Fuzzy Rule Control for an Unstable System *

CHENG Fuyan, ZHONG Guomin and LI Youshan
(Hebei Institute of Mechano-Electric Engineering · Shijiazhuang, 050054, PRC)
XU Zhengming
(Yanshan University · Qinhuangdao, 066000, PRC)

Abstract: A high accuracy and high resolution fuzzy controller is desinged to stabilize a double inverted pendulum at an upright position successfully. A new idea of dealing with multivariate system is described. The fuzzy control rules of a double inverted pendulum are given and a powerful fuzzy decision way is described to divide the output in a more delicate stage. And a high resolution fuzzy controller is designed by using that.

Key words: fuzzy control rule; fuzzy control; double inverted pendulum; composition coefficient

1 Introduction

The double inverted pendulum is a multivariate nonlinear fast reaction and unstable system. To stabilize a double inverted pendulum is not only a challenging problem but also a useful way to show the power of the control method. By using an analogue computer Shozo Mori[1] designed a controller in stabilizing an inverted pendulum hinged to a cart on a rail. K. Furuta[2] designed a controller for the double inverted pendulum by using of the state space approach and the minimal order observer. K. Furuta[3] have designed and developed a digital controller for a double inverted pendulum on an inclined rail by the same approach in 1980.

It has been known as a more difficult problem to design a fuzzy controller for a double inverted pendulum by using fuzzy control theory. In this paper, this problem is studied under the circumstance that a computer is employed in the control system design and the controller. The study based on the fuzzy control theory has been done by a controller for stabilizing a double inverted pendulum at a upright position. A high accuracy and high resolution fuzzy controller is set up.

Today, there is no simple and efficient way in dealing with the multivariate system with the fuzzy control theory. In this paper, a composition coefficient for the state variables is given by use of the computer aided design (CAD). and two new state variables, composition error "E" and composition rate of error "EC".

* This paper is supported by the Natural Science Foundation of Hebei Province, PRC.
Manuscript received Mar. 19, 1994, revised Oct. 17, 1994.

2 System Description and Linearized Model

As mentioned in the introduction, based on the composition error "E" and the composition rate of error "EC" the purpose of this paper is to design and construct a fuzzy controller to stabilize a double inverted pendulum. The control system of the double inverted pendulum is presented in Fig. 1.

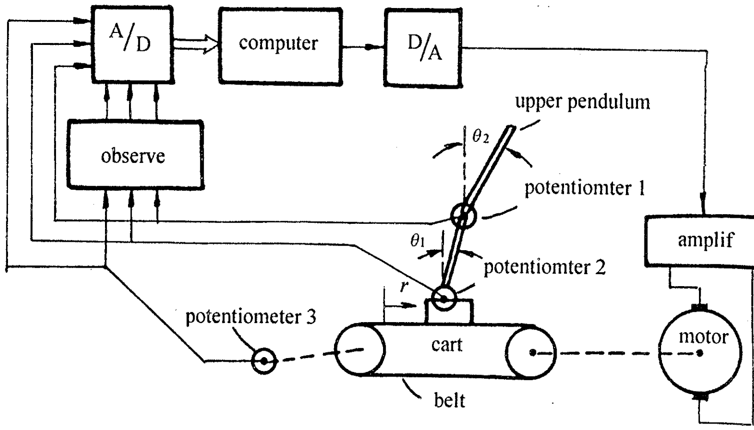

Fig. 1 The control system block diagram

The mathematical model for the system is realized and the following differential equation is obtained by the dynamics theory.

$$M(\theta_1,\theta_2)\begin{bmatrix}\ddot{r}\\ \ddot{\theta}_1\\ \ddot{\theta}_2\end{bmatrix}+F(\theta_1,\theta_2,\dot{\theta}_1,\dot{\theta}_2)\begin{bmatrix}\dot{r}\\ \dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix}=G(u,\theta_1,\theta_2). \tag{1}$$

where

$$M(\theta_1,\theta_2)=\begin{bmatrix}M_0+M_1+M_2 & (M_1l_1+M_2L_1)\cos\theta_1 & M_2l_2\cos\theta_2\\ (M_2L_1+M_1l_1)\cos\theta_1 & J_1+M_1l_1^2+M_2L_1^2 & M_2L_1l_2\cos(\theta_2-\theta_1)\\ M_2L_2\cos\theta_2 & M_2L_1l_2\cos(\theta_2-\theta_1) & J_2+l_2^2M_2\end{bmatrix},$$

$$F(\theta_1,\theta_2,\dot{\theta}_1,\dot{\theta}_2)=\begin{bmatrix}F_0 & -(M_1l_1+M_2L_1)\sin\theta_1\dot{\theta}_1 & -M_2l_2\sin\theta_2\dot{\theta}_2\\ 0 & F_1+F_2 & -L_1l_2M_2\sin(\theta_2-\theta_1)\dot{\theta}_2-F_2\\ 0 & M_2L_1l_2\sin(\theta_2-\theta_1)\dot{\theta}_2-F_2 & F_2\end{bmatrix},$$

$$G(u,\theta_1,\theta_2)=[G_0u \quad (M_1l_1+M_2L_1)g\sin\theta_1 \quad M_2l_2g\sin\theta_2]^{\mathrm{T}}.$$

Thus a mathematical model of the double inverted pendulum has been derived. The symbols are defined and the parameters are given in Table 1.

In the neighbourhood of unstable balance point ($r=\theta_1=\theta_2=0$ and $\dot{r}=\dot{\theta}_1=\dot{\theta}_2=0$), the following linear model is gained.

$$\dot{X}=AX+Bu,$$

$$Y = CX. \tag{2}$$

where

$$X = [x_1, x_2, x_3, x_4, x_5, x_6]^T = [r, \theta_1, \theta_2 - \theta_1, \dot{r}, \dot{\theta}_1, \dot{\theta}_2 - \dot{\theta}_1]^T,$$
$$Y = [y_1, y_2, y_3]^T = [r, \theta_1, \theta_2 - \theta_1]^T.$$

Table 1 Definition of symbols and the parameters

M_0	equivalent mass of the cart drive system	1.3282 kg
M_1	mass of the lower pendulum	0.22 kg
M_2	mass of the upper pendulum	0.187 kg
J_1	mass moment of inertia of the lower pendulum	0.004963 kg · m^2
J_2	mass moment of inertia of the lower pendulum	0.004824 kg · m^2
F_0	equivalent friction constant of the cart drive system	22.915 n · s/m
F_1	friction constant of the lower pendulum	0.007056 n · s/m
F_2	friction constant of the upper pendulum	0.002644 n · s/m
G_0	gain from the output voltage to the output of the motor	11.887 n/v
l_1	distance of the center-of-mass of the lower pendulum	0.304 m
l_2	distance of the center-of-mass of the upper pendulum	0.226 m

By substituting the parameters of Table 1. the following matrixes of coefficient are given.

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -2.569 & 0.16408 & -16.667 & 0.01718 & -0.0011 \\ 0 & 29.919 & -15.181 & 40.317 & -0.28268 & 0.0959 \\ 0 & -36.656 & 65.378 & -49.395 & 0.64301 & -0.41489 \end{bmatrix},$$

$$B = [0 \quad 0 \quad 0 \quad 8.6462 \quad -20.914 \quad 25.642]^T,$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}.$$

3 Composition Coefficient

The composition coefficient is derived by using state space theory. The quadratic criterion function is given by:

$$J = \int_0^\infty (\| X \|_Q^2 + \| u \|_R^2) dt.$$

Accroding to the quadratic criterion function the positive definite symmetric matrix P can be solved by the Riccati equation of linear optimal control system.

$$PA + A^T P - PBR^{-1}B^T P + Q = 0.$$

The composition coefficient K can be derived by

$$K^T = R^{-1}B^T P = [k_1, k_2, k_3, k_4, k_5, k_6].$$

And two new state variables the composition error E and rate of error EC are gained with the composition coefficient K. Let

$$E = [k_1, k_2, k_3]\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad E_C = [k_4, k_5, k_6]\begin{bmatrix} x_4 \\ x_5 \\ x_6 \end{bmatrix}. \tag{3}$$

If the positive definite matrix: $Q = \text{diag}(1, 50, 250, 0, 0, 0)$; $R = 0.1$; the composition coefficient K is: $K^T = (3.156, 46.956, 122.48, 3.974. 19.423, 16.381)$

4 Fuzzy Controller

In this part, a high accuracy and high resolution fuzzy controller is formed.

4.1 Fuzzy Sets

It is necessary to use more fuzzy lignuistic terms than the traditional way in stabilizing a double inverted pendulum. The specific terms for the error E and rate of error EC are as the following:

UB: Upper bound; PL: Positive large; PM: Positive medium; PS: Positive small; PO: Positive zero; OK: Zero; LB: Negative bound; NL: Negative large; NM: Negative medium; NS: Negative small; NO: Negative zero.

A high accuracy fuzzification algorithm of the output U is received by using more linguistic terms than E or EC.

UB: Upper bound; PVL: Positive very-large; PL: Positive large; PVM: Positive big medium; PM: Positive medium; PS: Positive small; PO: Positive zero; OK: Zero; LB: Lower bound; NVL; Negative very-larg; NL: Negative large; NVM: Negative big medium; NM: Negative medium; NS: Negative small; NO: Negative zero.

Where PO is the values slightly above zero. And PVM is the values between the PM and PL, and PVL between the PL and UB etc.

The upper boundary values of the composition error E, the composion rate of error EC and output U which are respectively definded as E_m, EC_m and U_m are given by the CAD and the experiments. The fuzzy sets are formed upon a discrete support universe of 11 elements for E and EC. The fuzzy sets of E and EC are detailed by Fig. 2.

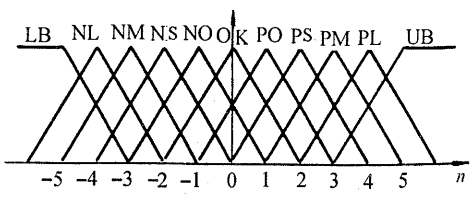

Fig. 2 The fuzzy sets of E and EC

It is necessary for a high resolution fuzzy controller to use more stages of output. The fuzzy sets are formed upon a discrets support universe of 21 elements for the output U. And an automatic performance the coarse-fine control is desinged by constructing the fuzzy sets of output U. When the output of the stage is smaller than 5 or bigger than -5, the change of output is more sensitive than others. So the fuzzy controller can control the double inverted pendulum in a more stable way. The fuzzy sets are detailed by Fig. 3.

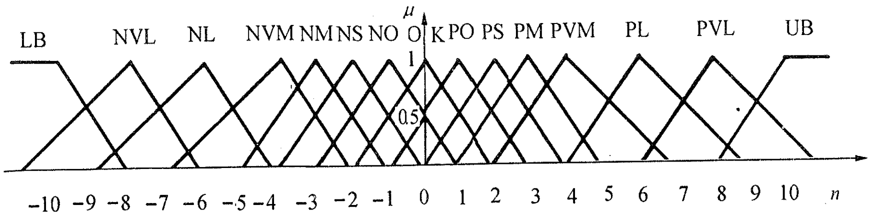

Fig. 3 The fuzzy set of the output

2 Control Rule

The core of the method is the resulting control rules which are used to form a look-up ble. The look-up table directly relates the controller inputs *E* and *EC* with the controller ıtput *U*. The control rules are designed by the computer aided design. And the double verted pendulum is stabilized by the control rules in Table 2 which are described below if is PB and *EC* is UB then *U* is UB, etc.

Table 2 Fuzzy control rule

U \ *EC* / *E*	LB	NL	NM	NS	NO	OK	PO	PS	PM	PL	UB
LB	LB	NVL	NVL	NL	NL	NVM	NM	NS	OK	PO	PS
NL	LB	NVL	NL	NL	NVM	NM	NS	NO	OK	PS	PM
NM	NVL	NVL	NL	NVM	NVM	NS	NO	OK	PO	PS	PVM
NS	NVL	NL	NL	NVM	NM	NS	OK	PO	PS。	PM	PVM
NO	NL	NL	NVM	NM	NS	NO	OK	PS	PM	PVM	PL
OK	NL	NVM	NVM	NS	NO	OK	PO	PS	PVM	PVM	PL
PO	NL	NVM	NM	NS	OK	PO	PS	PM	PVM	PL	PL
PS	NVM	NM	NS	NO	OK	PS	PM	PVM	PL	PL	PVL
PM	NVM	NS	NO	OK	PO	PS	PVM	PVM	PL	PVL	PVL
PL	NM	NS	OK	PO	PS	PM	PVM	PL	PL	PVL	UB
PB	NS	NO	OK	PS	PM	PVM	PL	PL	PVL	PVL	UB

.3 Fuzzy Decision and the Look-Up Table

A good resolution fuzzy controller is ɔrmed by the use of a powerful fuzzy decision hich divides the output *U* in a more delicate tage. The fuzzy decision is obtained by outting ff fuzzy membership of the output with a arameter λ and averaging the stages of the nembership are bigger than the parameter λ. This method is shown by Fig. 4.

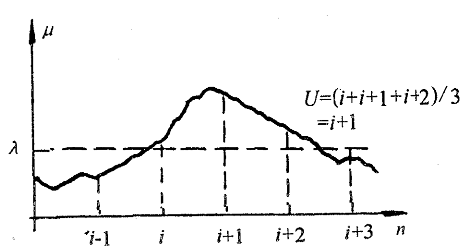

Fig. 4 The fuzzy partition and desision

According to the above description, the look-up table can be seen in Table 3.

Table 3 Look-up table

U (E \ EC)	−5	−4	−3	−2	−1	0	1	2	3	4	5
−5	−10	−8.5	−8.5	−6.5	−6.5	−4.5	−3	−2	0	1	2
−4	−10	−8.5	−6.5	−6.5	−4.5	−3	−2	−1	0	2	3
−3	−8.5	−8.5	−6.5	−4.5	−4.5	−2	−1	0	1	2	4.5
−2	−8.5	−6.5	−6.5	−4.5	−3	−2	0	1	2	3	4.5
−1	−6.5	−6.5	−4.5	−3	−2	−1	0	2	3	4.5	6.5
0	−6.5	−4.5	−4.5	−2	−1	0	1	2	4.5	4.5	6.5
1	−6.5	−4.5	−3	−2	0	1	2	3	4.5	6.5	6.5
2	−4.5	−3	−2	−1	0	2	3	4.5	6.5	6.5	8.5
3	−4.5	−2	−1	0	1	2	4.5	4.5	6.5	8.5	8.5
4	−3	−2	0	1	2	3	4.5	6.5	6.5	8.5	10
5	−2	−1	0	2	3	4.5	6.5	6.5	8.5	8.5	10

5 Conlusion

In this paper, a real system of the double inverted pendulum has been stabilized successfully by the fuzzy controller. The control rules of the double inverted pendulum are presented as well. A new idea of dealing with the multivariate system is displayed. Thus the optimal control thoery is employed in the fuzzy control system design. A powerful fuzzy decision way is given. The experiment of on-line control a double inverted pendulum and with the sampling interval of 4 ms has been done by the look-up table (Table 3).

References

[1] Mori, S. Nishinbara, H. and Furuta, K.. Control of Unstable Mechanical System Control of Pendulum. Int. J. Control, 1976, 23(5): 672—693

[2] Furuta, K. Okutani, T. and Sone, H. Computer Control of a Double Inverted Pendulum. Comput. & Elect. Engng, 5: 67—84

[3] Fruuta, K. Kajiwara, H. and Kosuga, K.. Digital Control of a Double Inverted Pendulum on An Inclined Rail. Int. J. Control, 1980, 32(5): 907—924

模糊规则控制一种绝对不稳定系统

程福雁　钟国民　李友善

（河北机电学院自动化系，石家庄市，050054）

徐征明

（燕山大学自动化系，秦皇岛市，0660000）

摘要：本文设计了一个高精度、高分辨率的模糊控制器，并用以控制二阶倒立摆获得成功．提出了一种处理多变量系统的新观点；给出了模糊控制二阶倒立摆的控制规则；和一种强有力的清晰化方法，从而使模糊控制器的输出更加细腻．应用上述理论设计的高精度、高分辨率的模糊控制器，对二阶倒立摆进行实时控制获得成功．

关键词：模糊控制规则；模糊控制；二阶倒立摆；综合系统

本文作者简介

程福雁　1957 年生．1982 年毕业于河北机电学院自动化系，并留校任教，1991 年考入燕山大学自动化系攻读硕士研究生．主要研究领域为模糊控制．近年来，在国内学术刊物及学术会议上发表数篇学术论文．

钟国民　1939 年生．现任河北机电学院自动化系主任、教授．主要研究领域为模糊控制和过程控制．在国内学术刊物及学术会议上发表二十多篇学术论文．

李友善　1929 年生．1955 年毕业于哈尔滨工业大学．现任河北机电学院自动化系教授．从事理论研究工作．代表性论著有《自动控制原理》、《离散系统理论》、《模糊控制理论及其过程控制中的应用》等．

徐征明　1938 年生．燕山大学自动化系教授．主要研究领域为模糊控制和过程控制．在国内外发表多篇学术论文．