Internal Model Control of Multivariable Nonlinear Processes with Time Delays

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Abstract: The nonlinear internal model control (NIMC) design concept is extended to MIMO nonlinear systems with time delays. For the process without time delays. the controller is designed to provide nominal performances, and the nonlinear filter is added to obtain robustness for the plant/model mismatch then to make the controller implementable. When the plant has multiple time delays, by adding time delays to some inputs of a plant and its model, NIMC based on the auxiliary model without time delays can be designed. The performance of the proposed control methodology is evaluated through simulation for a reactor example.

Key words: nonlinear control system; time delays; predictive control; process control.

1 Introduction

Many common chemical processes such as distillation columns and chemical reactions are inherently nonlinear and strongly interacting. For the process control, the customary model-based control strategies for nonlinear coupled processes have been to neglect nonlinearities by local linearization in the operating point, and to apply linear theory to design linear controllers. For highly nonlinear processes, however, linear feedback controllers must be detuned significantly to ensure their stability when the plant is beyond the small neighborhood of the operating point. Among several design methods of the controller based on the model, IMC is a powerful controller design strategy for linear systems[1]. An IMC strategy for nonliner SISO systems was proposed by Henson and Seborg[2], but their work is limited to minimum-phase processes. In this paper, the IMC controller design approach for multivariable nonlinear processes (MNIMC) is investigated. The SISO nonlinear IMC is extended to open-loop stable multivariable nonlinear systems with the stable inversion, and IMC scheme for nonlinear processes with time delays is suggested.

1.1 Process Model

We first consider a class of MIMO nonlinear plants with equal number of inputs and outputs of the form

$$\dot{X} = \hat{f}(X) + \sum_{j=1}^{m} \hat{g}_j(X)u_j, \quad \hat{y}_i = \hat{h}_i(X), \quad i = 1,2,\cdots,m, \tag{1}$$

where $X = (x_1, x_2, \cdots, x_n)$ is the n-dimensional state vector, $Y = (y_1, y_2, \cdots, y_m)$ is the

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m-dimensional output variable and $U = (u_1, u_2, \cdots, u_m)$ is the m-dimensional input vector respectively, $f(X)$ is a smooth vector field on R^n, $g_1(X), \cdots, g_m(X)$ are smooth vector fields on R^m and $m \leqslant n$. Now we assume that the available model for the controller design has the form

$$\dot{X} = f(X) + \sum_{j=1}^{m} g_j(X) u_j, \quad y_i = h_i(X), \quad i = 1, 2, \cdots, m, \tag{2}$$

where $X, Y, U, f(X), g(X)$ and $h(X)$ are defined similarly to the plant.

1.2 Relative Order and Characteristic Matrix[3]

Definition 1 Given a multivariable nonlinear model, we say that the ith output y_i has relative order r_i if

$$L_g L_f^k h_i(X) = 0, \quad i = 1, 2, \cdots, m, \quad k = 0, \cdots, r_i - 2, \tag{3}$$

$$L_g L_f^{r_i-1} h_i(X) = [L_{g_1} L_f^{r_i-1} h_i(X) L_{g_2} L_f^{r_i-1} h_i(X) \cdots L_{g_m} L_f^{r_i-1} h_i(X)] = 0. \tag{4}$$

By definition 1, the derivatives of the outputs can be represented as

$$y_i^{(k)} = L_f^{(k)} h_i(X), \quad k = 1, 2, \cdots, r_i - 1,$$
$$y_i^{(r_i)} = L_f^{(r_i)} h_i(X) + \sum_{j=1}^{m} L_{g_j} L_f^{(r_i-1)} h_i(X) u_j, \tag{5}$$

i.e, r_i is the smallest order of derivative of y_i that explicitly depends on the vector u_i.

Definition 2 Consider the model of the form of Eq. 2 and assume that each output y_i possesses relative order r_i, the matrix $c(X) = [c_{ij}]$ which the ith line elements are

$$[L_{g_1} L_f^{r_i-1} h_i(X) \cdots L_{g_m} L_f^{r_i-1} h_i(X)], \quad i = 1, 2, \cdots, m \tag{6}$$

is called the characteristic matrix of the model(2).

1.3 Inversion of Nonlinear Multivariable System

The inversion of nonlinear multivariable systems has been solved by Hirschorn[1]. An explicit formula for calculation of the inversion is given as follows[5]: Consider a MIMO nonlinear system and its model is given by the form of Eq. (2) with finite relative order r_i, $i = 1, \cdots, m$, and nonsingular characteristic matrix $c(X)$, then the dynamic system,

$$\dot{\eta} = f(\eta) + g(\eta) c(\eta)^{-1} ((y_1^{(r_1)}, \cdots, y_m^{(r_m)})^T - (L_f^{r_1} h_1(\eta), \cdots, L_f^{r_m} h_m(\eta))^T),$$
$$U = c(\eta)^{-1} ((y_1^{(r_1)}, \cdots, y_m^{(r_m)})^T - (L_f^{r_1} h_1(\eta), \cdots, L_f^{r_m} h_m(\eta))^T) \tag{7}$$

is a realization of the inverse of the model (2).

2 An Internal Model Controller Design for MIMO Nonlinear Systems

The SISO IMC structure[2] and the proposed MIMO IMC control scheme are shown in Fig. 1 and Fig. 2.

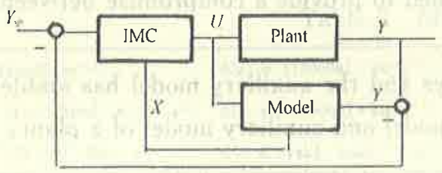

Fig. 1 SISO nonlinear IMC systems

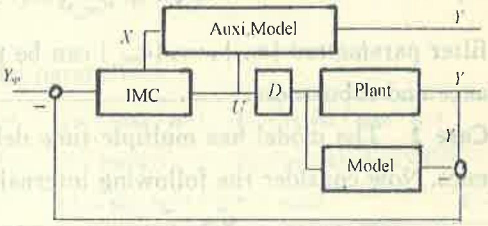

Fig. 2 General structure IMC for MIMO nonlinear systems

Where D is the time delay matrix, y^* is the output of model without time delays, and IMC is the controller, its design is following on the assumptions $1\sim5$[2]:

Case 1 The model has no time delays.

The dead time matrix is selected as $D(t) = \mathrm{diag}\{1,1,\cdots,1\}$ and auxiliary state variables and outputs have the form

$$\dot{X}^* = \dot{X} = f(X) + \sum_{j=1}^{m} g_j(X)u_j, \quad y^* = y_i = h_i(X), \quad i = 1,2,\cdots,m. \tag{8}$$

To track the setpoint, the model inverse controller IMC can be chosen as in Eq. 7 with $y_{sp}^{(r)}$ as the input

$$U = c(X)^{-1}((y_{sp1}^{(r_1)},\cdots,y_{spm}^{(r_m)})^{\mathrm{T}} - (L_f^{r_1}h_1(X),\cdots,L_f^{r_m}h_m(X))^{\mathrm{T}}). \tag{9}$$

If the following initial conditions are satisfied

$$y_i^{(k)}(0) = y_{spi}^{(k)}(0), \quad 0 \leqslant k \leqslant r_i - 1, \quad i = 1,2,\cdots,m, \tag{10}$$

and model/plant has no mismatch, then perfect control

$$y_i(t) = y_{spi}(t), \quad t > 0, \quad i = 1,2,\cdots,m \tag{11}$$

is obtained by the model inverse controller (9).

The controller (9) would yield the best output response. However, this controller cannot be implemented for the well-known limitation[2]. Therefore, the nonlinear filter is used to handle those limitations. The filter has the following form

$$v_i^{(r_i)} = -\alpha_{ir_i}y_i^{(r_i-1)} - \alpha_{ir_i-1}y_i^{(r_i-2)} - \cdots - \alpha_{i_1}y_i + \alpha_{i_1}e_i, \quad i = 1,2,\cdots,m \tag{12}$$

where $v_1^{(r_1)},\cdots,v_m^{(r_m)}$ are filters, $\{\alpha_{1r_1}\},\cdots,\{\alpha_{mr_m}\}$ are filter tuning parameters, and error signals $e_1,\cdots,e_m$ are given by $e_i = y_{spi} - \hat{y}_i + y_i$, $i = 1,\cdots,m$. Combining Eq. (5), (9) and (12), the multivariable nonlinear IMC controller

$$U = c(X)^{-1}((\alpha_{11}e_1,\cdots,\alpha_{m1}e_m)^{\mathrm{T}} - (\sum_{k=1}^{r_1+1}\alpha_{1k}L_f^{k-1}h_1(X),\cdots,\sum_{k=1}^{r_m+1}\alpha_{mk}L_f^{k-1}h_m(X))^{\mathrm{T}}) \tag{13}$$

is obtained, where $\alpha_{ir_i+1} = 1$. If the model is perfect and the following initial conditions are satisfied

$$y_i^{(k)}(0) = e^{(k)}(0), \quad i = 1,2,\cdots,m, \quad 0 \leqslant k \leqslant r_i - 1, \tag{14}$$

then by combining Eqs. (5), (12) and (13) a closed-loop transfer function matrix between the controlled output y and its setpoints y_{sp} can be obtained as follows:

$$Y = \mathrm{diag}\{(S^{r_1} + \alpha_{1r_1}S^{r_1-1} + \cdots + \alpha_{11}S + \alpha_{11})^{-1},\cdots,$$

$$(S^{r_m} + \alpha_{mr_m}S^{r_m-1} + \cdots + \alpha_{m1}S + \alpha_{m1})^{-1}\}Y_{sp}. \tag{15}$$

The filter parameters $\{\alpha_{1r_1}\},\cdots,\{\alpha_{mr_m}\}$ can be tuned to provide a compromise between performance and robustness.

Case 2 The model has multiple time delays and the auxiliary model has stable zero dynamics. Now consider the following internal model and auxiliary model of a plant:

$$\dot{X} = f(X) + \sum_{j=1}^{m} g_j(X)u_j(t-\tau_j), \quad y_i = h_i(X), \quad i = 1,2,\cdots,m, \tag{16}$$

$$\dot{X}^* = f(X^*) + \sum_{j=1}^{m} g_j(X^*)u_j(t), \quad y_i^* = h_i(X^*), \quad i = 1,2,\cdots,m. \tag{17}$$

For the system of Eq. (15), like linear GMDC[6], let

$$D(s) = \mathrm{diag}\{e^{-(\tau-\tau_1)s},\cdots,e^{-(\tau-\tau_m)s},\}, \quad \tau = \max\{\tau_i\}, \quad i = 1,2,\cdots,m. \tag{18}$$

For delayed system, its model which has the same dead-time for every input can be split into two parts: deadtime free defined by Eq. (17) and dead-time matrix, i. e. , $\mathrm{diag}\{e^{-\tau s},\cdots,e^{-\tau s}\}$, then use Eq. (13) as a controller by substituting X with X^*, i. e. ,

$$U = c(X^*)^{-1}((\alpha_{11}e_1,\cdots,\alpha_{m1}e_m)^{\mathrm{T}} - (\sum_{k=1}^{r_1+1}\alpha_{1k}L_f^{k-1}h_1(X^*),\cdots,\sum_{k=1}^{r_m+1}\alpha_{mk}L_f^{k-1}h_m(X^*))^{\mathrm{T}}). \tag{19}$$

We compare Eq. (16) with Eq. (17), then yields:

$$X^*(t-\tau) = X(t), \quad y_i^*(t-\tau) = y_i(t), \quad i = 1,2,\cdots,m. \tag{20}$$

Note that the relationship between y^* and y_{sp} can be expressed by Eq. (16) in which y_i are substituted with y_i^*. When the initial conditions, i. e. , $y_{i(0)}^*(k) = e_{i(0)}(k), 0 \leqslant k \leqslant r_i - 1, i = 1,2,\cdots,m$, and the model is pecfect, in terms of Eq. (20), the input/output behavior of the closed-loop system is governed by

$$Y = \mathrm{diag}\{((S^{r_1} + \alpha_{1r_1}S^{r_1-1} + \cdots + \alpha_{11}S + \alpha_{11})^{-1},\cdots,$$
$$(S^{r_m} + \alpha_{mr_m}S^{r_m-1} + \cdots + \alpha_{m1}S + \alpha_{m1})^{-1}\}\, e^{-\tau s}Y_{sp}. \tag{21}$$

3 A Simulation Example

Consider a non-isothermal CSTR with the consecutive reactions,

$$A \xrightarrow{k_1} B \xrightarrow{k_2} C$$

taking place, and the dynamic behavior of the process is described by the following mass and energy balances.

$$\frac{\mathrm{d}C_A}{\mathrm{d}t} = \frac{F(C_{Ai} - C_A)}{V} - k_1C_A^2,$$
$$\frac{\mathrm{d}C_B}{\mathrm{d}t} = -\frac{FC_B}{V} + k_1C_A^2 - k_2C_B, \tag{22}$$
$$\rho C_PV\frac{\mathrm{d}T}{\mathrm{d}t} = \rho C_PF(T_i - T) + k_1C_A^2(-\Delta H_1)V + k_2C_B(-\Delta H_2)V + Q(t-\tau_2),$$
$$k_1 = A_{10}e^{-E_1/RT}, \quad k_2 = A_{20}e^{-E_2/RT}.$$

Here V is the reactor volume which is kept constant during the operation, and C_a and C_b are the concentrations of A and B inside the reactor, respectively, and T is the temperature of the reactor, and F is the inlet flow rate of A, and Q is the heat added to the reactor. The kinetic and physical parameters of steady state operating conditions for the particular process are given in Table 2.

Table 2 Model parameters[7]

$C_A = 1\mathrm{kmol\cdot m^{-3}}$	$\Delta H_1 = 418000\mathrm{kJ\cdot kmol^{-1}\cdot K^{-1}}$	$\rho = 1000\mathrm{kg\cdot m^{-3}}$	$E_1 = 1840\mathrm{kJ\cdot kmol\cdot K^{-1}}$
$A_{10} = 11\mathrm{m^3\cdot kmol\cdot m^{-1}\cdot s^{-1}}$	$\Delta H_2 = 418000\mathrm{kJ\cdot kmol^{-1}\cdot K^{-1}}$	$F = 10\mathrm{m^3s^{-1}}$	$E_2 = 34833\mathrm{kJ\cdot kmol\cdot K^{-1}}$
$A_{20} = 172.2\mathrm{m^3\cdot kmol\cdot s^{-1}}$	$R = 8.314\mathrm{kJ\cdot kmol^{-1}\cdot K^{-1}}$	$C_P = 1\mathrm{kJ\cdot kg^{-1}C^{-1}}$ $V = 100\mathrm{m^3}$	$T_i = 25\mathrm{C}$

The model may be put into the standard state space form by letting

$$x_1 = C_a, \quad x_2 = C_b, x_3 = T, \quad u_1 = F/V, u_2 = Q/\rho C_p V, \quad y_1 = C_b, \quad y_2 = T,$$

which result in the standard state equation, i. e, Eq. (2). Where the manipulated input, u_2, is delayed by τ_2. The auxiliary state variable, X^*, and output, Y^*, have no deadtime. The relative orders are $r_1 = r_2 = 1$. The control law is defined by Eq. (19). The step response of the closed-loop systems is shown in Fig. 3. The good servo behavior and a decoupled response are observed, which the filter constants take $\alpha_{11} = \alpha_{21} = 1.0$.

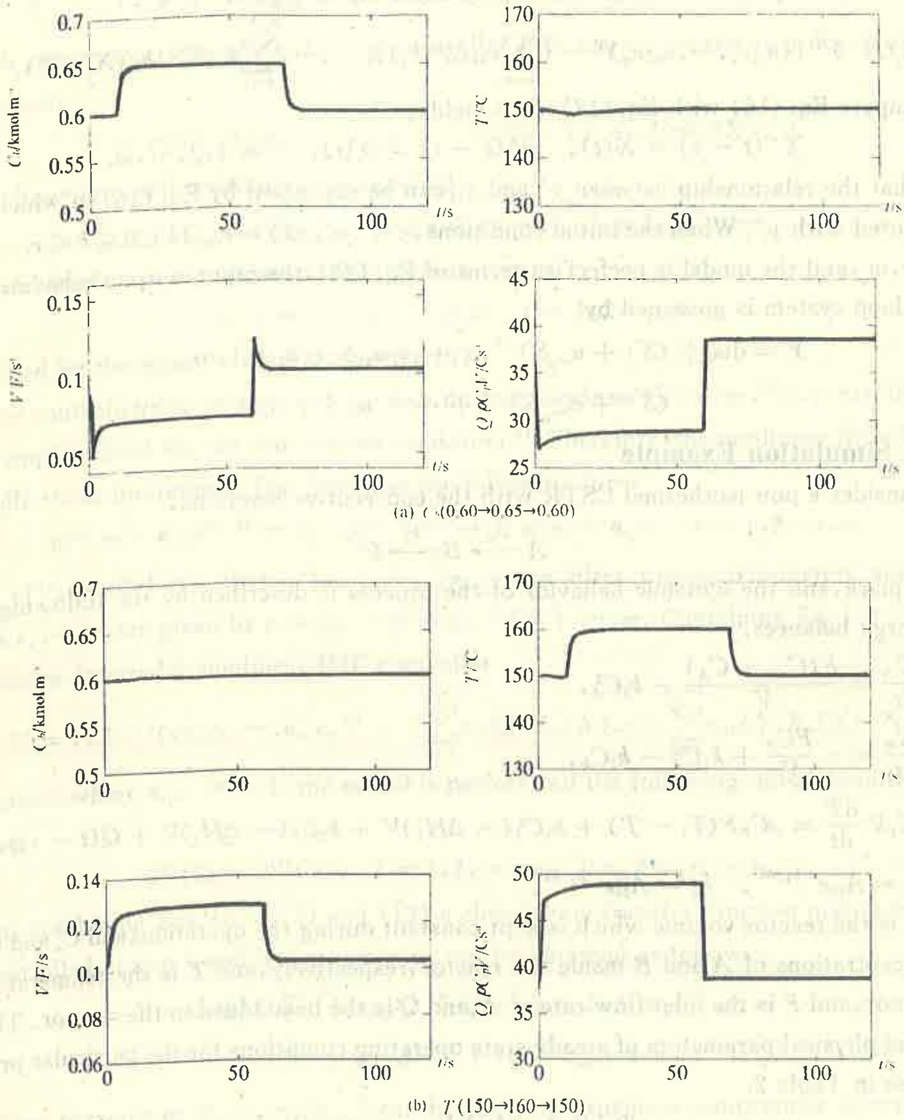

Fig. 3 The closed-loop responses for step changes in set points

4 Conclusion

An internal model control strategy for multivariable nonlinear processes with time delays has been developed. The proposed structure is an extension of nonlinear SISO IMC to nonlinear MIMO IMC. The delay matrix is added to some inputs to make the approach effective for the process with time delays, and a decoupled response is obtained. However, the multivariable IMC is restricted to the same assumptions as SISO nonlinear IMC. Simulation results for a reactor illustrate performances of the proposed multivariable nonlinear IMC strategy.

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多变量非线性时滞系统的内模控制

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摘要: 本文将单入单出非线性内模控制系统(NIMC)的设计方法推广到了含多步时滞的多入多出的非线性系统控制器设计中. 对于不含纯滞后的过程, 所设计的控制器能够实现所期望的常规性能, 其中通过非线性滤波器的加入, 能够获得当过程和模型存在失配时的鲁棒性, 并使控制器结构得以实现. 当过程含有多步时滞时, 通过在过程及模型的某些输入端增加适当的纯滞后, 就可基于不含时滞的辅助模型来设计 NIMC 控制器. 通过化学反应器控制的仿真评价了所提方法的性能.

关键词: 非线性系统; 纯滞后; 预估控制; 过程控制

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