BIBO Stabilization of Large-Scale Systems *

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Abstract: This paper first gives a decentralized scheme for bounded input bounded output (BIBO) stabilization of a class of large-scale systems without perturbations in each subsystem, the only stabilizability condition is the uniform complete controllability of the subsystems. Then the problem of robust BIBO stabilization of the large-scale systems with nonlinearly perturbations in the subsystems is considered and the bound of permissible perturbations is obtained by using a stabilizing local state feedback. Finally, we apply the obtained results to the single linear system and improve a previous result.

Key words: large-scale systems; BIBO stabilization; robustness

1 Introduction

The theory of large-scale systems has made great progress in the past twenty years, also many papers have been published on the stabilization of large-scale systems[1,2]. In particular, Ikeda and Siljak[2] proposed a decentralized control law to stabilize exponentially the linear time-varying interconnected systems. However, in general, it is expected that the control systems can track input signals. In this paper, we first discuss the BIBO stabilization problem of large-scale systems with reference input signals. Based on the matrix Riccati equation, by making use of the scalar Lyapunov function and combining with the Bihari-type inequality, we present a class of linear interconnected systems with reference input signals which can always be BIBO stabilized by the local feedback control[2]. The stabilizability condition is uniform complete controllability of the subsystems.

On the other hand, the robust stabilization problem of control systems in the presence of uncertainty has been receiving considerable attention, because in many practical control problems, uncertainty often occurs in systems due to modelling errors, measurement errors , linearization approximations , and so on . Many design techniques for uncertain

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* Part of work is supported by the Foundation of the Sichuan Education Commission and the National Natural Science Foundation of China.
Manuscript received Apr. 22, 1994, revised Dec. 13, 1994.

systems have been developed to guarantee a required degree of robustness[3,4]. However, most of these papers only considered the robust stability and stabilization in Lyapunov meaning. Wu and Mizukami[3] dealt with the robust BIBO stabilization of single uncertain linear systems. By using the presented approach, we discuss the robust BIBO stabilization of large-scale system with nonlinearly perturbation, By a robust linear state feedback controller we can guarantee that the large-scale system is BIBO stable. The obtained result is applied to the single system[3] and enlarge the allowable perturbation bound given by Wu and Mizukami[3].

2 BIBO stabilization

Consider a large-scale system S composed of N interconnected subsystems S_i described by the equations

$$S_i: \qquad \dot{x}_i = A_i x_i + \sum_{j=1}^{N} B_i F_{ij} C_j x_j + B_i u_i, \quad y_i = C_i x_i, \quad i = 1,2,\cdots,N, \tag{1}$$

where $x_i(t), u_i(t), y_i(t)$ are n_i, r_i, m_i vectors, $A_i(t), B_i(t), C_i(t)$, and $F_{ij}(t)$ are matrices of appropriate dimensions with time-varying elements which are measurable and bounded on every finite subinterval of time t.

Ikeda and Siljak[2] proposed the scheme for exponential stabilization of the large-scale system S. In general, it is expected that the control systems can track input singals, such as step function. The main objective of this section is to find the decentralized local state feedback control

$$u_i = K_i x_i + r_i, \quad i = 1,2,\cdots,N, \tag{2}$$

where $r_i(t)$ is the reference input signal and $K_i(t)$ is the feedback gain matrix of appropriate dimension with the elements having the same properties as those of $A_i(t)$, such that the large-scale system S is BIBO stable. Applying the state feedback controller (2) to the system (1) yields a closed-loop subsystem as follows:

$$\hat{S}_i: \qquad \dot{x}_i = (A_i + B_i K_i) x_i + \sum_{j=1}^{N} B_i F_{ij} C_j x_j + B_i r_i, \quad i = 1,2,\cdots,N. \tag{3}$$

Definition 1 The signal vector $r(t) = [r_1, \cdots, r_N]^{\mathrm{T}} \in L_\infty^n$ if $\| r(t) \|_\infty \triangleq \sup_{(t_0,\infty)} \| r(t) \| < \infty$.

Definition 2 The system S is said to be bounded input bounded output (BIBO) stabilizable by the local control law (2) if there exists a feedback gain matrix K such that every solution $x(t)$ of the closed-loop system $\hat{S}$ satisfies

$$\| x(t) \| \leqslant \theta_1 \| r(t) \|_\infty + \theta_2, \tag{4}$$

where θ_1 and θ_2 are positive numbers.

Theorem 1 The system S is BIBO stabilizable by the local state feedback (2) if every subsystem S_i is uniformly completely controllable.

Proof According to the results in [2], The uniform complete controllability of the subsystem S_i implies that the matrix Riccati equation

$$\dot{P}_i + A_i^{\mathrm{T}}P_i + P_iA_i - P_iB_iB_i^{\mathrm{T}}P_i + NI_i = 0, \quad i = 1,2,\cdots,N \tag{5}$$

has a solution $P_i(t)$ such that

$$\xi_iI_i \leqslant P_i \leqslant \eta_iI_i \tag{6}$$

holds for some positive numbers ξ_i, η_i, and all t. The inequalities (6) mean that $P_i - \xi_iI_i$ and $\eta_iI_i - P_i$ are symmetric nonnegative definite matrices, where I_i is the $n_i \times n_i$ identical matrix. With this P_i, we have the local state feedback gain

$$K_i = -\frac{h_i}{2}B_i^{\mathrm{T}}P_i, \quad i = 1,2,\cdots,N, \tag{7}$$

where $h_i(t)$ is an arbitrary scalar function, which is measurable and bounded on every finite subinterval of time, and which satisfies the inequality

$$h_i \geqslant 1 + \sum_{j=1}^{N}\delta^2 \| F_{ij}C_j \|^2 \quad (\delta > 1 \text{ constant}) \tag{8}$$

for all t. Define a scalar Lyapunov function:

$$V(t,x) = \sum_{i=1}^{N}x_i^{\mathrm{T}}(t)P_i(t)x_i(t), \tag{9}$$

for the overall closed-loop system $\hat{S}$ given by (3). The function V of (9) satisfies $\xi \| x \|^2 \leqslant V \leqslant \eta \| x \|^2$ for all t and x, where $\xi = \min_i\{\xi_i\}, \eta = \max_i\{\eta_i\}$.

Now, we calculate the total time derivative of the function $V(t,x)$ with respect to (3) as follows

$$\begin{aligned}
\dot{V} = &\sum_{i=1}^{N}\{x_i^{\mathrm{T}}[\dot{P}_i + A_i^{\mathrm{T}}P_i + P_iA_i - h_iP_iB_iB_i^{\mathrm{T}}P_i]x_i + \sum_{j=1}^{N}(B_iF_{ij}C_jx_j)^{\mathrm{T}}P_ix_i \\
&+ x_i^{\mathrm{T}}P_i\sum_{j=1}^{N}B_iF_{ij}C_jx_j + (B_ir_i)^{\mathrm{T}}P_ix_i + x_i^{\mathrm{T}}P_iB_ir_i\} \\
\leqslant &\sum_{i=1}^{N}\{x_i^{\mathrm{T}}[-NI_i - (h_i - 1)P_iB_iB_i^{\mathrm{T}}P_i]x_i + 2\| P_iB_i \| \ \| x_i \| \ \| r_i \| \\
&+ 2\sum_{j=1}^{N}x_j^{\mathrm{T}}(F_{ij}C_j)^{\mathrm{T}}(B_i^{\mathrm{T}}P_ix_i)\} \\
\leqslant &\sum_{i=1}^{N}\{-N\| x_i \|^2 - \sum_{j=1}^{N}\delta^2 \| F_{ij}C_j \|^2 x_i^{\mathrm{T}}P_iB_iB_i^{\mathrm{T}}P_ix_i + 2\| P_iB_i \| \ \| x_i \| \ \| r_i \| \\
&+ 2\sum_{j=1}^{N}\| x_j \| \ \| F_{ij}C_j \| \ \| B_i^{\mathrm{T}}P_ix_i \| \} \\
= &-\sum_{i=1}^{N}\sum_{j=1}^{N}[\delta^{-1}\| x_j \| - \delta \| F_{ij}C_j \| \ \| B_i^{\mathrm{T}}P_ix_i \|]^2 \\
&+ \sum_{i=1}^{N}[-N(1-\delta^{-2})\| x_i \|^2 + 2\| P_iB_i \| \ \| x_i \| \ \| r_i \|] \\
\leqslant &\sum_{i=1}^{N}[-N(1-\delta^{-2})\| x_i \|^2 + 2\| P_iB_i \| \ \| x_i \| \ \| r_i \|].
\end{aligned} \tag{10}$$

Noting that $\sum_{i=1}^{N}\| x_i \| \leqslant \sqrt{N\sum_{i=1}^{N}\| x_i \|^2} = \sqrt{N}\| x \|$, we get $\sum_{i=1}^{N}\| x_i \| \leqslant$

$\sqrt{N\xi^{-1}}V^{1/2}$ and

$$\dot{V} \leqslant -2\lambda_1 V + \lambda_2 V^{1/2}, \tag{11}$$

where $2\lambda_1 = N(1-\delta^{-2})/\eta, \lambda_2 = 2\sqrt{N/\xi}\max_i\{\|P_iB_i\|\}\|r(t)\|_\infty$. This yields that

$$e^{2\lambda_1(t-t_0)}V \leqslant V(t_0) + \lambda_2\int_{t_0}^{t} e^{\lambda_1(s-t_0)}[e^{2\lambda_1(s-t_0)}V(s)]^{1/2}\mathrm{d}s. \tag{12}$$

By the Bihari-type inequality [5, Theorem 1. 3. 1]. we obtain

$$e^{2\lambda_1(t-t_0)}V \leqslant \{[V(t_0)]^{1/2} + \frac{1}{2}\lambda_2\int_{t_0}^{t} e^{\lambda_1(s-t_0)}\mathrm{d}s\}^2. \tag{13}$$

After simple manipulations, we finally obtain for any $t \geqslant t_0$

$$\begin{aligned}\|x\| &\leqslant \sqrt{\frac{V}{\xi}} \leqslant \frac{1}{\sqrt{\xi}}\{[V(t_0)]^{1/2}e^{-\lambda_1(t-t_0)} + \frac{\lambda_2}{2\lambda_1}(1-e^{-\lambda_1(t-t_0)})\} \\ &\leqslant \frac{\sqrt{V(t_0)}}{\sqrt{\xi}} + \frac{\lambda_2}{2\lambda_1\sqrt{\xi}} \leqslant \theta_1\|r(t)\|_\infty + \theta_2,\end{aligned} \tag{14}$$

where $\theta_1 = \sqrt{N}\max_i\{\|P_iB_i\|\}/\xi\lambda_1, \theta_2 = \sqrt{\eta/\xi}\|x(t_0)\|$. This proves Theorem 1.

3 Robust BIBO stabilization

Now, we consider an uncertain large-scale dynamical system with nonlinearly perturbations in each subsystem. Then the state space description of the closed-loop system can be rewritten as

$$\dot{x}_i = (A_i + B_iK_i)x_i + \sum_{j=1}^{N}B_iF_{ij}C_jx_j + (B_i + \Delta B_i)r_i + g_i, \quad i = 1,2,\cdots,N \tag{15}$$

where ΔB_i is the input uncertainty and bounded [3], and $g_i(t,x_i)$ is a nonlinear perturbation vector caused by the input uncertainty and others and satisfies:

$$\|g_i(t,x_i)\| \leqslant b_i\|x_i\|. \tag{16}$$

A system is said to be robustly BIBO stabilizable, if it is tolerant to change in certain specific bounds of perturbation.

Theorem 2 Let P_i with the condition (6) be a solution of the following Riccati equation

$$\dot{P}_i + (A_i + \alpha I_i)^{\mathrm{T}}P_i + P_i(A_i + \alpha I_i) - \mu P_iB_iB_i^{\mathrm{T}}P_i + Q_i = 0, \quad i = 1,2,\cdots,N, \tag{17}$$

where α and μ are positive numbers, $Q_i(t)$ is a symmetric matrix and satisfies $\tau_iI_i \leqslant Q_i \leqslant \xi_iI_i$ for some positive numbers τ_i, ξ_i, and all t. Suppose s is the number of $\max_j\{\|F_{ij}C_j\|\} \neq 0$ for $i = 1,\cdots,N$. If every subsystem S_i is uniformly completely controllable, and if the perturbation $g_i(t,x_i)$ of each S_i satisfies (17) and

$$\min_i\{2\xi_i\alpha + \tau_i - s - 2b_i\|P_i\|\} \triangleq \omega > 0 \tag{18}$$

then the system (16) is robustly BIBO stabilizable by the local state feedback (2).

Proof This proof follows directly from the proof of the Theorem 1. We choose h_i in (7) such that

$$h_i - \mu \geqslant \sum_{j=1}^{N} \| F_{ij}C_j \|^2. \tag{19}$$

Without loss of generality, let $\max_j\{ \| F_{ij}C_j \| \} \neq 0$ for $i = 1,\cdots,s$ and $\max_j\{ \| F_{ij}C_j \| \} = 0$ for $i = s+1,\cdots,N$. Imitating the calculation of (11) and using (19) and (20), the derivative of $V(t,x)$ satisfies

$$\begin{aligned} \dot{V} \leqslant & - 2\alpha V - \sum_{i=1}^{s}\sum_{j=1}^{N}[\, \| x_j \| - \| F_{ij}C_j \| \; \| B_i^{\mathrm{T}}P_i x_i \| \,]^2 \\ & + \sum_{i=1}^{N}[(-\tau_i + s + 2b_i \| P_i \|) \| x_i \|^2 \\ & + 2(\| P_iB_i \| + \| \Delta B_i \| \; \| P_i \|) \| x_i \| \; \| r_i \| \,] \\ \leqslant & \sum_{i=1}^{N}[- \omega \| x_i \|^2 + 2(\| P_iB_i \| + \| \Delta B_i \| \; \| P_i \|) \| x_i \| \; \| r_i \| . \end{aligned} \tag{20}$$

The proof of the rest is similar to that of the remainder of Theorem 1 and omitted.

Finally, we consider the linear system [3]:

$$\dot{x} = (A + \Delta A)x + (B + \Delta B)u + (B + \Delta B)r. \tag{21}$$

where $x \in \mathrm{R}^n, u \in \mathrm{R}^m, A, B$, and C are known constant matrices of appropriate dimensions, ΔA is the system matrix uncertainty, ΔB is the input matrix uncertainty.

By the linear state feedback controller

$$u = -\frac{k}{2}B^{\mathrm{T}}Px, \tag{22}$$

where k is a positive number and P is a symmetric positive definite matrix satisfying the following Riccati equation

$$(A + \alpha I)^{\mathrm{T}}P + P(A + \alpha I) - kPBB^{\mathrm{T}}P = - 2\gamma Q, \tag{23}$$

Wu et al. gave the following sufficient condition [3, Theorem 2] to stabilize (22) robustly

$$\left(\beta_1 + \frac{k}{2}\beta_2 \| B \|_E \| P \|_E\right) \| P \| < \alpha\lambda_m(P) - \gamma\lambda_m(Q). \tag{24}$$

where $\beta_1 \geqslant \| \Delta A \|_E$ and $\beta_2 \geqslant \| \Delta B \|_E$.

Applying Theorem 2, we get the following corollary which improve the condition (25).

Corollary If we choose parameters α, γ, k and a matrix Q such that for every reference input $r(t) \in L_\infty^n$

$$\left(\beta_1 + \frac{k}{2}\beta_2 \| B \| \; \| P \|\right) \| P \| < \alpha\lambda_m(P) + \gamma\lambda_m(Q), \tag{25}$$

then, by using the linear state feedback controller (23), the system (22) is robustly BIBO stabilizable.

Remark According to the compatibility of the norms of matrix and vector, we may take the norm $\| \cdot \|_E$ in (25). Therefore, the Corollary enlarge the allowable perturbation bound of ΔA and ΔB given by Theorem 2 in [3].

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大系统的BIBO稳定性

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摘要:本文首先给出了一类大系统有界输入有界输出(BIBO)稳定化的一种分散控制方案.其可稳定化的条件仅仅是其子系统一致完全可控.然后考虑了每个子系统含有非线性摄动的大系统鲁棒稳定化问题,其摄动的允许界通过局部状态反馈获得.最后应用获得的结果到线性单系统,改进了 Wu 等人[3]的一些近期结果.

关键词:大系统;有界输入有界输出稳定化;鲁棒性

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