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Causal Reachability of 2D System *

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Abstract: A new definition of controllability for 2D Roesser Model is proposed, and it is shown that the new definition is dual to the well known concept of causal observability proposed by Bisiacco[1]. Finally the similar results are also generalized to the 2D general state-space model.

Key words: 2D systems; 2D reachability; linear systems

1 Introduction

2D discrete state-space theory has a wide range of applications in digital filtering, digital picture processing, seismic data processing and many other areas[1], and it has received increasing attention. In last decade, a great deal of extensive research has been made for the controllability and observability theory to 2D discrete state-space models, such as Roesser model[2] and the general model[3], etc. Among these works, several of the most remarkable progress are involved in the local controllability/observability[2], global controllability (and reachability)/observability (and reconstructibility)[4~6], and also the causal observability and reconstructibility[5]. However, the local notion is proved to be not very important[4], and the global concepts are usually difficult to test although their applications in observer design[7] and feedback stabilizability[8] are very significant. Therefore, those causal concepts appear of considerable interest[4~13]. Bisiacco[5] and Kurek[12] have also proposed a notion of controllability to be dual to causal reconstructability, which is very useful in deadbeat control. But unfortunetely, there exists not any appropriate known discrete state-space definition for the notion of controllability or reachability that is dual to causal observability. This notion will be of very importance to the synthesis of 2D systems[13].

In this note, the main purpose is just to set up a new definition of reachability for the dual concept of causal observability to 2D Roesser model as well as general model in discrete state-space.

2 Problem Statement and Preliminaries

Consider the following 2D Roesser model (2-DRM).

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$$\begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} = Ax(i,j) + Bu(i,j),$$
$$y(i,j) = Cx(i,j), \tag{1}$$

where
$$x(i,j) = \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} \in \mathrm{R}^n$$

is a local state vector, $x^h(i,j) \in \mathrm{R}^{n_1}, x^v(i,j) \in \mathrm{R}^{n_2}$ and $n = n_1 + n_2, u \in \mathrm{R}^m$ is an input vector, $y \in \mathrm{R}^p$ is an output vector and

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \quad C = [C_1, C_2]$$

are real matrices of appropriate dimensions.

Model (1) is a local only[12]. Hence, the boundary condition set (BCS) will be assumed to be the so-called standard BCS, i. e.

$$x^h(0,j) = x_{0,j}, \qquad x^y(i,0) = x_{i,0}. \tag{2}$$

In order to illustrate the problem under discussion clearly, we introduce the following definitions and theorems.

Definition 1[11] The 2-DRM is said to be causally observable/reconstructable, if one can calculate its local state $x(k,l)$ based on future/past outputs and inputs of the systems.

Definition 2[12] The 2-DRM is said to be causally controllable, if for any $x(0,0) = x_0$ and other standard BCS equal to zero there exist (k,l) and an appropriate input sequence $\{u(i,j); i,j \geqslant 0\}$ such that $x(i,j) = 0$ for $i > k$ or $j > l$.

Theorem 1[11] The 2-DRM is causal observable/causal reconstructable if and only if for any finitely large complex numbers z_1, z_2,

$$\mathrm{rank}\begin{bmatrix} I(z_1,z_2) - A \\ C \end{bmatrix} = n \Big/ \mathrm{rank}\begin{bmatrix} I - I(z_1,z_2)A \\ C \end{bmatrix} = n. \tag{3}$$

Here $I(z_1,z_2) = \mathrm{diag}\{I_{n_1}z_1, I_{n_2}z_2\}$.

Theorem 2[12] The 2-DRM is causal controllable if and only if for any finitely large complex numbers z_1, z_2,

$$\mathrm{rank}[I - AI(z_1,z_2), B] = n. \tag{4}$$

Obviously, it follows from theorem 2 that:

i) Causal controllability is dual to causal reconstructibility.

ii) 2D causal controllability/observability (reconstructibility) is a well reasonable generalization of the corresponding 1D concept. However, it is different from 1D case that causal observability is no longer implies causal reconstructability and vice versa even if A, A_{11} and A_{22} are simultaneously nonsingular[14].

Thus, by the duality between causal reconstructibility and causal controllability of 2-DRM it can be foreseen that the dual notion of causal observability, i. e. the reachability corresponding to the criterion

$$\text{rank}[I(z_1,z_1) - A,\ B] = n \quad \forall\ z_1,z_2 \neq \infty \tag{5}$$

might have been much different from the causal controllability. Now we can formulate the problem under study as: To find a definition called causal reachability in discrete state-space for 2-DRM such that condition (5) is a sufficient and necessary criterion of it.

3 Causal Reachability

Definition 3 2-DRM is said to be causal reachable, if for zero BCS and an arbitrary $x_0 \in R^n$ there exist $(k,l) > (0,0)$ and input sequence $u(i,j), (0,0) \leqslant (i,j) \leqslant (k,l)$, such that $\begin{bmatrix} x^h(k+1,l) \\ x^v(k,l+1) \end{bmatrix} = x_0$ and

$$x^h(k+1,j) = 0, \quad x^v(i,l+1) = 0; \quad (0,0) \leqslant (i,j) \leqslant (k-1,l-1).$$

Theorem 3 2-DRM is causal reachable if and only if (5) holds.

Proof For 2-DRM with BCS being equal to zero one gets after 2D z transformation that

$$I(z_1,z_2)X(z_1,z_2) = AX(z_1,z_2) + BU(z_1,z_2), \tag{6}$$

where
$$X(z_1,z_2) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} x(i,j) z_1^{-i} z_2^{-j}, \tag{7a}$$

$$U(z_1,z_2) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} u(i,j) z_1^{-i} z_2^{-j}. \tag{7b}$$

Mutiplying (6) both sides by $z_1^k z_2^l$, one gets from (7) that

$$[I(z_1,z_2) - A]\hat{X}(z_1,z_2) + B\hat{U}(z_1,z_2) + \hat{Y}(z_1,z_2) + S(z_1,z_2) = 0, \tag{8}$$

where

$$\hat{X}(z_1,z_2) = \sum_{i=-k}^{0}\sum_{j=-l}^{0} x(i+k,j+l) z_1^{-i} z_2^{-j}, \tag{9a}$$

$$\hat{U}(z_1,z_2) = -\sum_{i=-k}^{0}\sum_{j=-l}^{0} U(i+k,j+l) z_1^{-i} z_2^{-j}, \tag{9b}$$

$$\hat{Y}(z_1,z_2) = \sum_{j=-l}^{0}\begin{bmatrix} x^h(k+1,j+l) \\ 0 \end{bmatrix} z_2^{-j} + \sum_{i=-k}^{0}\begin{bmatrix} 0 \\ x^v(k+i,j+1) \end{bmatrix} z_1^{-i} \tag{9c}$$

and $S(z_1,z_2)$ is the series of z_1^{-i} or z_2^{-j} with $i > 1$ or $j > 1$. Therefore from (8) it follows that

$$[I(z_1,z_2) - A]\hat{X}(z_1,z_2) + B\hat{U}(z_1,z_2) = -\hat{Y}(z_1,z_2). \tag{10}$$

Now, by definition 3 it is obvious that 2-D RM is causal reachable if and only if for any $x_0 \in R^n$ there exists a polynomial solution X and U to equation

$$[I(z_1,z_2) - A]X + BU = x_0, \tag{11}$$

i.e. (5) holds[15].

Example 1 Let

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \tag{12}$$

Then for any $z_1, z_2 \neq \infty$,

$$\mathrm{rank}[I(z_1,z_2) - A, B] = \mathrm{rank}\begin{bmatrix} z_1 & -1 & 1 \\ -1 & z_2 & 0 \end{bmatrix} = 2.$$

So (12) is causal reachable.

Example 2 Let

$$A = \left[\begin{array}{c:c} 0 & 0 \\ \hdashline -1 & 0 \end{array}\right], \qquad B = \left[\begin{array}{c} 1 \\ \hdashline 1 \end{array}\right]. \tag{13}$$

Then

$$\mathrm{rank}\ [I(1,0) - A, B] = \mathrm{rank}\begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} = 1 < 2.$$

So (13) is not causal reachable although it is both local reachable[2] and modal controllable[1]. In fact in this example,

$$x^h(i+1,j) = u(i,j), \quad x^v(i,j+1) = u(i,j) - u(i-1,j),$$
$$x^v(0,j+1) = -x^h(0,j) + u(0,j), \quad \forall\ (i,j) \geqslant (1,0). \tag{14}$$

Therefore the same conclusion can be easily derived from the definition directly.

4 Generalization in 2D General Model

Consider the following 2D linear shift-invariant discrete state-space general model (2-DGM)

$$x(i+1,j+1) = A_0x(i,j) + A_1x(i+1,j) + A_2x(i,j+1)$$
$$+ B_0u(i,j) + B_1u(i+1,j) + B_2u(i,j+1), \tag{15}$$

where $x(i,j) \in \mathrm{R}^n$ and $u(i,j) \in \mathrm{R}^m$ are the local state and input of the system respectively. A_k, B_k, $(k = 0,1,2)$, are real matrices of appropriate dimensions. The boundary condition set of (15) is

$$\mathrm{BCS} = \{x(i,0), x(0,j) : i,j \geqslant 0\}. \tag{16}$$

Definition 4 2-D GM is said to be causal reachable, if for any $x_0 \in \mathrm{R}^n$ and zero BCS there exist $(k,l) \geqslant (0,0)$ and an appropriate input sequence $\{u(i,j), (0,0) \leqslant (i,j) \leqslant (k,l)\}$ such that $x(k+1,l+1) = x_0$ and $x(k,j) = x(i,l) = 0$ for any $(0,0) \leqslant (i,j) \leqslant (k,l)$.

Theorem 4 2-D GM is causal reachable if and only if

$$\mathrm{rank}\ [A(z_1,z_2), B(z_1,z_2)] = n, \qquad \text{for } \forall\ z_1,z_2 \neq \infty, \tag{17}$$

where

$$A(z_1,z_2) = z_1z_2I - z_1A_1 - z_2A_2 - A_0, \tag{18a}$$
$$B(z_1,z_2) = B_0 + z_1B_1 + z_2B_2. \tag{18b}$$

Proof It is very simillar to the proof of thorem 3 and is omitted hence.

Finally, as the end of this section we introduce a consequence to illustrate the importance of the causal reachability.

Theorem 5 Let (1) be a single input and single output (SISO) 2-DRM. Then there exists a compensator $G_c(z_1,z_2) = V(z_1,z_2)/w(z_1,z_2)$ for (1) such that the transfer function of the closed loop has any given denominator $\varphi(z_1,z_2)$ if (1) is causal inreducible (i. e. both causal reachable and causal observable).

Proof It is a directly corrollary of the theorem 5 in [13].

5 Conclusion Remarks

In this note the new definition of reachability, a dual notion to causal observability, for 2-DRM as well as 2-DGM is proposed. Obviously, this concept implies the modal controllability/modal reachability[7], and local controllability/local reachrability. However the relationship between causal reachability and global reachability is still unclear.

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2D 系统的因果能达性

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摘要:本文给出了 2D Roesser 模型的一种新的能达性定义,并证明了这一新概念与 Bisiacco[1]在1985 年提出的因果能观性是对偶的.从而最终完善了现有 2D 离散系统能达(能观)性与能控(可重构)性理论.最后本文还将结果推广到 2D 一般离散模型,并利用陈文德在 1984 年的结果给出了这种新的能达性的一类应用.

关键词:2D 系统理论;2D 能达性;线性系统理论

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