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Simultaneous Identification of ARMA Model Parameters and Orders Using Bi-Channel Lattice Filter*

LI Weihua, XIAO Deyun and FANG Chongzhi
(Department of Automation, Tsinghua University · Beijing, 100084, PRC)

Abstract: This paper presents a new derivation of least squares bi-channel lattice filters, and applies the filters to simultaneously identify ARMA model parameters and order. Numerical results for simulated model show that the identification algorithms presented here have many advantages, such as, less computations, high accuracy and etc.

Key words: lattice filter; parameter estimation; order identification

1 Introduction

Many approaches to estimate ARMA model parameters have appeared, while the orders of the model has been known. However, to our knowlege, how to simultaneously estimate the parameters and identify the orders of an ARMA model is still difficult.

In this paper, a special bi-channel lattice filter (denoted as SBCLF) is constructed for the synchro identification of ARMA model orders and parameters. SBCLF originates from the vector channel lattice filter, which is due to the work of F. Jabbari and J. S. Gibson[1]. In vector channel lattice filters, provided input and output are specified, both time and order updated algorithms to identify ARMA model can be derived. Without question, the methods to be stated following are useful.

Consider ARMA model

$$A(z^{-1})y(k) = B(z^{-1})u(k) + v(k),$$

where, $u(k)$ is input, $y(k)$ is output, and $v(k)$ is white noise. In addition,

$$A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_p z^{-p},$$

$$B(z^{-1}) = 1 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_q z^{-q}.$$

2 Double Channel Lattice Filter

2.1 Order Update

First of all, necessary notations and definitions are given. The history space, in this paper, is the following Hilbert space of infinite column vectors,

$$l_2(R,\lambda) = \{\Psi = (x_1\ x_2\ \cdots)^{\mathrm{T}}, \langle\Psi,\Psi\rangle = \|\Psi\|^2 < \infty\},$$

where $$\langle\Psi,\Psi\rangle = \sum_{i=1}^{n}\lambda^{-1}x_i x_i,$$

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and λ, the forgetting factor, is a positive real number, between 0 and 1. Throughout this paper, $<, >$ represents the inner product of two vectors. A double channel sampled process is specified by two infinite history vectors of $l_2(R,\lambda)$

$$\Psi^1(k) = [y(k)\ y(k-1)\ \cdots\ y(1)\ y(0)\ 0\ \cdots\ 0]^{\mathrm{T}},$$

and

$$\Psi^2(k) = [u(k)\ u(k-1)\ \cdots\ u(1)\ u(0)\ 0\ \cdots\ 0]^{\mathrm{T}},$$

where, $\{u(k-i)\}$ and $\{y(k-i)\}$, $(i = 0,1,\cdots,k)$ are input and output series, alternatively. Note that under the consideration of engineering application, the dimension of $\Psi^i(k)$ for $i = 1,2$, is limited to N, a large enough positive integer.

$\Psi^1(k)$ and $\Psi^2(k)$ consist of a data matrix

$$\Psi(k) = [\Psi^1(k)\ \Psi^2(k)],$$

and span a subspace of $l_2(R,\lambda)$, that is

$$S(k) = \mathrm{span}\{\Psi^1(k), \Psi^2(k)\}. \tag{2.1}$$

While the sum of a series of subspace is denoted as

$$H_n(k) = S(k-1) \oplus S(k-2) \oplus \cdots \oplus S(k-n); \quad H_0(k) = \{0\}, \tag{2.2}$$

and the orthogonal projection operator onto subspace $H_n(k)$ and $P_n(k)$, the forward and backward error spaces can be expressed sequentially as

$$E_0^f(k) = S(k), \quad E_n^f(k) = [I - P_n(k)]S(k),$$

$$E_0^b(k) = S(k), \quad E_n^b(k) = [I - P_n(k+1)]S(k-n).$$

Similarly, if forward and backward error vectors are defined, respectively, as

$$f_0^i(k) = \Psi^i(k), \quad f_n^i(k) = [1 - P_n(k)]\Psi^i(k), \quad i = 1,2, \tag{2.3}$$

$$b_0^i(k) = \Psi^i(k), \quad b_n^i(k) = [1 - P_n(k)]\Psi^i(k), \quad i = 1,2, \tag{2.4}$$

then, $E_n^f(k)$ and $E_n^b(k)$ can be rewritten as

$$E_n^f(k) = \mathrm{span}\{f_n^1(k), f_n^2(k)\}, \quad E_n^b(k) = \mathrm{span}\{b_n^1(k), b_n^2(k)\}.$$

Two $N \times 2$ matrices

$$f_n(k) = [f_n^1(k), f_n^2(k)] \quad \text{and} \quad b_n(k) = [b_n^1(k), b_n^2(k)],$$

as well as three 2×2 matrices $R_n^r(k-1)$, $R_n^e(k)$, and $K_{n+1}(k)$, with their ij th elements

$$\{R_{ij}^r\} = \langle b_n^i(k-1), b_n^j(k-1)\rangle, \quad i = 1,2, \quad j = 1,2,$$

$$\{R_{ij}^e\} = \langle f_n^i(k), f_n^j(k)\rangle, \quad i = 1,2, \quad j = 1,2,$$

$$\{K_{ij}\} = \langle f_n^i(k), b_n^j(K-1)\rangle, \quad i = 1,2, \quad j = 1,2,$$

will be needed for applying the orthogonal projection algorithms.

A infinite history vector of $l_2(R,\lambda)$

$$\Phi = [1\quad 0\quad 0\quad \cdots\quad 0]^{\mathrm{T}}.$$

is defined, in order to 'choose' the variables in which we are interested, correspondingly, the first element of $f_{n+1}(k)$ and $b_{n+1}(k)$ would be

$$e_{n+1}(k) = \Phi^{\mathrm{T}} f_{n+1}(k),$$

and

$$r_{n+1}(k) = \Phi^{\mathrm{T}} b_{n+1}(k).$$

After these definitions are accomplished, the lattice filter algorithms for order update can be derived

$$e_{n+1}(k) = e_n(k) - r_n(k-1)R_n^{-r}(k-1)K_{n+1}^{\mathrm{T}}(k), \tag{2.5}$$

$$r_{n+1}(k) = r_n(k-1) - e_n(k)R_n^{-e}(k)K_{n+1}(k), \tag{2.6}$$

$$R_{n+1}^{e}(k) = R_n^{e}(k) - K_{n+1}(k)R_n^{-r}(k-1)K_{n+1}^{\mathrm{T}}(k), \tag{2.7}$$

$$R_{n+1}^{r}(k) = R_n^{r}(k-1) - K_{n+1}^{\mathrm{T}}(k)R_n^{-e}(k)K_{n+1}(k). \tag{2.8}$$

2.1 Time Update

To accquire the time updated algorithms of $K_{n+1}(k)$, several definitions are added up

$$\hat{\Phi}_n(k+1) = [1 - P_n(k+1)]\Phi, \tag{2.9}$$

$$\hat{H}_n(k+1) = \mathrm{span}\{\Psi^1(k), \Psi^2(k), \Psi^1(k-1), \Psi^2(k-1), \cdots, \Psi^1(k-n+1), \Psi^2(k-n+1), \Phi\},$$

$$G_n^{-1}(k) = \langle \hat{\Phi}_n(k+1), \hat{\Phi}_n(k+1)\rangle^{-1}, \quad i = 1,2.$$

So that, by means of the orthogonality of Hilbert space, the following equations can be gotten[1]

$$K_{n+1}(k+1) = \lambda K_{n+1}(k) + e_n^{\mathrm{T}}(k+1)G_n^{-1}(k)r_n(k), \tag{2.10}$$

$$G_{n+1}(k) = G_n(k) - r_n(k)R_n^{-r}(k)r_n^{\mathrm{T}}(k), \tag{2.11}$$

by the way, in accordance with the original definition of $G_n(k)$, it is obvious that, for $k>0$,

$$G_0(k) = 1.$$

(2.5)～(2.8), (2.10), and (2.11) compose a complete a algorithm set. Provided a series of samples, this set will generate forward and backward residuals subsequently.

3 The Simultaneous Identification of ARMA Model Orders and Parameters

Because lattice filter is able to generate residual series, this feature will be utilized to identify ARMA model orders and the parameters.

From (2.3) and (2.4), it follows that

$$\begin{aligned} f_n(k) &= \Psi(k) - P_n(k)\Psi(k), \\ b_n(k) &= \Psi(k-n) - P_n(k+1)\Psi(k-n), \end{aligned} \tag{3.1}$$

obviously, for $i = 1,2, P_n(k)\Psi^i(k) \in H_n(k), P_n(k+1)\Psi^i(k-n) \in H_n(k+1)$.

Additionally, by means of (2.1) and (2.2), there exist

$$f_n(k) = \Psi(k) - \sum_{i=1}^{n}\Psi(k-j)A_{nj}(k), \tag{3.2}$$

and

$$b_n(k) = \Psi(k) - \sum_{i=1}^{n}\Psi(k-j+1)B_{nj}(k), \tag{3.3}$$

where, $A_{nj}(k)$ and $B_{nj}(k)$ are 2×2 matrices. Because of the specific data matrices, the parameters of ARMA model are included in $\mathrm{A_{nj}(k)}$ *and* $\mathrm{B_{nj}(k)}$. This conclusion is to be proven as follows.

Multiplying the both sides of (3.3) and (3.4) by Φ^{T} yields

$$e_n(k) = [y(k), u(k)] - \sum_{j=1}^{n}[y(k-j), u(k-j)]A_{nj}(k), \tag{3.4}$$

$$r_n(k) = [y(k-n), u(k-n)] + \sum_{j=1}^{n}[y(k-j+1), u(k-j+1)]B_{nj}(k). \quad (3.5)$$

Assume that

$$A_{nj}(k) = \begin{bmatrix} a_{11}^j & a_{12}^j \\ a_{21}^j & a_{22}^j \end{bmatrix}, \quad B_{nj}(k) = \begin{bmatrix} b_{11}^j & b_{12}^j \\ b_{21}^j & b_{22}^j \end{bmatrix},$$

then, after substituting them into (3.4) and (3.5), separately, we get the first element of $e_n(k)$, which takes the form of

$$e_n^1(k) = y(k) - \sum_{j=1}^{n} y(k-j)a_{11}^j - \sum_{j=1}^{n} u(k-j)a_{21}^j. \quad (3.6)$$

Clearly, the right hand side of the last equation is a standard ARMA (n,n) model, and the left hand side is the forward error. Furthermore, even if q is not equal to p, (3.6) can still be used to express an ARMA (p,q) model. For instance, without the loss of generality, suppose that p be not less than q. Several authors have suggested criteria to estimate p, the order of lattice filter, see also [2] and [3]. After p is determined, q will be identified automatically. This inference will be rectified by numerical simulations.

From what stated above, it is infered that, the estimation of ARMA model parameters means to derive the recursive algorithm of $A_{nj}(k)$ and $B_{nj}(k)$.

The substitution of (3.4) and (3.5) into (2.5) and (2.6), yields

$$A_{(n+1)j}(k) = A_{nj}(k) - B_{nj}(k-1)R_n^{-r}(k-1)K_{n+1}^{\mathrm{T}}(k), \quad (3.7)$$

$$B_{(n+1)(j+1)}(k) = B_{nj}(k-1) - A_{nj}(k)R_n^{-e}(k)K_{n+1}(k),$$

$$j = 1,2,3,\cdots,n; \quad n = 1,2,3,\cdots, \quad (3.8)$$

with the initial and end conditions.

$$A_{(n+1)(n+1)}(k) = R_n^{-r}(k-1)K_{n+1}^{\mathrm{T}}(k), \quad n = 0,1,2, \quad (3.9)$$

$$B_{(n+1)1}(k) = R_n^{-e}(k-1)K_{n+1}(k), \quad n = 0,1,2. \quad (3.10)$$

Although the AR coefficients $A_{(n+1)j}(k)$ can be generated with (3.7) ~ (3.10), this algorithm requires that the AR coefficients be calculated at every sampling time, since $B_{nj}(k-1)$ is needed to compute. The following derivation provides an algorithm for computing the AR coefficients at any time t without the value of $B_{nj}(k-1)$ and $A_{nj}(k-1)$.

From the definitions of $P_n(k)$ and $P_n^b(k)$ in section 2, it follows that

$$P_{n+1}(k+1) = P_n(k+1) + P_n^b(k).$$

Combining this equation with (2.9) and using the orthogonality of Hilbert space will generate

$$\hat{\Phi}_{n-1}(k+1) = [1 - P_n(k+1)]\Phi - P_n^b(k)\Phi$$
$$= \hat{\Phi}_n(k+1) - b_n(k)R_n^{-r}(k)r_n^{\mathrm{T}}(k). \quad (3.11)$$

Since $\quad P_n(k+1)\Phi \in H_n(k+1), \quad P_{n+1}(k+1)\Phi \in H_{n+1}(k+1),$

besides, according to (2.1), (2.2) and (2.9), there exist

$$\hat{\Phi}_n(k+1) = \Phi - \sum_{j=1}^{n} \Psi(k-j+1)C_{nj}(k); \quad n = 1,2,\cdots, \tag{3.12}$$

$$\hat{\Phi}_{n+1}(k+1) = \Phi - \sum_{j=1}^{n+1} \Psi(k-j+1)C_{(n+1)j}(k); \quad n = 1,2,\cdots. \tag{3.13}$$

After the substitution of (3. 12) and (3. 13) into (3. 11), the recursive algorithm of $C_{nj}(k-1)$ will be

$$C_{(n+1)j}(k) = C_{nj}(k) - B_{nj}(k)R_n^{-r}(k)r_n^{\mathrm{T}}(k); \quad j = 1,2,\cdots,n, \quad n = 0,1,\cdots \tag{3.14}$$

with end conditions

$$C_{(n+1)(n+1)}(k) = R_n^{-r}(k)r_n^{\mathrm{T}}(k); \quad n = 0,1,2,\cdots. \tag{3.15}$$

The expression $b_n(k)$ will denote the part of $\hat{b}_n(k)$ below the first row. Then, from (3. 3)

$$\hat{b}_n(k) = \Psi(k-n-1) - \sum_{j=1}^{n} \Psi(k-j)B_{nj}(k), \tag{3.16}$$

next, the development in secton 2, and the definition of $b_n(k)$ leads to

$$b_n(k) = \begin{bmatrix} 0 \\ b_n(k-1) \end{bmatrix} + \hat{\Phi}_n(k+1)G_n^{-1}(k)r_n(k), \tag{3.17}$$

morever, in (3. 17), replacing $\hat{\Phi}_n(k+1)$ by (3. 12), we get new equation from the second row down

$$\hat{b}_n(k) = b_n(k-1) - \sum_{j=1}^{n} [\Psi(k-1)]C_{nj}(k)G_n^{-1}(k)r_n(k). \tag{3.18}$$

Finally, equating the right sides of (3. 16) and (3. 18), using the right side of (3. 3) for $b_n(k-1)$, and matching the coefficients of the history space, yield

$$B_{nj}(k-1) = B_{nj}(k) - C_{nj}(k)G_n^{-1}(k)r_n(k);$$
$$j = 1,2,\cdots,n; \quad n = 1,2,\cdots,k-1. \tag{3.19}$$

The identification of an ARMA model can be carried out based on (3. 7), (3. 8), (3. 14) and (3. 19), where (3. 9), (3. 10) and (3. 15) are used as end conditions.

4 The Implementation

4.1 Initialization

The first row of matrix $\Psi(k)$ is denoted by

$$X(k) = [y(k) \quad u(k)].$$

We start the lattice filter at time $k = 0$, with the following initialization:

$$R_0^e(-1) = R_0^r(-1) = \sigma I,$$

where σ is a very small number, and

$$K_{n+1}(k) = 0, \quad \text{for} \quad n+1 > k > 0.$$

4.2 Double Channel Lattice Filter

For each $k > 0$,

$$e_0(k) = r_0(k) = X(k),$$

$$R_0^e = R_0^r(k) = X^{\mathrm{T}}(k)X(k) - \lambda R_0^{-r}(k-1), \quad G_0(k) = 1,$$

for each $k \geq 1, 0 < n \leq k-1$, (2.5)～(2.8), (2.10) and (2.11) generate a series of forward and backward error vectors.

4.3 The Identification of ARMA Model

(3.7), (3.8), (3.14) and (3.19) form a complete algorithms set, with end conditions: (3.9), (3.10) and (3.15).

The order of lattice filter can be determined according to a criterion, for example, Akaike Criterion.

5 Simulation

example 1 $z(k)[1-0.7z^{-1}+0.1z^{-2}]=[z^{-1}+0.4z^{-2}]u(k)+\lambda v(k)$.

example 2 $z(k)[1-0.8z^{-1}+0.15z^{-2}]=[0.45z^{-1}]u(k)+\lambda v(k)$.

Where, $z(k)$ is output, input $u(k)$ and interference $v(k)$ are white noise sequences, whose magnitudes are, on the average, between 15 to 20% of their corresponding outputs. In example 1, p and q are determined as 2. The identified parameters are shown in Fig. 1. and Fig. 2 In example 2, we can determine that p is 2, but q is 1.

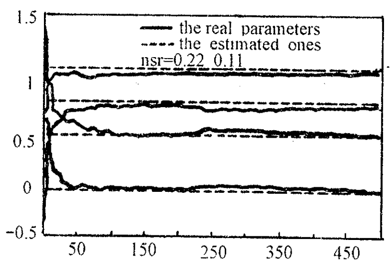

Fig. 1 The identified results of example 1

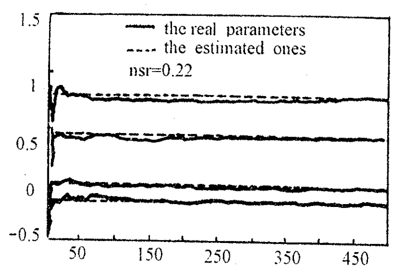

Fig. 2 The identified results of example 2

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利用双通道格形滤波器同时辨识 ARMA 模型的参数和阶次

李渭华　萧德云　方崇智

(清华大学自动化系·北京, 100084)

摘要: 本文重新构造了一种最小二乘通道格形滤波器,并将它用于同时辨识 ARMA 模型的参数和阶

次.数值仿真结果表明,这种辨识算法具有计算量小、精度高等优点.

关键词: 格形滤波器; 参数估计; 阶次辨识

本文作者简介

李渭华 1963年生.1982年获华南工学院自动化系学士学位,1988年获江西工业大学电机系硕士学位,1994年获清华大学自动化系博士学位,1995年到加拿大 Western Ontario 大学从事博士后工作,主要致力于故障诊断、智能信号处理的研究.

萧德云 1945年生.1970年毕业于清华大学,现为清华大学自动化系教授,出版译著有《过程辨识》、《系统辨识》、《过程控制系统》等,在国内外发表论文30余篇,主要从事过程控制、辨识建模、故障诊断、大型过程工业CIMS的研究.

方崇智 1921年生.重庆中央大学机械工程系工学士,英国伦敦马丽皇后学院哲学博士.现任清华大学自动化系教授,学术方向为自动控制理论及应用,尤其是工业过程的建模、控制与优化.

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会议期间,吴刚、伍清河、陈善本、慕小武、吴志华、洪奕光等还组织了"先进控制理论在实际应用中若干问题"、"鲁棒控制"和"混沌控制"专题讨论会,代表们踊跃参加,积极发表自己的见解,对这些方向今后的发展和组织学术活动提出了很多很好的建议.

会议期间,控制理论专业委员举行了两次工作会议,讨论了这次会议的组织安排及《中国控制会议论文集》的出版、印刷等问题,并就明年可能承办《中国控制会议》单位、地点等事项交换了意见.

第二届关肇直奖评奖委员们认真听取了四名关肇直奖候选论文作者的学术报告,经无记名投票,一致同意将第二届关肇直奖授予东南大学的田玉平博士和黄一博士合作的论文《具有线性和非线性不确定性的多变量系统的鲁棒稳定性》.

中国自动化学会控制理论专业委员会主任秦化淑研究员主持了大会的闭幕式.在闭幕式上,第二届关肇直奖评奖委员会主任黄琳教授宣布了第二届关肇直奖获奖论文.中国科学院院士、中国自动化学会理事长杨嘉墀研究员向获奖者颁发了证书和奖金并讲了话.他鼓励控制界的青年们要在攀登控制科学高峰、促进我国自动化技术的发展及其转化为生产力的过程中作出积极的贡献.第二届关肇直奖获奖论文作者之一田玉平博士及老一辈自动控制科学家、清华大学方崇智教授也在闭幕式上讲了话.方先生勉励青年人要深入实际,那里是大有可为的,一定能为控制理论的应用做出更有效的成果.中国自动化学会控制理论专业委员会副主任郑应平研究员在闭幕词中充分肯定了大会取得的一系列成果,并希望今后更好地调动青年的积极性,进一步提高会议质量、加速与国际接轨.

(张纪峰)