), \\ u_d^j(t) &= k_p e^j(t) + k_v \dot{e}^j(t), \\ j &= 1, 2, 3, \dots \end{aligned}$$

the system (1) (2) converges as follows:

- i) $V^{j+1}(t) \leq V^j(t)$,
- ii) $\lim_{j \rightarrow \infty} y^j(t) = y_d(t)$ or $\lim_{j \rightarrow \infty} X^j(t) = X_d(t)$,
- iii) $\lim_{j \rightarrow \infty} u^j(t) = u_d(t)$.

where $e^j(t) = y_d(t) - y^j(t)$, $u_d(t)$ denotes the desired control input for the desired trajectory $y_d(t)$. the index function $V^j(t)$ is defined as

$$V^j(t) = \int_0^t \bar{u}^{jT}(\tau) \bar{u}^j(\tau) d\tau > 0 \tag{8}$$

for all $t \in [0, t_f]$ in which $\bar{u}^j(t) = u_d(t) - u_f^j(t)$. $u_f^j(t) = 0$ for all $t \in [0, t_f]$ and $u_d^j(0) = 0$ for all j . β is a continuous variable with respect to the tracking error $e(t)$.

Proof of theorem Let $y_d(t) = C^T X_d(t)$, $y(t) = C^T X(t)$ and $\Delta \bar{w}^j(t) = \bar{w}^{j+1}(t) - \bar{w}^j(t)$. Then learning rule is described as

$$\Delta \bar{w}^j(t) = u_f^j(t) - u_f^{j+1}(t) = -\beta e^j(t). \tag{9}$$

Now, $e^j(t) = y_d(t) - y^j(t) = C^T \bar{X}^j(t) = \sum_{i=1}^n c_i |\bar{X}_i(t)| > 0$, where $e^j(t)$ is continuous function for all t , but it is not differentiable at $\bar{X}(t) = 0$. If $\bar{x}_i(t) \neq 0$, we get

$$\begin{aligned} e^j(t) &= C^T \dot{\bar{X}}^j(t) \\ &= -C^T g(\cdot) k_p e^j(t) - C^T g(\cdot) k_v e^j(t) + C^T \bar{f}(\cdot) + C^T \bar{g}(\cdot) u_d + C^T g(\cdot) \bar{w}^j(\cdot) \end{aligned} \quad (10)$$

where $\bar{f}(\cdot) = f(x_d, t) - f(x^j(t), t)$, $\bar{g}(\cdot) = g(x_d, t) - g(x^j(t), t)$.

Now, when $j=1$, $u_f(t) = 0$ and

$$V^j(t) = \int_0^t u_d(\tau) u_d(\tau) d\tau < \infty \quad \text{for all } t \in [0, t_f].$$

If we define $\Delta V^j(t) = V^{j+1}(t) - V^j(t)$, then

$$\begin{aligned} \Delta V^j(t) &= \int_0^t (\Delta \bar{w}^j(\tau) \Delta \bar{w}^j(\tau) + 2\Delta \bar{w}^j(\tau) \bar{w}^j(\tau)) d\tau \\ &= \beta^2 \int_0^t e^{2j}(\tau) d\tau - 2\beta \int_0^t e^j(\tau) \bar{w}^j(\tau) d\tau. \end{aligned} \quad (11)$$

Because $e(t)$ is not differentiable at $\bar{x}(0) = 0$, (11) can be described as follows

$$\begin{aligned} \Delta V^j(t) &= \lim_{\epsilon \rightarrow 0} \int_{\epsilon}^t (\Delta \bar{w}^j(\tau) \Delta \bar{w}^j(\tau) + 2\Delta \bar{w}^j(\tau) \bar{w}^j(\tau)) d\tau \\ &= \lim_{\epsilon \rightarrow 0} \int_{\epsilon}^t (\beta^2 e^{2j}(\tau) - 2\beta e^j(\tau) \bar{w}^j(\tau)) d\tau. \end{aligned} \quad (12)$$

Here, note that $e^j(t)$ and $w^j(t)$ are the continuous function for all t , we get

$$\begin{aligned} \Delta V^j(t) &= \beta^2 \int_0^t e^{2j}(\tau) d\tau - 2\beta \int_0^t e^j(\tau) (C^T g(\cdot))^{-1} [(1 + C^T g(\cdot) k_v) e^j(t) \\ &\quad + C^T g(\cdot) k_p e^j(\tau) - C^T \bar{f}(\cdot) - C^T \bar{g}(\cdot) u_d] d\tau \\ &= -\beta(k_v + (C^T g(\cdot))^{-1}) e^{2j}(t) + \beta \int_0^t e^{2j}(\tau) \frac{d}{d\tau} (C^T g(\cdot))^{-1} d\tau \\ &\quad - \int_0^t (2\beta k_p - \beta^2) e^{2j}(\tau) d\tau - \int_0^t 2\beta (C^T g(\cdot))^{-1} [C^T \bar{f} + C^T \bar{g} u_d] e^j(\tau) d\tau \\ &\leq -\beta(k_v - |(C^T g(\cdot))^{-1}|) e^{2j}(t) - \int_0^t \Delta w^j(\tau) d\tau, \end{aligned}$$

$$\begin{aligned} \Delta w^j(\tau) &= (2\beta k_p - \beta^2) e^{2j}(\tau) + 2\beta e^j(\tau) (C^T g(\cdot))^{-1} (C^T \bar{f} + C^T \bar{g}(\cdot) u_d) - \beta \frac{d}{d\tau} (C^T g(\cdot))^{-1} e^{2j}(\tau) \\ &\geq e^{2j}(\tau) \beta(2k_p - \beta - \tau_1) - 2\beta r_0 \|C^T\|_{\infty} e^j(t) (f_0 + g_0 u_0) \|\bar{X}\|. \end{aligned}$$

Note $e^j(t) = \sum_{i=1}^n c_i |\bar{x}_i(t)| > \|\bar{X}(t)\|_{\infty}$, when $c_i = 1$.

Therefore, we obtain

$$\Delta w^j(\tau) \geq e^{2j}(t) \beta(2k_p - \beta - \tau_1 - 2r_0 \|C^T\| (f_0 + g_0 u_0)). \quad (13)$$

If $0 < \beta < 1$, $k_v > |(C^T g(\cdot))^{-1}|$,

$$2k_p > \beta + \tau_1 + 2r_0 \|C^T\| (f_0 + g_0 u_0)$$

we obtain

$$\Delta w(\tau) \geq 0, \quad \Delta V^j(\tau) < 0.$$

Hence, the sequence $\{V^j(t)\}$ is monotonically decreasing, which confirms i). Because $V^1(t)$ is bounded, the monotonically decreasing nonnegative sequence $\{V^j(t)\}$ converges to a nonnega-

tive fixed value. $V^j(t) \rightarrow 0$ as $j \rightarrow \infty$. This implies that $e^j(t)$ converge to zero for all $t \in [0, t_f]$, because $\Delta V^j(t) \leq 0$, this implies ii). Finally because $V^j(t) \rightarrow 0$ as $j \rightarrow \infty$, we have iii). Furthermore, we have

$$\lim_{j \rightarrow \infty} X^j(t) = X_s(t).$$

On the other hand, β is a continuous variable with respect to the tracking error $e(t)$, and $0 < \beta < 1$. we obtain

$$\beta = \begin{cases} \beta_0 < 1, & e^j(t) \geq 1, \\ \beta_0 (e^j(t))^2, & e^j(t) < 1. \end{cases} \tag{14}$$

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Note that, in order to avoid overshoot, we have (14).

4 Application

The system with PD learning controller was implemented digitally as the following form:

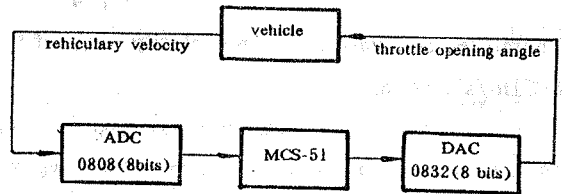


Fig. 2 Digital control system

We use a Mcs-51 microcomputer. At the beginning of a typical sampling control period, the speed v was sampled and converted to a digital format.

Using PD learning control law, the samples were processed. After a processed time, the processed sampled signal was converted to an analog signal, held and applied to the plant. After a time of some 400ms, the process above is repeated. Generally the same time T was selected by considering the closed-loop bandwidth of the system. Here the sampling time T was chosen as 400ms which is well.

“WS631” automobile was employed for the full-scale testing. The control functions of bracking and accelerating were accomplished via electrohydraulic control systems. In particular, the actuator which controlled the position of the throttle value, was characterized by a corner frequency of some 7rad/s. A computer, 18 potentiometers, and other necessary components was installed over the seat. The computing elements were used for control and data collection. The velocity v is measured via a carefully calibrated tachometer connected to the drive shaft. All testing was conducted on a $50 \times 50 \text{ m}^2$ square.

The vehicular longitudinal dynamics, which has been experimentally verified, is given by [5]

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3(v) & -a_2(v) & -a_1(v) \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \xi(v) \end{bmatrix} u. \tag{15}$$

Where $x_1 = v$, $x_2 = \dot{v}$, $x_3 = \ddot{v}$, v is the forward velocity of the vehicle. u is a voltage, applied to an electrohydraulic actuator, which controls the throttle value.

$$\begin{aligned} a_1(v) &= 0.05 + \xi(v) + 1/t_p(v), \\ a_2(v) &= 0.05\xi(v) + (\xi(v) + 0.05)/t_p(v), \end{aligned}$$

$$a_3(v) = 0.05\xi(v)/t_p(v),$$

where $t_p(v)$ is a function associated with the propulsion system and its interaction with the roadway interface and $\xi(v)$ is a function associated with the tire-roadway interface. The parameters $t_p(v)$ and $\xi(v)$ vary substantially over the velocity range of interest $0 < v \leq 10\text{m/s}$ and may be represented functionally as

$$\xi(v) = \frac{12}{1 + 0.25v}, \quad t_p(v) = \frac{12}{1 + 0.5v}$$

which gives

$$a_1(v) = 0.4167 \frac{v^2 + 6v + 123}{v + 4},$$

$$a_2(v) = 0.0208 \frac{v^2 + 966v + 2043}{v + 4},$$

$$a_3(v) = \frac{v + 2}{v + 4}.$$

Therefore, as v varies over the velocity range of the interest, $0 < v \leq 10\text{m/s}$, $0 < \dot{v} \leq 4\text{m/s}^2$, $0 < \ddot{v} < 1\text{m/s}^3$, we have

$$C^T = [\text{sgn}(v - v_d) \quad \text{sgn}(\dot{v} - \dot{v}_d) \quad \text{sgn}(\ddot{v} - \ddot{v}_d)],$$

$$r_0 = 0.5, \quad r_1 = 0.06, \quad u_0 = 8, \quad f_0 = 47.2, \quad g_0 = 3.$$

The gains of the PD learning controller were chosen to be

$$k_p = 45.0, \quad k_v = 42.0, \quad \beta_0 = 0.69.$$

In fact, Lyapunov's direct method is very conservative in general, which in turn implies that inequalities in theorem are conservative. Thus, reasonable gains work well even if they may not satisfy some of those three inequalities. On the other hand, if the gains k_p , k_v , β of the PD learning controller were chosen to be too large, the larger overshoot would be generated. Therefore, we should choose reasonably large gains.

$$\begin{cases} k_p = 45.0, & k_v = 42.0, \\ \beta = \begin{cases} 0.69, & e^j(t) \geq 1, \\ 0.69(e^j(t))^2, & e^j(t) < 1. \end{cases} \end{cases} \quad (16)$$

Fig. 3 contains the experimental results, which show asymptotic stability of tracking a given velocity.

First, $v_d = 1.7\text{m/s}$ was given. Second, $v_d = 3.8\text{m/s}$ was given. It shows good tracking performance over a variety of operating point and external conditions. The overshoot is very small and $e(\infty) = 0$. the vehicular velocity $v(t)$ and the revs of the engine $v_e(t)$ are represented by plot (1) and plot (2) respectively. Some tests were conducted on country roads, experimental results were

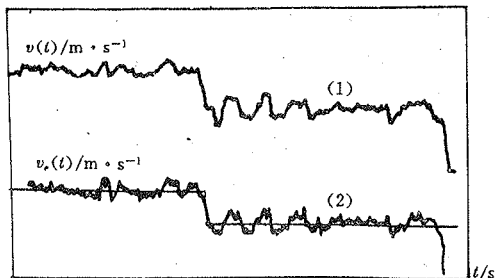


Fig. 3 Experiment results

very closed to the results in Fig. 3. Therefore, it is shown to be an adaptive and robust control scheme for the tracking problem of the vehicle velocity.

5 Conclusion

The PD learning controller is shown to be an effective controller for the complicated nonlinear system. In this paper, we show a theoretical proof. It guarantees that a learning PD control law can be used for the complicated nonlinear system. This result also provides a constructive algorithm for choosing the control gains. Experiment results are presented to validate the conclusions.

The results in this paper significantly improve previous results.

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无人陆地车辆的自学习 PD 迭代纵向控制器

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摘要: 本文研究自动汽车速度跟踪问题, 由于汽车纵向动力学中存在严重非线性和不确定性, 本文提出设计一种自学习 PD 迭代非线性控制器。只要选择合适的控制器增益和训练因子, 可使跟踪误差足够小。增益的选择几乎不依赖于动力学, 只与状态量变化范围有关。因此, 此方法对许多控制对象均可施用。有理论和实验结果证明此方法的有效性。

关键词: 学习控制; 陆地车辆; 纵向控制

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