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Robust Controller Design for Uncertain Systems with Interval Parameters*

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Abstract: A method to design a robust stabilization controller for uncertain systems with interval parameters which can be time-varying is presented. Only the bounds of the system parameters are required to design the robust controller. The design procedure is iterative in nature and the obtainded robust controller is a linear and fixed one but can tolerate all admissible uncertainties. The method is demonstrated by a power system load-frequency control example.

Key words: robust control; systems with interval prarmeters; power systems

1 Introduction

The robust control of uncertain systems is of practical importance since the plants to be controlled are often modelled approximately and may be subject to parameter changes[1]. Many design techniques for uncertain systems have been developed to deal with parameter variations, such as sliding mode control technique based on the theory of variable-structure systems[2] and various adaptive control techniques[3]. In this paper, based on the analysis theory given in [4,5], a general method is presented for designing a robust stabilization controller for an uncertain system with interval parameters. The uncertain parameters can be time-varying and only the lower and upper bounds of these parameters are required to design the robust controller. A step-by-step iterative design procedure is proposed. The obtained controller is a linear and fixed one but can tolerate all admissible uncertainties. The method is demonstrated through a power system load-frequency control example. The organization of this paper is as follows: the main theory and a step-by-step design procedure are developed in Section 2, as an illustration of the proposed method, a load-frequency control example with some simulation results is provided in Section 3, and the conclusion is given in the last section.

2 Main Results

2.1 Preliminaries

The following notations are used in this paper: $\Rightarrow$: belongs to; $\forall$: for all; $\lambda(\cdot)$: eigenvalues of matrix$(\cdot)$; $\sigma(\cdot)$: singular value of matrix$(\cdot)$; $(\cdot)_s$: symmetric part of matrix $(\cdot)$; $|(\cdot)|$: modulus of the entry$(\cdot)$; $(\cdot)_m$: modulus matrix, i.e. the matrix with modulus entries of a matrix$(\cdot)$.

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Let us consider a dynamic system described by the following equation

$$\dot{x}(t) = f(x(t)) + E(t)x(t), \tag{1}$$

where $f(x):\mathrm{R}^n \to \mathrm{R}^n$ is a continuous function, $x \in \mathrm{R}^n$ is the state vector, and $E(t) = (E_{ij}(t))$ is an $n \times n$ time-varying matrix which denotes the parameter uncertainty of the system.

Assumption 1 There exists an $n \times n$ positive definite symmetric matrix solution P for the following algebraic nonlinear matrix equation

$$F(P) + 2I_n = 0, \tag{2}$$

where I_n is an $n \times n$ identity matrix, $F(P):\mathrm{R}^{n\times n} \to \mathrm{R}^{n\times n}$ is the nonlinear term which satisfies $x^{\mathrm{T}}F(P)x = 2x^{\mathrm{T}}Pf(x)$.

Let $V(x) = x^{\mathrm{T}}Px$, where P is the solution of the equation (2). The following preliminary lemma can be proved.

Lemma 1 The system (1) is asymptotically stable if

$$\varepsilon < \frac{1}{\sigma_{\max}(P_m U_e)_s}, \tag{3}$$

where $P_m = (P)_m$, $U_e = (U_{eij}) = (\varepsilon_{ij}/\varepsilon)$, $E_{ij}(t) < \varepsilon_{ij} = |E_{ij}(t)|_{\max}$, $\forall\, t$ and $\varepsilon = \varepsilon_{ij\max}$.

Proof Let us choose $V(x) = x^{\mathrm{T}}Px$ as a Lyapunov candidate. By deriving the derivative of $V(x)$ along the trajectory of the system (1) and using Assumption 1, we obtain

$$\begin{aligned} \dot{V}(x(t)) &= 2x^{\mathrm{T}}(t)Pf(x(t)) + 2x^{\mathrm{T}}(t)PE(t)x(t) \\ &= x^{\mathrm{T}}(t)F(P)x(t) + x^{\mathrm{T}}(t)(E^{\mathrm{T}}(t)P + PE(t))x(t) \\ &= x^{\mathrm{T}}(t)(-2I_n + E^{\mathrm{T}}(t)P + PE(t))x(t). \end{aligned} \tag{4}$$

Now from (3), similarly to the proof of Theorom 1 in [4] and [5], we can get

$$\begin{aligned} &\sigma_{\max}(P_m\varepsilon U_e)_s < 1, \forall\, t, \Rightarrow \sigma_{\max}(|PE(t)|)_s < 1, \forall\, t, \Rightarrow \sigma_{\max}(PE(t))_s < 1, \forall\, t, \\ &\Rightarrow |\lambda(PE(t))_s|_{\max} < 1, \forall\, t, \Rightarrow \lambda_i[(PE(t))_s - I_n] < 0, \forall\, t \\ &\Rightarrow [-I_n + (PE(t))_s] < 0, \quad \forall\, t \\ &\Rightarrow [-2I_n + E^{\mathrm{T}}(t)P + PE(t)] < 0, \forall\, t. \end{aligned}$$

Then, from (4), we further have

$$\dot{V}(x(t)) < 0. \tag{5}$$

Therefore, the system (1) is asymptotically stable and we complete the proof.

2.2 Robust Control Analysis and Design

Let us consider the following uncertain system

$$\dot{x} = (A_0 + \Delta A(t))x(t) + (B_0 + \Delta B(t))u(t), \tag{6}$$

where A_0 and B_0 are the nominal constant matrices, $\Delta A(t) = (\Delta A_{ij}(t))$ is an $n \times n$ time-varying matrix which describes the system matrix parameter uncertainty, $\Delta B(t) = \beta(t)B_0$ is an $m \times m$ time-varying matrix, where $\beta(t)$ denotes the input matrix parameter uncertainty, $\beta(t) + 1 \leq \delta$ for some constant $\delta > 0$ and for all t.

Assumption 2 The pair (A_0, B_0) is controllable.

Let $W_e = (W_{eij})$, $W_{eij} = (\Delta\alpha_{ij}/\alpha)$, where $\Delta\alpha_{ij} = |\Delta A_{ij}(t)|_{\max}$, $\forall\, t$, and $\alpha = \alpha_{ij\max}$, $i, j = 1, 2, \cdots, n$, Let $P_m = (P)_m$, where P is the $n \times n$ postitive definite symmetric matrix solution of the following Riccati equation

$$A_0^T P + PA_0 - PB_0 R_1^{-1} B_0^T P + 2I_n = 0, \tag{7}$$

where $R_1 = (1/(2\delta))R_0$, R_0 is an $m \times m$ positive definite symmetric matrix which is given by designers, and I_n denotes the $n \times n$ identity matrix.

Remark 1 Equation (7) is a standard Riccati equation. Since (A_0, B_0) is controllable and R_1 and I_n are positive definite symmetric matrices, the Riccati eqnation (7) has a unique positive definite symmetric matrix solution P.

Theorem 1 If the system (6) subjecting to Assumption 2 satisfies

$$\alpha < \frac{1}{\sigma_{\max}(P_m W_e)}, \tag{8}$$

where P is the solution of the Riccati equation (7), then the system (6) is asymptotically stable for all admissible uncertainties via the following linear feedback control law

$$u(t) = - R_0^{-1} B_0^T P x(t) = - Kx(t). \tag{9}$$

Proof Let a Lyapunov function candidate be

$$V(x(t)) = x^T(t) P x(t). \tag{10}$$

Then the derivative of V along the trajectory of (6) with the control law (9) gives

$$\begin{aligned}
\dot{V}(x(t)) &= \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) \\
&= x^T(t)A_0^T Px(t) + x^T \Delta A^T(t)Px(t) + u^T(t)(B_0 + \Delta B(t))^T Px(t) \\
&\quad + x^T(t)PA_0 x(t) + x^T(t)P\Delta A(t)x(t) + x^T(t)P(B_0 + \Delta B(t))u(t) \\
&= x^T(t)A_0^T Px(t) + x^T(t)PA_0 x(t) - 2(1+\beta(t))x^T(t)PB_0 R_0^{-1}B_0^T Px(t) \\
&\quad + x^T(t)(\Delta A^T(t)P + P\Delta A(t))x(t) \\
&\leq x^T(t)(A_0^T P + PA_0 - 2\delta PB_0 R_0^{-1} B_0^T P)x(t) \\
&\quad + x^T(t)(\Delta A^T(t)P + P\Delta A(t))x(t) \\
&= x^T(t)(A_0^T P + PA_0 - PB_0 R_1^{-1} B_0^T P)x(t) \\
&\quad + x^T(t)(\Delta A^T(t)P + P\Delta A(t))x(t) \\
&= x^T(t)(-2I_n + \Delta A^T(t)P + P\Delta A(t))x(t).
\end{aligned} \tag{11}$$

By Lemma 1, the proof of this theorem is completed.

Remark 2 For the nominal parameters (A_0, B_0), the control (9) is optimal with respect to the linear quadratic index function whose weighting matrices are $(2I_n, R_0)$, i. e.

$$J = \int_0^\infty (2x^T(t)x(t) + u^T(t)R_0 u(t))\mathrm{d}t.$$

2.3 Robust Controller Design Procedure

Based on the theory of Section 2.1 and 2.2, we propose the following method to design a robust stabilization controller for uncertain systems with interval parameters.

Step 1 Find the range of system parameters for a given uncertain system and decide the lower and upper bounds of the uncertain parameters.

Step 2 Decide the nominal parameters of the system (One can choose the central points of the uncertain intervals as the design nominal parameters) and calculate the constant α.

Step 3 Choose the design matrix R_0 and solve the Riccati equation (7) to obtain the solution P.

Step 4 Construct the feedback gain matrix

$$K = R_0^{-1}B_0^{T}P.$$

Step 5 Construct the matrix W_e and calculate $\sigma_{\max}(P_m W_e)_s$. Then check the conditon (8). If the inequality (8) holds, then stop. Otherwise, go to Step 3.

Remark 3 Like most practical design methods, the proposed procedure is iterative in nature and several designs must be completed before the results can be judged acceptable. In Step 5, by repeating Step 3 (i. e. rechoose R_0), we can obtain larger control gain so that the condition (8) is satisfied.

The proposed design method is applicable to any uncertain system which can be written in the form of (6). In the next section, this method is illustrated by a power system load frequency control example.

3 Application to a Power System Load-Frequeny Control Example

3.1 Power System Model

In general, electrical power systems are complex nonlinear dynamical systems. For the load-frequency control (LFC) problem, the usual practice is to linearize a power system model around an operating point and then, based on the linearized model, develop control laws. Since the power system is only expossed to small changes in load during its normal operating, the linearized model can be used and will be sufficient to represent the power system dynamics around the operation point. The block diagram of the linearized model for a single control area is shown in Fig. 1[6,7].

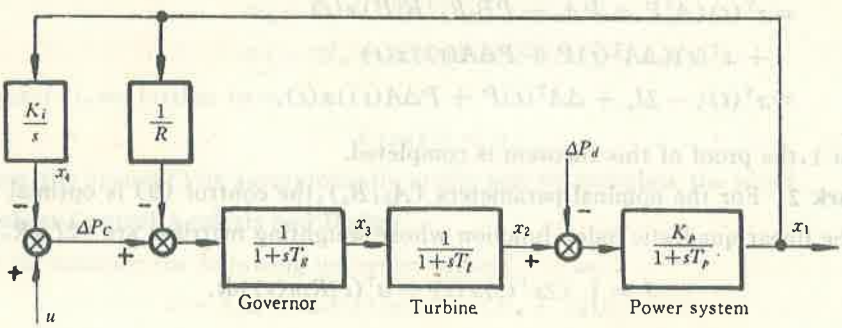

Fig. 1 Block diagram of single control area

In Fig. 1, the block K_i/s is a classical integral control unit which is a part of the conventional tie-line bias control in the case more than one control area[8]. The variables and parameters in Fig. 1 are defined as follows: $x_1=\Delta f$: incremental frequency deviation in Hz; $x_2=\Delta P_g$: incremental change in generator output in p. u. MW; $x_3=\Delta X_g$: incremental change in governor valve position in p. u. MW; $x_4=\int \Delta f dt$: incremental change in voltage angle in radians; ΔP_d: load disturbance in p. u. MW; ΔP_c: incremental change in the speed-changer position in p. u MW; T_g: governor time constant in seconds; T_t: trubine time constant in seconds; T_p: plant time constant in secounds; K_p: plant gain; R: speed regulation due to governor action in Hz/p. u, MW; K_i: integral control gain; u: control input from the designed robust controller.

From the schematic diagram, the system dynamics can be described by the following state differential equation

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \end{bmatrix} = \begin{bmatrix} -1/T_p & K_p/T_p & 0 & 0 \\ 0 & -1/T_t & 1/T_t & 0 \\ -1/RT_g & 0 & -1/T_g & -1/T_g \\ K_i & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/T_g \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} -K_p/T_p \\ 0 \\ 0 \\ 0 \end{bmatrix} \Delta P_d(t). \quad (12)$$

However, as system parameters can not be completely known and only their so called nominal values are known, a controller designed based on a fixed-parameter model may not work properly for the actual plant.. For LFC problem, several authors have dealt with the model parameter variations by using various control techniques such as variable-structrue control[6,7,9] and adaptive control[10,11]. In the following, the interval parameter uncertainties are taken into account. Suppose that the power system (12) has the following time-varying parameter range: $1/T_p \in [\underline{a}_1, \bar{a}_1]$, $K_p/T_p \in [\underline{a}_2, \bar{a}_2]$, $1/T_p \in [\underline{a}_3, \bar{a}_3]$, $1/T_p \in [\underline{a}_4, \bar{a}_4]$, and $1/RT_g \in [\underline{a}_5, \bar{a}_5]$ for all t where $\underline{a}_i$ and $\bar{a}_i$, $i=1,2,\cdots,5$, are the known lower and upper bounds of uncertain interval parameters, respectively. Then we obtain the following matrix form equation in the form of (6)

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) + F(t)\Delta P_d(t), \quad (13)$$

where $x(t) = [x_1(t)x_2(t)x_3(t)x_4(t)]^T$, $A(t)$, $B(t)$ and $F(t)$ are time-varying matrices with the appropriate dimensions.

3.2 Design and Simulation Results

Basing on the system parameters given in [7] and [12], we consider the following parameter range: $1/T_p \in [0.078, 0.089]$, $K_p/T_p \in [9.433, 10.639]$, $1/T_t \in [3.144, 3.547]$, $1/T_g \in [11.160, 14.205]$, $1/RT_g \in [3.207, 10.762]$, and $|\beta(t)| \leqslant 0.13$ for all t. Let us choose the central point of each parameter interval as the design nominal value. Then the nominal matrices A_0, B_0 and F_0 of the system (13) are as follows:

$$A_0 = \begin{bmatrix} -0.0835 & 10.0360 & 0 & 0 \\ 0 & -3.3455 & 3.3455 & 0 \\ -6.9845 & 0 & -12.6825 & -12.6825 \\ 0.6 & 0 & 0 & 0 \end{bmatrix},$$

$$B_0=\begin{bmatrix}0\\0\\12.6825\\0\end{bmatrix},\quad F_0=\begin{bmatrix}-10.0360\\0\\0\\0\end{bmatrix},$$

respectively. By calculating, we obtain the constant $\alpha=3.7775$. According to the robust controller design procedure given in Section 2.3, when choosing $R_0=7.2702E-02$, we solve the Riccati equation (7) and obtain the solution

$$P=\begin{bmatrix}6.7254E-01 & 7.2874E-01 & 2.1574E-02 & 6.1745E-01\\ 7.2874E-01 & 1.2692E+00 & 4.0337E-02 & 6.3690E-01\\ 2.1574E-02 & 4.0337E-02 & 1.8921E-02 & 1.7624E-02\\ 6.1745E-01 & 6.3690E-01 & 1.7624E-02 & 3.9156E+00\end{bmatrix}$$

Through calculation, we have $\sigma_{\max}(P_m W_e)_s=0.2632$ so that $\alpha=3.7775<\frac{1}{\sigma_{\max}(P_m W_e)_s}=3.7993$ and the condition (8) holds. We obtain the following robust controller $u(t)=-R_0^{-1}B_0^T Px(t)=-Kx(t)$, where $K=[k_1\ \ k_2\ \ k_3\ \ k_4]=[3.7633\ \ 7.0363\ \ 3.3005\ \ 3.0743]$.

Remark 4 By Theorem 1, the power system (13) is asymptotically stable for all admissable uncertainties via the linear and fixed controller. This means, under all admissable uncertainties, $\lim_{t\to\infty}\Delta f(t)=0$.

To show the control effectiveness and the comparision results for the cases with and without the proposed robust controller, two test cases are considered under 0.01 p.u. MW load change, i.e. $\Delta P_d=0.01$.

Case 1 Four different groups of system parameters are chosen in the parameter region:

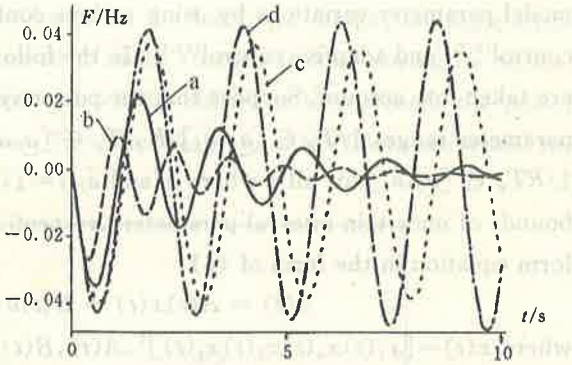

Fig. 2 Case 1: $\Delta f(t)$ responses versus time for groups a~d.

a: the nominal parameters; b: $1/T_p=0.089, K_p/T_p=10.639, 1/T_t=3.547, 1/T_g=14.205, 1/RT_g=10.762$; c: $1/T_p=0.078, K_p/T_p=9.433, 1/T_t=3.144, 1/T_g=11.160, 1/RT_g=3.207$; d: $1/T_p=0.089, K_p/T_t=10.639, 1/T_t=3.144, 1/T_g=11.160, 1/RT_g=3.207$.

The simulations results of $\Delta f(t)$ versus time for only having the integral controller are shown in Fig. 2.

Remark 5 The simulation results in Fig. 2 show that under the parameter uncertainties the classical integral controller can not ensure a good control performance for the system. In fact, the eigenvalues of the system matrix A_d which is corresponding to the parameter group d are $\{-12.0, -2.421, 0.0138\pm j2.777\}$. This implies that, without the proposed robust controller, i. e. $u(t)\equiv 0$, the system is unstable for the parameter group d. This can also be seen in Fig. 2.

Case 2 With the same four groups of system parameters a~d as in Case 1, but the obtained robust controller is used. The simulation results of $\Delta f(t)$ versus time are shown in Fig. 3.

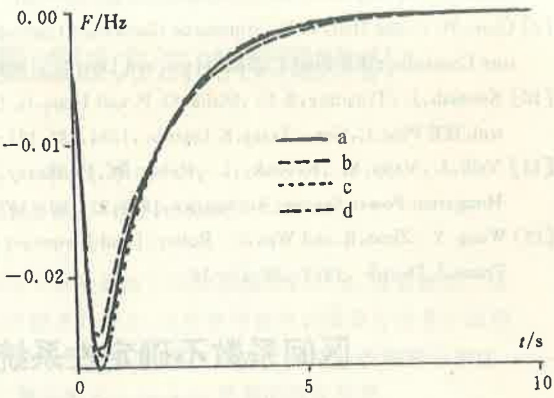

Fig. 3 Case 2: $\Delta f(t)$ responses versus time for groups a)~d).

Remark 6 The simulation results in Case 1 and 2 indicate that the obtained robust load-frequency controller can ensure the asymptotical stability of the overall system for all admissible uncertainties and also improve the control performance of the system.

4 Conclusion

In this paper, a general method to design a robust stabilization controller for uncertain systems with interval parameters is established. A step-by-step iterative robust controller design procedure is also proposed. During the design, the true system parameters are not needed and only the lower and upper bounds of the plant unknown and time-varying parameters are required. The obtained robust controller is simple linear and fixed one but it can tolerate all admissible uncertainties and ensure the asymptotical stability of the overall system. The method is demonstrated by a power system load-frequency control example. The simulation results have shown that the proposed robust controller ensure the asymptotical stability of the uncertain power system under the tested load change and improve the control performance of the systm and all of these can not be guaranteed by the conventional integral controller.

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区间系数不确定性系统的鲁棒控制器设计

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摘要: 本文提出了一种方法用于设计区间系数不确定性系统的鲁棒镇定控制器. 所考虑的区间系数可以是时变的,控制器的设计仅依赖于区间系数的上下界,而设计步骤具有迭代特征. 虽然所提到的控制器是线性的和确定性的,但能够承受所有容允不确定性. 所提出的方法通过一个电力系统负荷频率控制例子加以说明.

关键词: 鲁棒控制; 区间系数系统; 电力系统

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