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Robust Excitation Controllers Design for Power Systems

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Abstract: In this paper, a kind of robust excitation controllers for power systems is proposed. The design principle is based on nonlinear H_∞ control theory for affine systems. The controller involves two parts: one is linear optimal excitation controller and the other is an extra compensation part. The simulation show that the transient properties of power systems are greatly improved by using H_∞ controllers.

Key words: robust control; power system; excitation system

1 Introduction

It is well known that any dynamical systems can not be explicitly described by a set of differential equations, because uncertainty always exists. Uncertainties include variation of dynamics and external disturbances. In power systems, apart from the modelling error of dynamics, external disturbances may caused by suddenly applied loads, occurrence of faults, the loss of excitation in the field of a generator and switching, etc.. Such disturbances are very destructive to the stability of power system operation. Thus, besides developing protection devices, improving control schemes is important and necessary. In this paper, a new robust excitation controller of power systems is designed which is based on the nonlinear H_∞ control theory. The structure of the proprosed controller in the paper consists of two parts: one is a linear quadratic optimal control and the other is a compensation part which guarantees the closed loop excitation system has an L_2 gain as small as possible.

2 Nonlinear H_∞ Control

In this paper, a vector x denotes a column vector and its transposition is written to be x^{T}. The norm of x is denoted by $\|x\|$ which is Euclidian. Let A be an $n \times n$ matrix. Its induced norm is denoted by $\|A\|$ defined as follows:

$$\|A\| = \sqrt{\lambda_{\max}(A^{\mathrm{T}}A)},$$

where $\lambda(\cdot)$ denotes the eigenvalue set of a matrix.

Consider a smooth affine nonlinear control system

$$(\Sigma)\qquad \begin{aligned} \dot{x} &= f(x) + g_1(x)\omega + g_2(x)u, \\ z &= h(x) + K(x)u, \end{aligned}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $\omega \in \mathbb{R}^r$, $z \in \mathbb{R}^s$ are called state, control, disturbance and output (or

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where $x \in R^n$, $u \in R^m$, $\omega \in R^r$, $z \in R^s$ are called state, control, disturbance and output (or penalty) variables respectively; f is a smooth vector field and $g_1 \in R^{n\times r}$, $g_2 \in R^{n\times m}$, $h(x) \in R^s$, $K(x) \in R^{s\times m}$ are smooth maps. Assume $f(0)=0$, $h(0)=0$.

Nonlinear H_∞ control requires that designing a controller and finding a smallest positive number γ_* such that

1) Closed loop system is asymptotic stable.

2) $$\int_0^T \| z \|^2 dt \le \gamma^2 \int_0^T \| \omega \|^2 dt, \quad \forall T \ge 0, \quad \gamma > \gamma_*.$$

From the matured literatures [1,2,4], we have known that above H_∞ control problem can be answered by using dissipative system and differential game theories. Assume matrix $R(x)=K^T(x)K(x)$ is nonsingular for $\forall x \in R^n$, and write $V_x = \frac{\partial V}{\partial x}$. We have the following important results.

Proposition 2.1 The nonlinear H_∞ control problem of (Σ) is solvable iff

1) there exists an C^1 nonnegative function $V(x)$, $V(0)=0$, such that

$$V_x \hat{f}(x) + \hat{h}(x)^T \hat{h}(x) + \frac{1}{4} V_x \hat{R}(x) V_x^T(x) \le 0, \tag{1}$$

where

$$\hat{f}(x) = f(x) - g_2(x) R^{-1}(x) K^T(x) h(x),$$
$$\hat{h}(x) = (I - K(x) R^{-1}(x) K^T(x)) h(x),$$
$$\hat{R}(x) = \frac{1}{\gamma^2} g_1(x) g_1^T(x) - g_2(x) R^{-1}(x) g_2^T(x),$$

2) Set $u = - R^{-1}(x)\left(\frac{1}{2} g_2^T(x) V_x^T + K^T(x) h(x)\right)$.

The inequality (1) is called a Hamilton-Jacoby-Issacs inequality (Abbr. HJI inequality). Write $\| x \| = \rho$.

Proposition 2.2[2] Suppose there exist nonnegative real value functions $F(\rho)$, $G_1(\rho)$, $G_2(\rho)$, $H(\rho)$ in the neighborhood of $x = 0$ such that

$$x^T \hat{f}(x) \le - F(\rho), \quad x^T g_1 g_1^T x \le G_1(\rho),$$
$$x^T g_2 R^{-1} g_2^T x \ge G_2(\rho), \quad \hat{h}^T \hat{h} \le H(\rho),$$

then the L_2 gain γ has the following estimation:

$$\gamma^* = \sqrt{G_1/G_2}, \quad \gamma_* = \sqrt{G_1 H/(F^2 + G_2 H)}.$$

When $\gamma \in [\gamma^*, \infty)$, HJI inequality has the following nonnegative solution

$$V(\rho) = \int_0^\rho p(s) ds, \quad p(\rho) \ge 2 \frac{F - \sqrt{\Delta}}{G} \rho.$$

When $\gamma \in [\gamma_*, \gamma^*]$, HJI inequality has the following nonnegative solution

$$V(\rho) = \int_0^\rho p(s) ds, \quad 2 \frac{F - \sqrt{\Delta}}{G} \rho \le p(\rho) \le 2 \frac{F + \sqrt{\Delta}}{G} \rho,$$

where $$G = \frac{1}{\gamma^2} G_1 - G_2, \quad \Delta = F^2 - GH.$$

The proposition 2. 2 can be proven by setting $V_x = \frac{\mathrm{d}V}{\mathrm{d}\rho}\frac{x^{\mathrm{T}}}{\rho}$ in(1).

3 The Design Method of H_∞ Excitation Controller

Consider an one-machine infinite-bus excitation system, its dynamical equation is

$$\begin{cases} \dot{\delta} = \omega - \omega_0, \\ \dot{\omega} = \frac{\omega_0}{M}P_m - \frac{D}{M}(\omega - \omega_0) - \frac{\omega_0}{M}\frac{E'_q V_s}{x'_{d\Sigma}}\sin\delta + \varepsilon_1, \\ \dot{E}'_q = -\frac{1}{T'_d}E'_q + \frac{1}{T_{d0}}\frac{x_d - x'_d}{x'_{d\Sigma}}V_s\cos\delta + \frac{1}{T_{d0}}V_f + \varepsilon_2, \end{cases} \tag{2}$$

where

δ: rotor angle, in radian; ω: rotor speed, in rad/s, $\omega_0 = 2\pi f_0$; E'_q: internal transient voltage; V_f: voltage of the field circuit of a generator; M: inertia coefficient of a generator set, in seconds; D: damping constant, in per unit; V_s: voltage of infinite-bus, in per uint; T_{d0}: field circuit time constant, in seconds; x_d: d-axis synchronous reactance of a generator, in per unit ; x'_d: d-axis transient reactance, in per unit; $x'_{d\Sigma}: = x'_d + x_T + x_{TL}$, $x_{d\Sigma}: = x_d + x_T + x_{TL}$, x_T: transformer reactance; x_{TL}: transmission line reactance, in per unit; P_m: mechanical power, assumed to be constant, in per unit; $P_e: = \frac{E'_q V_s}{x'_{d\Sigma}}\sin\delta$, active electrical power, in per uint; $T'_d: = T_{d0}\frac{x'_{d\Sigma}}{x_{d\Sigma}}$; ε_1, ε_2: external disturbances.

In order to match the system (Σ), let

$$x_1 = \delta - \delta_0, \quad x_2 = \omega - \omega_0, \quad x_3 = E'_q - E'_{q0},$$

where δ_0, $E'_{q0} = \frac{T'_d}{T_{d0}}\frac{x_d - x'_d}{x'_{d\Sigma}}V_s\cos\delta_0 + \frac{T'_d}{T_{d0}}V_{f0}$ are equilibrium point and steady-state of (2). V_{f0} is the voltage of the field circuit in equilibrium point.

In the new coordinates, the dynamical equation is

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -\frac{D}{M}x_2 + \frac{\omega_0}{M}(P_m - P_e) + \varepsilon_1, \\ \dot{x}_3 = -\frac{1}{T'}x_3 + \frac{1}{T_{d0}}\frac{x_d - x'_d}{x'_{d\Sigma}}V_s\cos(\delta_0 + x_1) \\ \qquad - \frac{1}{T_{d0}}\frac{x_d - x'_d}{x'_{d\Sigma}}V_s\cos\delta_0 + \frac{1}{T_{d0}}V_{f1} + \varepsilon_2, \end{cases} \tag{3}$$

where

$$V_f = V_{f0} + V_{f1}.$$

Suppose in steady-state, $P_{e0} = P_m$, then in the neighborhood of equilibrium point, (3) can be approximated as follows:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -ax_2 - bx_1 - cx_3 + \omega_1, \\ \dot{x}_3 = -dx_3 - ex_1 + \omega_2 + fu, \end{cases} \tag{4}$$

where

$$a=\frac{D}{M},\quad b=\frac{\omega_0}{M}\frac{E'_{q0}V_s\cos\delta_0}{x'_{d\Sigma}},\quad c=\frac{\omega_0}{M}\frac{V_s\sin\delta_0}{x'_{d\Sigma}},$$

$$d=\frac{1}{T'_d},\quad e=\frac{1}{T_{d0}}\frac{x_d-x'_d}{x'_{d\Sigma}}V_s\sin\delta_0,\quad f=\frac{1}{T_{d0}},$$

$$\omega_1=O(\|x_1\|^2,\|x_3\|^2)+\varepsilon_1,\quad \omega_2=O(\|x_1\|^2)+\varepsilon_2,\quad u=V_{f1}.$$

For (4), select an output as follows

$$z=\begin{bmatrix} l_1x_1\\ l_2x_2\\ l_3x_3\\ \Sigma_{i=1}^3 r_0k_ix_i\end{bmatrix}+\begin{bmatrix}0\\0\\0\\r_0\end{bmatrix}u,$$

where l_1,l_2,l_3,k_1,k_2,k_3 are constants, $r_0\neq 0$.

$$A=\begin{bmatrix}0&1&1\\-b&-a&-c\\-e&0&-d\end{bmatrix},\quad C=\begin{bmatrix}l_1&0&0\\0&l_2&0\\0&0&l_3\\r_0k_1&r_0k_2&r_0k_3\end{bmatrix}.$$

Then $R=r_0^2$. It is important to choose k_1,k_2,k_3.

According to the Proposition 2.1, the H_∞ controller $u=-\frac{1}{r_0^2}(\frac{1}{2}g_2^TV_x^T+K^TCx)$, and $V(x)$ should satisfy the following HJI inequality

$$V_x\hat{f}(x)+\hat{h}(x)^T\hat{h}(x)+\frac{1}{4}V_x\hat{R}(x)V_x^T\leq 0, \tag{5}$$

where

$$\hat{f}(x)=\begin{bmatrix}0&1&0\\-b&-a&-c\\-e&0&-d\end{bmatrix}x-g_2(k_1,k_2,k_3)x,$$

$$\hat{h}(x)=\begin{bmatrix}l_1x_1\\l_2x_2\\l_3x_3\\0\end{bmatrix},\quad \hat{R}(x)=\frac{1}{\gamma^2}\begin{bmatrix}0&0&0\\0&1&0\\0&0&1\end{bmatrix}-\begin{bmatrix}0&0&0\\0&0&0\\0&0&f^2/r_0^2\end{bmatrix}.$$

It is easy to verify that $[A,g_2]$ in (4) is controllable, so we can design $K_1=(k_1,k_2,k_3)$ such that

$$K_1=R_0^{-1}g_2^TP,$$

where P satisfies a following Riccati matrix equation

$$PA+A^TP-Pg_2R_0^{-1}g_2^TP+Q=0, \tag{6}$$

in which Q is a positive definite matrix, R_0 is a positive number.

Then we have

$$x^T\hat{f}(x)=-\frac{1}{2}x^T(Q+Pg_2R_0^{-1}g_2^TP)x\leq-\frac{1}{2}\|x\|_Q^2\leq-\lambda\|x\|^2,$$

$$\begin{aligned} &\lambda = \frac{1}{2}\min(\lambda_i(Q), i = 1,2,3), \\ &x^{\mathrm{T}} g_1 g_1^{\mathrm{T}} x \leq \beta_1 \| x \|^2, \quad \beta_1 = \| g_1 g_1^{\mathrm{T}} \|, \\ &x^{\mathrm{T}} g_2 \frac{1}{r_0^2} g_2^{\mathrm{T}} x \geq 0, \\ &\hat{h}^{\mathrm{T}} \hat{h} \leq \beta_2 \| x \|^2, \quad \beta_2 = \| (I - (0,0,0,1)^{\mathrm{T}}(0,0,0,1))C \|^2. \end{aligned} \tag{7}$$

Let $\rho = \sqrt{x_1^2 + x_2^2 + x_3^2}$, $p(\rho) = \frac{\mathrm{d}V}{\mathrm{d}\rho}$, and substitute $V_x = p(\rho)\frac{x^{\mathrm{T}}}{\rho}$ and (7) into (1), by simple manipulation, we get an inequality

$$\frac{\beta_1}{4_\gamma{}^2 \rho^2} p^2 - \frac{\lambda}{\rho} p + \beta_2 \leq 0. \tag{8}$$

If (8) has a nonnegative solution, the following condition should be satisfied,

$$\lambda^2 - \frac{\beta_1 \beta_2}{\gamma^2} \geq 0.$$

We thus have

$$\gamma_* = \frac{\sqrt{\beta_1 \beta_2}}{\lambda},$$

when $\gamma \in [\gamma_*, \infty)$,

$$\frac{2\gamma^2\lambda - 2\gamma\sqrt{\gamma^2\lambda^2 - \beta_1\beta_2}}{\beta_1}\rho \leq p \leq \frac{2\gamma^2\lambda - 2\gamma\sqrt{\gamma^2\lambda^2 - \beta_1\beta_2}}{\beta_1}\rho.$$

For the smallest γ, i. e., $\gamma = \gamma_*$,

$$p_* = 2\frac{\beta_2}{\lambda}\rho.$$

The feedback control law is

$$u = -\frac{1}{r_0^2}\left(\frac{1}{2}(0,0,f)p\frac{x}{\rho} + (0,0,0,r_0)C_x\right).$$

When $p = p_*$, the H_∞ excitation controller is

$$u_* = -\frac{1}{r_0^2}\left((0,0,f)\frac{\beta_2}{\lambda}x + (0,0,0,r_0)Cx\right).$$

It is well known that by using a simple linear transformation, the feedback variables $\delta - \delta_0$ and $E_q' - E_{q0}'$ in H_∞ controller can be replaced by the deviation of terminal voltage ΔV_t and active power ΔP_e which can be measured directly.

Remark Since $\gamma_* = \sqrt{\beta_1\beta_2}/\lambda$, β_1 is an invariant in the excitation system, γ_* only depends on two parameters β_2 and λ. Hence the main designing task is to select suitable output and Q.

4 Simulations

Consider the following one-machine infinite bus system (Fig. 1).

The disturbance is assumed to be a three-phase short cut with clearing out time 0. 1s.

The H_∞ excitation controller (HEC) is designed with $l_1 = 40, l_2 = 40, l_3 = 40, r_0 = 1$. the linear optimal excitation controller (LOEC) is designed under the constraint dynamics (6) and the quadratic performance index

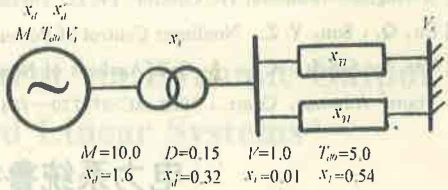

Fig. 1 One-machine infinite bus system

$$J = \int_0^\infty (x^TQx + u^TR_0u)\mathrm{d}t,$$

where $Q = \mathrm{diag}(25, 144, 20000), \quad R_0 = 1.$

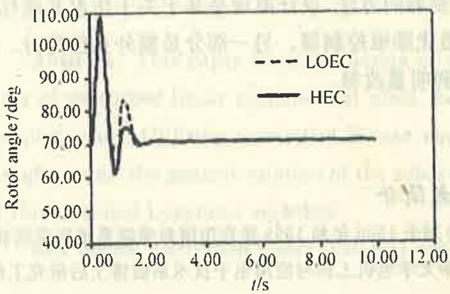

Fig. 2 The transient response of angle(deg).

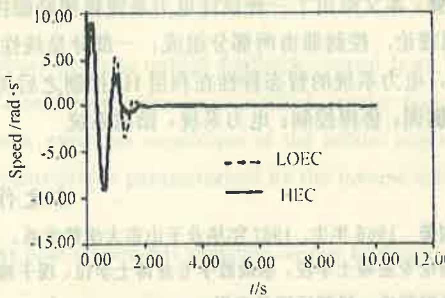

Fig. 3 The transient response of speed.

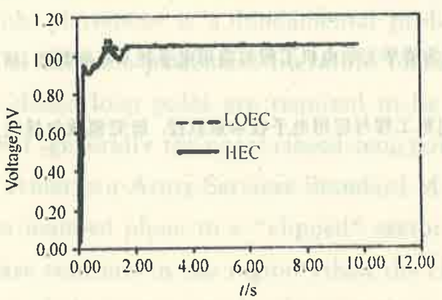

Fig. 4 The transient response of terminal voltage

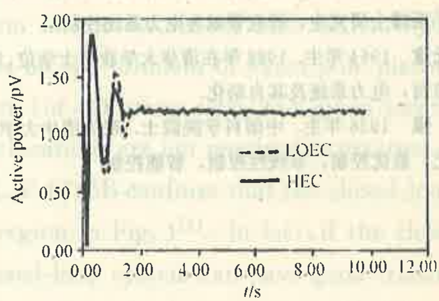

Fig. 5 The transient response of active power

5 Conclusions

Similar to the linear optimal excitation control law, the H_∞ excitation controller designed in this paper is also a linear state feedback. Hence the realization of HEC is much easier than that of nonlinear excitation controllers[3]. The H_∞ controller consists of two part: $-\frac{1}{r_0^2}(0,0,0,r_0)Cx$, which is a linear quadratic optimal controller; $-\frac{1}{r_0^2}(0,0,f)\frac{\beta_2}{\lambda}x$, which can be viewed as a compensation part such that the closed loop system has an L_2 gain γ..

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电力系统鲁棒励磁控制器设计

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摘要：本文给出了一种设计电力系统鲁棒励磁控制器的方法．设计原理是基于关于仿射非线性系统的H_∞控制理论．控制器由两部分组成：一部分是线性最优励磁控制器，另一部分是额外补偿部分．仿真结果表明，电力系统的暂态特性在利用H_∞控制之后得到明显改善．

关键词：鲁棒控制；电力系统；励磁系统

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