

H_∞ Output Control of Discrete-Time Systems via Convex Optimization

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Abstract: This paper presents a convex optimization approach to (nearly) optimal H_∞ output control problem of discrete-time systems. By using nonlinear matrix valued mapping, it is shown that the H_∞ output problem can be formulated as minimization of a linear objective function over a convex set in $n(n+1)+1$ dimensional space where n is the system dimension. Under the detectability and stabilizability conditions, the set is bounded. The H_∞ norm of the closed-loop system yielded by this method can come arbitrarily close to the minimum H_∞ norm. The method proposed can handle the additional convex constraints.

Key words: H_∞ control; output feedback; convex optimization; discrete-time systems

1 Introduction

It has been shown many control problems such as quadratic stabilization of uncertain systems^[6], H^2 control problems^[7], generalized H^2 control problem^[6] and mixed H^2/H_∞ control problems^[5,9] (H^2 control problems with the H_∞ norm bound) can be formulated as convex programming problems because a convex programming problem can be solved efficiently by well developed optimization algorithms and the global optimal solution can be obtained. Another important advantage is that other requirements and limitations can be directly tackled with if they can be converted to convex constraints. For H_∞ control problems of discrete-time systems, [3] has shown design of state feedback controllers to achieve the prescribed H_∞ norm bound can be solved by convex optimization based on a sufficient condition.

This paper addresses H_∞ output feedback control problems for discrete-time systems. The optimal H_∞ control problems are formulated as nonlinear optimization problems with five coupling nonlinear matrix inequalities (with respect to the cone of positive definite matrices). By changes of the variables, we show the H_∞ output control problem can be solved by a convex optimization approach. A new procedure for design of H_∞ output controllers is presented.

2 Problem Statement

Consider the discrete-time feedback system shown in Fig. 1. The plant P is described by

$$\begin{cases} x(t+1) = Ax(t) + B_1w(t) + B_2u(t), \\ z(t) = C_1x(t) + D_{11}w(t) + D_{12}u(t), \\ y(t) = C_2x(t) + D_{21}w(t) + D_{22}u(t) \end{cases} \quad (1)$$

where $x \in R^n, w \in R^{m_1}, u \in R^{m_2}, z \in R^{p_1}$ and $y \in R^{p_2}$ are state, noise, control, regulated output and measurement output vectors respectively, The following assumptions are imposed on the system(1).

A1) For every θ in $[0, 2\pi]$,

$$\begin{cases} \text{rank} \begin{vmatrix} A - \exp(j\theta)I & B_2 \\ C_1 & D_{12} \end{vmatrix} = n + m_2, \\ \text{rank} \begin{vmatrix} A - \exp(j\theta)I & B_1 \\ C_2 & D_{21} \end{vmatrix} = n + p_2. \end{cases}$$

A2) $D_{11} = 0, D_2 = 0.$

A3) $D_{12}'[C_1, D_{12}] = [0 \ I]. D_{21}'[C_2, D_{21}] = [0 \ I].$

A4) A is non-singular.

Some above assumptions are not necessary and can be removed^[1]. We impose these assumptions to make the results as simple as possible. This paper considers the following problem; to design a linear output feedback controller C such that closed-loop system is internally stable and the H_∞ norm of the closed-loop system is minimized.

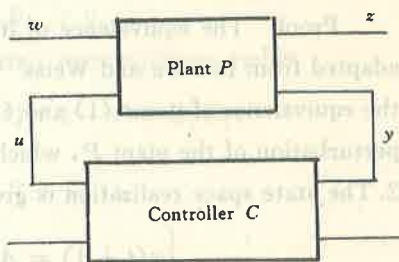


Fig. 1 Feedback system

In the following $A > (\geq) B$ denotes $A - B$ is positive definite (or positive semidefinite, respectively). A' refers to the transpose of a matrix $A.$ $\rho(\cdot)$ is the spectral radius of a matrix.

Let

$$J = \begin{vmatrix} -I_{m_1} & 0 \\ 0 & I_{m_2} \end{vmatrix}, \hat{J} = \begin{vmatrix} -I_{p_1} & 0 \\ 0 & I_{p_2} \end{vmatrix},$$

$$B(\gamma) = \begin{bmatrix} \frac{1}{\gamma}B_1 & B_2 \end{bmatrix}, C(\gamma) = \begin{bmatrix} \frac{1}{\gamma}C_1 \\ C_2 \end{bmatrix},$$

where m_1, m_2, p_1 and p_2 are the dimensions of the noise, control, regulated output and measurement output vectors, respectively.

Theorem 1 Suppose the discrete-time system (1) satisfies Assumptions A1)~A4). For a given $\gamma > 0,$ the following statements are equivalent.

1) There exists a stabilizing controller such that $\|T_{zw}\|_\infty < \gamma.$

2) a) There exists stabilizing $P \geq 0$ such that

$$R_1(P, \gamma) := A'PA - P - A'PB(\gamma)(J + B(\gamma)'PB(\gamma))^{-1}B(\gamma)'PA + C_1'C_1 = 0,$$

$$R_2(P, \gamma) := -\gamma^2I + B_1'PB_1 < 0.$$

b) There exists stabilizing $Q \geq 0$ such that

$$R_3(Q, \gamma) := AQA' - Q - AQC(\gamma)'(J + C(\gamma)QC(\gamma)')^{-1}C(\gamma)QA' + B_1B_1' = 0,$$

$$R_4(Q, \gamma) := -\gamma^2 I_{p_1} + C_1QC_1' - C_1QC_2'(I + C_2QC_2')^{-1}C_2QC_1' < 0.$$

c)
$$R_5(P, Q, \gamma) := \rho(PQ) - \gamma^2 < 0.$$

3) There exist $P > 0$ and $Q > 0$ such that

$$R_i(P, Q, \gamma) < 0, \quad i = 1, \dots, 5.$$

Moreover, the H_∞ output controller is given by

$$\begin{cases} \xi(t+1) = A_D \xi(t) + B_D y(t), \\ u(t) = C_D \xi(t) + D_D y(t). \end{cases} \tag{2}$$

$$A_D = (A + B(\gamma)F)(I + EC_2'C_2)^{-1}, \quad B_D = (A + B(\gamma)F)(I + EC_2'C_2)^{-1}EC_2'$$

$$C_D = -F_2(I + EC_2'C_2)^{-1}, \quad D_D = F_2(I + EC_2'C_2)^{-1}EC_2',$$

$$F = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = -(J + B(\gamma)'PB(\gamma))^{-1}B(\gamma)PA,$$

$$E = (Q^{-1} - P/\gamma^2)^{-1}.$$

Proof The equivalence of items (1) and (2) is adapted from Ionescu and Weiss^[1]. We need to prove the equivalence of items (1) and (3). Consider a small perturbation of the plant P , which is depicted in Fig.

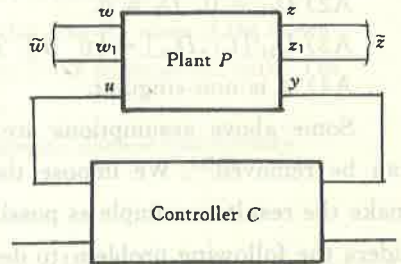


Fig. 2 Perturbed system

2. The state space realization is given by

$$\begin{cases} x(t+1) = Ax(t) + B_1w(t) + \sqrt{\beta_1}w_1 + B_2u(t), \\ z(t) = C_1x(t) + D_{11}w(t) + D_{12}u(t), \\ z_1(t) = \sqrt{\beta_2}x(t), \\ y(t) = C_2x(t) + D_{21}w(t) + D_{22}u(t), \end{cases} \tag{3}$$

where β_1, β_2 are small positive scalar.

Suppose there exists a controller such that the closed-loop system is asymptotically stable and $\|T_{zw}\|_\infty < \gamma > 0$. Then from the the continuity property of H_∞ (for example see [5]), there exist $\beta_{1+} > 0$ and $\beta_{2+} > 0$ such that for all $\beta_1 \in [0, \beta_{1+}]$ and $\beta_2 \in [0, \beta_{2+}]$ such that $\|T_{z\tilde{w}}\|_\infty < \gamma$, Therefore, following item (2) there exist $P \geq 0$ and $Q \geq 0$ satisfying

$$R_{\beta_1}(P, \gamma) := A'PA - P - A'PB_\beta(\gamma)(J_1 + B_\beta(\gamma)'PB_\beta(\gamma))^{-1}B_\beta(\gamma)'PA + C_1'C_1 = -\beta_2I,$$

$$R_{\beta_2}(P, \gamma) := -\gamma^2I + [B_1 \quad \sqrt{\beta_1}I]^p [B_1 \quad \sqrt{\beta_1}I] < 0,$$

$$R_{\beta_3}(Q, \gamma) := AQA' - Q - AQC_\beta(\gamma)'(J_1 + C_\beta(\gamma)QC_\beta(\gamma)')^{-1}C_\beta(\gamma)QA' + B_1B_1' = -\beta_1I,$$

$$R_{\beta_4}(Q, \gamma) := -\gamma^2I_{p_1+n} + \begin{bmatrix} C_1 \\ \sqrt{\beta_1}I \end{bmatrix} (Q - QC_2'(I + C_2QC_2')^{-1}C_2Q) \begin{bmatrix} C_1 \\ \sqrt{\beta_1}I \end{bmatrix} < 0,$$

$$R_5(P, Q, \gamma) := \rho(PQ) - \gamma^2 < 0.$$

where

$$J_1 = \begin{bmatrix} -I_{m_1+n} & 0 \\ 0 & I_{m_2} \end{bmatrix}, \quad \hat{J}_1 = \begin{bmatrix} -I_{p_1+n} & 0 \\ 0 & I_{p_2} \end{bmatrix},$$

$$B_\beta(\gamma) = \left[\frac{1}{\gamma} B_1 \quad \sqrt{\beta_1} I B_2 \right], \quad C_\beta(\gamma) = \begin{bmatrix} \frac{1}{\gamma} C_1 \\ \sqrt{\beta_2} I \\ C_2 \end{bmatrix}.$$

Because $([C_1' \quad \beta_2 I]', A)$ is detectable for $\beta_2 > 0$, it follows from $R_{\beta_1}(P, \gamma) + \beta_2 I = 0$ that $P > 0$. It also follows from $\beta_2 > 0$ that $R_{\beta_1} < 0$. Since $P > 0, R_{\beta_2} < 0$ implies

$$P^{-1} - (B_1 B_1' + \beta_2 I) / \gamma^2 > 0. \tag{4}$$

It also means

$$P^{-1} - \frac{1}{\gamma^2} B_1 B_1' > 0.$$

Simple matrix manipulations yield $R_2(P, \gamma) < 0$.

It follows from (4) that

$$P^{-1} - (B_1 B_1' + \beta_1 I) / \gamma^2 + B_2 B_2' > 0.$$

Then using the definition of J_1 and $B_\beta(\gamma)$, simple algebraic manipulations yields

$$\begin{aligned} R_{\beta_1} &= A'(P^{-1} - \frac{1}{\gamma^2}(B_1 B_1' + \beta_1 I) + B_2 B_2')^{-1} A - P + C_1' C_1, \\ &> A'(P^{-1} - \frac{1}{\gamma^2} B_1 B_1' + B_2 B_2')^{-1} A - P + C_1' C_1 \\ &= R_1(P, \gamma). \end{aligned}$$

Remember $R_{\beta_1} < 0$. Thus $R_1(P, \gamma) < 0$.

The proof of $R_3 < 0$ and $R_4 < 0$ is similar to that of $R_1 < 0$ and $R_2 < 0$. $R_5 < 0$ follows from the item (2) directly.

Conversely, suppose there exist $P > 0$ and $Q > 0$ satisfying $R_i < 0, i = 1, \dots, 5$. Obviously, there exist the matrices $M_1 \in R^{m \times n}$ and $M_2 \in R^{n \times n}$ satisfying

$$\begin{aligned} R_{M_1}(P, \gamma) + M_2 M_2' &= 0, \quad R_{M_2}(P, \gamma) < 0, \\ R_{M_3}(P, \gamma) + M_1 M_1' &= 0, \quad R_{M_4}(Q, \gamma) < 0, \quad R_5(P, Q, \gamma) < 0 \end{aligned}$$

where R_{M_i}, B_M, C_M are obtained by replacing $\beta_1 I$ and $\beta_2 I$ with M_1 and M_2 in $R_{\beta_1}, B_\beta, C_\beta$. Since the items (1) and (2) are equivalent, it implies $\|T_{z\bar{w}}\|_\infty < \gamma$ where

$$\begin{aligned} x(t+1) &= Ax(t) + B_1 w(t) + M_1 w_1 + B_2 u(t), \\ z(t) &= C_1 x(t) + D_{11} w(t) + D_{12} u(t), \\ z_1(t) &= M_2 x(t), \\ y(t) &= C_2 x(t) + D_{21} w(t) + D_{22} u(t). \end{aligned}$$

According to the definition of H_∞ norm^[5], we obtain $\|T_{z\bar{w}}\|_\infty < \gamma$. Using the equivalence of the items (1) and (2) again, the items (3) and (2) are equivalent.

The discrete-time H_∞ controller follows from [1] and the equivalence of items (2) and

(3). Q. E. D.

Define the set

$$S_D = \{(P, Q, \gamma); R_i(P, Q, \gamma) < 0, i = 1, 2, \dots, 5, P > 0, Q > 0\}. \quad (5)$$

The (nearly) optimal H_∞ output control problem for discrete-time systems is formulated as

$$(DP_1) \quad \min_{(P, Q, \gamma) \in S_D} \gamma.$$

Obviously, the problem (DP_1) is a highly nonlinear optimization problem. There are five coupling nonlinear matrix inequality constraints. In addition, these matrix inequalities contain the inverses of the matrices.

3 Convex Optimization Approach

Define the set

$$N_D = \{(X, Y, \gamma); H_i(X, Y, \gamma) < 0, i = 1, 2, 3, 4\} \quad (6)$$

where

$$\begin{aligned} H_1 &= \begin{vmatrix} XC_1' C_1 X - X & - XA' \\ - AX & \mu^2 B_1 B_1' - X - B_2 B_2' \end{vmatrix}, \\ H_2 &= \mu^2 B_1 B_1' - X < 0, \\ H_3 &= \begin{vmatrix} YB_1' B_1 Y - Y & - YA \\ - A^T Y & \mu^2 C_1' C_1 - C_2^T C_2 - Y \end{vmatrix}, \\ H_4 &= \begin{vmatrix} - X & - \mu I \\ - \mu I & - Y \end{vmatrix}. \end{aligned}$$

Next we will show the set N_D is equivalent to the set S_D , which is based on a series of lemmas.

Lemma 1 There exist $P > 0$ and $\gamma > 0$ such that $R_1(P, \gamma) < 0$ and $R_2(P, \gamma) < 0$ if and only if there exist $X > 0$ and $\mu > 0$ such $H_1(X, \mu) < 0$ and $H_2(X, \mu) < 0$.

Proof Suppose that there exist $P > 0$ and $\gamma > 0$ satisfying $R_1(P, \gamma) < 0$ and $R_2(P, \gamma) < 0$. Since $P > 0$, $R_2(P, \gamma) < 0$ implies

$$P^{-1} > \frac{1}{\gamma^2} B_1 B_1'. \quad (7)$$

Let $X = P^{-1}$ and $\mu = 1/\gamma$. $H_2(X, \mu) < 0$ is yielded.

The inequality (7) also means

$$P^{-1} - \frac{1}{\gamma^2} B_1 B_1' + B_2 B_2' > 0. \quad (8)$$

Hence the inverse of the matrix $P^{-1} - \frac{1}{\gamma^2} B_1 B_1' + B_2 B_2'$ exists. It follows from $P > 0$ and $R_1(P, \gamma)$ that

$$P - C_1' C_1 > A' (P^{-1} - \frac{1}{\gamma^2} B_1 B_1' + B_2 B_2')^{-1} A. \quad (9)$$

Combining (8) and (9) yields

$$\begin{vmatrix} P - C_1' C_1 & A' \\ A & P^{-1} - \frac{1}{\gamma^2} B_1 B_1' + B_2 B_2' \end{vmatrix} > 0.$$

Premultiply and postmultiply it by $\text{diag}\{P^{-1}, I\}$ and let $X=P^{-1}$ and $\mu=1/\gamma$. After simple algebraic manipulations we obtain $H_1(X, \mu) < 0$ directly.

Conversely, suppose that there exist $X > 0$ and $\mu > 0$ such that $H_i(X, \mu) < 0, i=1, 2$. Let $P=X^{-1}$ and $\gamma=1/\mu$. $R_2(P, \gamma) < 0$ follows from $H_2(X, \mu) < 0$ by simple algebraic manipulations. Using the same changes of variables $H_1(X, \mu) < 0$ can be written as

$$\begin{vmatrix} P^{-1} - P^{-1}C_1' C_1 P^{-1} & P^{-1}A' \\ AP^{-1} & -\frac{1}{\gamma^2}B_1 B_1' + P^{-1} + B_2 B_2' \end{vmatrix} > 0.$$

Premultiplying and postmultiplying it by $\text{diag}\{P, I\}$ yields

$$\begin{vmatrix} P - C_1' C_1 & A' \\ A & -\frac{1}{\gamma^2}B_1 B_1' + P^{-1} + B_2 B_2' \end{vmatrix} > 0.$$

It implies

$$P - C_1' C_1 > A'(P^{-1} + B(\gamma)JB(\gamma)')^{-1}A.$$

Following the matrix inverse lemma and the definition of $B(\gamma)$ and J , the desired result is obtained. Q. E. D.

Lemma 2 There exist $Q > 0$ and $\gamma > 0$ such that $R_3(Q, \gamma) < 0$ and $R_4(Q, \gamma) < 0$ if and only if there exist $Y > 0$ and $\mu > 0$ such that $H_3(Y, \gamma) < 0$.

Proof The proof is similar to that of Lemma 4. It should be noticed that the condition $H_3 < 0$ is equivalent to two conditions $R_i < 0, i=3, 4$.

Lemma 3 There exist $P > 0, Q > 0$ and $\gamma > 0$ such that $R_5(P, Q, \gamma) < 0$ if and only if there exists $\mu > 0$ such that $H_4(X, Y, \mu) < 0$.

Proof Suppose there exists $\mu > 0$ satisfying $H_4(X, Y, \mu) < 0$. It implies that $X > 0$ and $Y > 0$. It also means $Y > \mu_2 X^{-1}$. Let $P=X^{-1}, Q=Y^{-1}$ and $\gamma=1/\mu$. It follows that $P > 0, Q > 0$ and $P < \gamma^2 Q^{-1}$. Hence $R_5(P, Q, \gamma) < 0$. Conversely, suppose there exist $P > 0, Q > 0$ and $\gamma > 0$ such that $R_5(P, Q, \gamma) < 0$. Let $X=P^{-1}, Y=Q^{-1}$ and $\mu=1/\gamma$. It implies $X > 0, Y > 0$ and $Y > \mu^2 X^{-1}$. We obtain $H_4(X, Y, \mu) < 0$.

Theorem 2 The sets N_D defined in (5) and S_D defined in (6) are equivalent.

Proof Note that the same changes of the variables are used in the proof of Lemma 1, 2 and 3. The result follows from the above lemmas. Q. E. D.

Corollary 1 The optimal problem (DP_1) is equivalent to the following problem

$$(DP_2) \quad \min_{(X, Y, \mu) \in N_D} -\mu.$$

Moreover, if the optimal solution is achieved on X_0, Y_0 and μ_0 . The minimum H_∞ norm is given by $1/\mu_0$ and the associated (nearly) optimal H_∞ controller is given in (2) by replacing P, Q and γ with X_0^{-1}, Y_0^{-1} and $1/\mu_0$, respectively.

Theorem 3 The set N_D defined by (6) is convex.

Proof We only need to prove the constraints in the set N_D are convex. At first, we will prove $H_1(X, \mu); R^{n \times n} \times R^1 \rightarrow R^{n \times n}$ defines a convex matrix valued mapping. The map-

ping H_1 can be divided as

$$H_1(X, \mu) = \begin{vmatrix} 0 & 0 \\ 0 & B_2 B_2' \end{vmatrix} + \begin{vmatrix} -X & XA \\ -A'X & -X \end{vmatrix} + \begin{vmatrix} XC_1' C_1 X & 0 \\ 0 & \mu^2 B_1 B_1' \end{vmatrix}$$

The last part, depending on (X, μ) , is convex. The first part is constant and the second part linearly depends on (X, μ) . Hence the mapping H_1 is convex. The convexity of the rest constraints can be shown using the similar method. Q. E. D.

It should be noticed that the matrices X and Y are symmetric. The set N_D belongs to an $n(n+1)+1$ dimensional space. This set is bounded, which is an important property for numerical computation.

Theorem 4 The set N_D is bounded if the pair (C_1, A) is detectable and the pair (A, B_1) is stabilizable.

Proof It follows from $H_1 > 0$ that

$$X - XC_1 C_1' X > 0.$$

Since $X > 0$, it implies

$$\begin{vmatrix} X & XC_1 \\ C_1 X & I \end{vmatrix} > 0.$$

Thus $I - C_1 X C_1' > 0$. By simple algebraic manipulations, it follows from $H_1 < 0$ that

$$X - \mu^2 B_1 B_1' + B_2 B_2' - AXA' + AXC_1(I - C_1 X C_1')^{-1} C_1' X A' > 0.$$

That is

$$X + B_2 B_2' - AXA' + AXC_1'(I - C_1 X C_1')^{-1} C_1 X A' > \mu^2 B_1 B_1' \geq 0.$$

Since (C_1, A) is detectable, using the simple extension of Theorem 3.1^[4], there exists X , depending on (A, B_2, C_1) , such that $-X \geq X^-$ or $X \leq -X^-$ (see Lemma 4.6 in [9]). Since $X > 0$, $\|X\| \leq \|X^-\| < \infty$. Hence X is bounded. Since X and $B_2 B_2'$ are bounded. It follows from $\mu^2 B_1 B_1' - X - B_2 B_2' < 0$ that μ being bounded. We also can show Y is bounded by the similar method if (A, B_1) is bounded. Hence the set N_D is bounded. Q. E. D.

4 Example

Consider the following discrete-time system

$$\begin{cases} x(t+1) = \begin{vmatrix} -0.1 & 0 \\ 0 & -0.3 \end{vmatrix} x(t) + \begin{vmatrix} 1 \\ 1 \end{vmatrix} w_1(t) + \begin{vmatrix} 1 \\ 0 \end{vmatrix} u(t), \\ z(t) = \begin{vmatrix} X(t) \\ u(t) \end{vmatrix}, \\ y(t) = x_1(t) + w_2(t). \end{cases}$$

We want to design output feedback controller such that the H_∞ norm of the closed-loop transfer function matrix is minimized.

According to the design procedure developed in this paper, the convex optimization problem (DP₂) is solved with the existing optimization algorithm (e. g., "optimization toolbox" in Matlab 4.0) and the global optimal solution is yielded by

$$\mu = 0.6029,$$

$$X = \begin{bmatrix} 0.9915 & -0.0242 \\ -0.0242 & 0.7129 \end{bmatrix}, \quad Y = \begin{bmatrix} 7.3242 & -6.6586 \\ -6.6586 & 6.9599 \end{bmatrix}.$$

From Corollary 1, the optimal closed-loop norm is given by $\gamma = 1.6106$ and the optimal output H_∞ controller is given by

$$\begin{aligned} \dot{\xi}(t) &= \begin{bmatrix} -0.4041 & 0.3887 \\ 0.3052 & -0.2936 \end{bmatrix} \xi(t) + \begin{bmatrix} 0.0169 \\ -0.2050 \end{bmatrix} y(t), \\ u(t) &= [0.3974 \quad -0.3834] \xi(t) + 0.01y(t). \end{aligned}$$

5 Conclusion

This paper extends the existing results for the H_∞ output control problem in several aspects. First, the optimal H_∞ output control is formulated as a convex programming problem for discrete-time systems. More specially, it is a problem of minimization of a linear objective function over a bounded convex set in $n(n+1)+1$ dimensional space. This convex programming problem can be solved by well developed convex optimization algorithms and the global optimal solution can be obtained. Hence a new procedure for design of H_∞ output controllers is presented. Secondly, it is shown that the H_∞ norm performance requirement can be considered as a convex constraint. It is very useful for multi-objective robust control design. How to extend this approach to the H_∞ control for uncertain systems is being undertaken.

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凸规划与离散系统 H_∞ 输出控制

陈文华

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摘要: 本文提出了一种求解离散系统最优 H_∞ 输出控制器的凸优化方法. 利用非线性矩阵映射, 文中表明 H_∞ 输出控制问题可以转化为在 $n(n+1)+1$ 维空间内凸集上的线性目标函数优化问题, 这里 n 为系统阶次. 在可镇定、可检测的条件下, 该凸集为有界的. 由本文给出设计方法得到的输出 H_∞ 控制器可以使得闭环 H_∞ 范数任意接近最优. 本文提出方法可以处理具有凸约束的 H_∞ 输出控制问题.

关键词: H_∞ 控制; 输出反馈; 凸优化; 控制系统设计; 离散系统

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陈文华 1965 年生. 1991 年毕业于东北大学自动控制系, 获工学博士学位, 同年到南京航空航天大学自动控制系任教. 现为副教授. 曾在国内外杂志发表论文二十余篇. 现从事鲁棒控制, 控制系统设计等方面研究, 特别是在航空、航海、机械、工业过程等领域的应用研究.

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Title	1996	Place	Deadline	Further Information
IFAC Workshop Intelligent Manufacturing Systems	July 21-23	Seoul Korea	30 Nov. 1996	Prof. Hyung Suck Cho Dept. of Mechanical Engineering Korea Advanced Inst. of Science & Technology 373-1, Kusong-dong, Yuseong-gu Taejon 305-701, Korea FAX+82/42 869 3210 e-mail:hscho@lca.kaist.ac.kr
IFAC Workshop Distributed Computer Control Systems	July 28-30	Seoul Korea	30 Nov. 1996	Prof. Wook Hyun Kwon Dept. of Control & Instrumentation Engg. Seoul National University San 56-1, Shillim-dong, Kwanak-gu Seoul 151-742, Korea FAX+82/2/888 4182 e-mail:whkwon@cisl.snu.ac.kr
IFAC Symposium Robot Control	Sept. 3-5	Nantes France	30 Jan. 1997	Prof. W. Khalil LAN-Ecole Centrale de Nantes F-44072 Nantes Cedex, France FAX+33 7657 4754 e-mail:khalil@lan.ec.nantes.fr
IFAC Workshop New Trends in Design of Control Systems	Sept. 10-12	Smolenice Slovakia	31 Jan. 1997	FIAC Workshop'97 Ms. Alena Kozakove, Fac. of El. Engg. llkovicova 3 SK-81219 Bratislava, Slovakia FAX+42/71/72 97 34 e-mail:ntdcs@kasr.elf.stuba.sk

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