第14卷第1期 控制理论与应用 Vol. 14, No. 1
1997年2月 CONTROL THEORY AND APPLICATIONS Feb., 1997

Robustness of Higher-Order P-Type Learning Control *

SUN Mingxuan
(Department of Electrical Engineering, Xi'an Institute of Technology · Xi'an, 710032, PRC)

Abstract: A feedback-assisted iterative learning control scheme for general nonlinear systems is proposed. With the aid of linearization along the desired trajectories, the asymptotic boundedness of the control process is proved in the presence of reinitialization errors and state and output periodic disturbances. the uniform convergence of the output error is given under asymptotically repetitive initial conditions and disturabance constraints. A simulation example is presented to demonstrate the robustness performance of the proposed learning controller.

Key words: iterative learning control; robustness; initial condition; nonlinear systems

1 Introduction

The past decade has been characterized by growing interest in the study of iterative learning control systems (ILCS)[1~4]. In the existing theory, from practical point of view, the robustness problem of ILCS is critical in the presence of the system state and output disturbances and errors of initialization. Such problem was first treated by Arimoto et al[5] for the case of PID-type updating law with the aid of linearization of robotic dynamics, and then by Heinzinger et al [6] using D-type updating law for a class of nonlinear systems. It was showed that the iterative trajectories converge to a neighborhood of the desired trajectories as the initial state errors, the state and output disturbances are bounded. However, the system convergence can not be guaranteed unless the bounds on the initial state errors and the state and output disturbances are zero. In previous papers we have extended the result to a feedback-assisted PI-type learning scheme[7]. It has been shown that this kind of updating law can be used for tracking control of a wider variety of nonlinear systems. In this paper, with respect to the existence of disturbaces and perturbed intial conditions, we still devote to considering robustness problem of learning control for general nonlinear systems using a feedback-assisted higher-order P-type updating law.

2 Problem Formulation and Preliminaries

Consider a general nonlinear dynamic system

$$\dot{x}_k(t) = f(x_k(t), u_k(t), t) + w_k(t), \tag{1a}$$

$$y_k(t) = g(x_k(t), u_k(t), t) + v_k(t), \tag{1b}$$

where k is the iterative index. For all $t \in [0, T]$ and $\forall\, k$, $x_k(t) \in \mathbb{R}^n$, $u_k(t) \in \mathbb{R}^m$, $y_k(t) \in \mathbb{R}^m$, $w_k(t) \in \mathbb{R}^n$ and $v_k(t) \in \mathbb{R}^m$. In addition, the following assumption are made:

A1) Along a desired trajectory pair $(x_d(t), u_d(t))$, the function $g(\cdot,\cdot,\cdot)$ in (1b) can be represented as

* The project is supported by National Natural Science Foudation of China.
Manuscript received Aug. 28, 1995, revised May 6, 1996.

$$g(x_k(t),u_k(t),t) = C(t)x_k(t) + D(t)u_k(t) + h(x_k(t),u_k(t),t),$$

where $C(t)$ and $D(t)$ are the partial derivatives

$$C(t) = \left(\frac{\partial g}{\partial x}\right)_{x_d,u_d}, \quad D(t) = \left(\frac{\partial g}{\partial u}\right)_{x_d,u_d}.$$

A2) The functions $f(\cdot,\cdot,\cdot)$ and $h(\cdot,\cdot,\cdot)$ are uniformly globally Lipschitz in x and u on the interval $[0,T]$ for some constants k_f and k_h.

Use the feedback-assisted learning controller of the form

$$u_k(t) = u_{fb,k}(t) + u_{ff,k}(t), \tag{2a}$$

$$u_{fb,k}(t) = Q_0(t)e_k(t), \tag{2b}$$

$$u_{ff,k+1}(t) = \sum_{i=1}^{N}\{P_i(t)u_{k-i+1}(t) + Q_i(t)e_{k-i+1}(t)\}, \tag{2c}$$

where $e_k(t) = y_d(t) - y_k(t)$, and $y_d(t)$ is the desired output. In the sequel, the vector with suffix d represents the correspondent desired vector. To assist the presentation of our results we need the following lemma:

Lemma 1 Let $\{a_k\}$ be a positive sequence defined as

$$a_k \leqslant \rho_1 a_{k-1} + \rho_2 a_{k-2} + \cdots + \rho_N a_{k-N} + d_k, \quad k = 1,2,\cdots$$

with given initial conditions $a_i, i = 1-N, 2-N, \cdots, -1, 0$, and $\{d_k\}$ is a given positive sequence. If

$$\rho_i \geqslant 0, \quad i = 1,2,\cdots,N, \quad \text{and} \quad \sum_{i=1}^{N}\rho_i < 1 \tag{3}$$

holds, then

$$d_k \leqslant \bar{d} \text{ implies that } \limsup_{k\to\infty}\{a_k\} \leqslant \frac{\bar{d}}{1-\sum_{i=1}^{N}\rho_i}, \tag{4a}$$

$$\lim_{k\to\infty}\{d_k\} = d_\infty \text{ implies that } \limsup_{k\to\infty}\{a_k\} \leqslant \frac{d_\infty}{1-\sum_{i=1}^{N}\rho_i}. \tag{4b}$$

Proof The proof of (4a) is omitted. Consider a sequence $\{b_k\}$ defined by

$$b_k = \rho_1 b_{k-1} + \rho_2 b_{k-2} + \cdots + \rho_N b_{k-N} + d_k \tag{5}$$

with initial conditions $b_i = a_i, i = 1-N, 2-N, \cdots, -1, 0$. Apparently, one has

$$a_k \leqslant b_k, k = 1,2,\cdots \tag{6}$$

due to $\rho_i \geqslant 0, i = 1,2,\cdots,N$. The condition $\sum_{i=1}^{N}\rho_i < 1$ implies that the discrete time system described by (5) is asymptotically stable, and hence

$$(1 - \sum_{i=1}^{N}\rho_i)\lim_{k\to\infty} b_k = d_\infty. \tag{7}$$

Combining (6) and (7), one can conclude that (4b) holds.

For brevity, in this paper we denote $\Delta(\cdot)_k = (\cdot)_d - (\cdot)_k, \delta(\cdot)_k = (\cdot)_{k+1} - (\cdot)_k$.

3 ILCS with Uncertain Initial Conditions and Periodic Disturbances

In this section we consider the case:

A3) The controlled system (1) is perturbed by the disturbances with period T, namely, $w_k(t) = w(t), v_k(t) = v(t), w_d(t) = w(t), v_d(t) = v(t), t \in [0,T]$, and the uncertain but bounded reinitialization error $\| \Delta x_k(0) \| \leqslant b_{x0}, \forall k$.

Theorem 1 Suppose that iterative learning control system (1),(2) satisfies Assumptions A1)~A3),and the learning gains $P_i(t), Q_i(t), i = 1,2,\cdots,N$ and $Q_0(t)$ are chosen such that

$$\sum_{i=1}^{N} P_i(t) = I, \tag{8a}$$

$$\bar{\alpha}_0 > 0, \quad \sum_{i=1}^{N} \bar{a}_i/\bar{a}_0 < 1, \tag{8b}$$

where $\bar{\alpha}_0 = \inf_{t\in[0,T]} \{\alpha_0(t)\}, \alpha_0(t) = \| [I + Q_0(t)D(t)]^{-1} \|^{-1} - k_h \| Q_0(t) \|, \bar{\alpha}_i = \sup_{t\in[0,T]} \{\alpha_i(t)\}$, $\alpha_i(t) = \| P_i(t) - Q_i(t)D(t) \| + k_h \| Q_i(t) \|, i = 1,2,\cdots,N$, then the control input $u_k(t)$ converges to a neighborhood of the desired control input $u_d(t)$ for $t \in [0,T]$ as $k \to \infty$. Furthermore, the radius of the neighborhood is zero if the bound b_{x0} is zero.

Proof Denoting $j = k - i + 1$ and choosing $P_i(t)$ such that $\sum_{i=1}^{N} P_i(t) = I$, we get

$$\{I + Q_0(t)D(t)\}\Delta u_{k+1}(t) = \sum_{i=1}^{N} \{P_i(t) - Q_i(t)D(t)\}\Delta u_j(t) - \sum_{i=0}^{N} Q_i(t)\{C(t)\Delta x_j(t) + \Delta h_j + \Delta v_j(t)\}.$$

Taking the norm in both sides gives

$$\alpha_0(t) \| \Delta u_{k+1}(t) \| \leqslant \sum_{i=1}^{N} \alpha_i(t) \| \Delta u_j(t) \| + \sum_{i=0}^{N} \| Q_i(t) \| \{(\| C(t) \| + k_h) \| \Delta x_j(t) \| + \| \Delta v_j(t) \| \}, \tag{9}$$

and further taking the λ-norm of (9) yields

$$\bar{\alpha}_0 \| \Delta u_{k+1} \|_\lambda \leqslant \sum_{i=1}^{N} \bar{\alpha}_i \| \Delta u_j \|_\lambda + \sum_{i=0}^{N} q_i\{(c + k_h) \| \Delta x_j \|_\lambda + \| \Delta v_j \|_\lambda\}, \tag{10}$$

where $c = \sup_{t\in[0,T]} \| C(t) \|, q_i = \sup_{t\in[0,T]} \| Q_i(t) \|$. Now, writing the integral expression for $x_j(t)$ and using the Bellman-Gronwall lemma

$$\| \Delta x_j(t) \| = \| \Delta x_j(0) + \int_0^t \{\Delta f_j + \Delta w_j(\tau)\}d\tau \|$$

$$\leqslant \| \Delta x_j(0) \| e^{k_f t} + \int_0^t e^{k_f(t-\tau)}(k_f \| \Delta u_j(\tau) \| + \| \Delta w_j(\tau) \|)d\tau.$$

Multipling both sides of above eqn. by $e^{-\lambda t}, t \in [0,T]$, we have

$$\| \Delta x_j(t) \| e^{-\lambda t} \leqslant \| \Delta x_j(0) \| e^{(k_f-\lambda)t} + (k_f \| \Delta u_j \|_\lambda + \| \Delta w_j \|_\lambda)\int_0^t e^{k_f(t-\tau)}d\tau.$$

Defining $\lambda_f = \dfrac{1 - e^{(k_f-\lambda)T}}{\lambda - k_f}$, and choosing λ such that $\lambda > k_f$ give rise to

$$\| \Delta x_j \|_\lambda \leqslant \| \Delta x_j(0) \| + \lambda_f(k_f \| \Delta u_j \|_\lambda + \| \Delta w_j \|_\lambda). \tag{11}$$

Substituting (11) into (10) gives

$$\mu_0 \| \Delta u_{k+1} \|_\lambda \leqslant \sum_{i=1}^{N} \mu_i \| \Delta u_j \|_\lambda + \xi_{k+1}, \tag{12}$$

where $\mu_0 = \bar{\alpha}_0 - q_0\lambda_f k_f(c+k_h)$, $\mu_i = \bar{a}_i + q_i\lambda_f k_f(c+k_h)$, $\xi_{k+1} = \sum_{i=0}^{N} q_i\{(c+k_h)\|\Delta x_j(0)\| + \lambda_f(c+k_h)\|\Delta w_j\|_\lambda + \|\Delta v_j\|_\lambda\}$. According to (8b), we can choose λ large enough such that $\sum_{i=1}^{N}\mu_i/\mu_0 < 1$. By Lemma 1 and Assumption A3), the control input error is thus bounded as

$$\limsup_{k\to\infty}\|\Delta u_k\|_\lambda \leqslant \frac{(c+k_h)b_{x0}\sum_{i=0}^{N}q_i}{\mu_0 - \sum_{i=1}^{N}\mu_i}. \tag{13}$$

In addition, we obtain that the right-hand term of eqn. (13) is zero as $b_{x0} = 0$. This completes the proof.

Remark 1 Using eqn. (11) we can obtain that $\|\Delta x_k\|_\lambda$ is asymptotically bounded with the bound $b_x = b_{x0} + \lambda_f k_f b_u$ where b_u is the right-hand term of eqn. (13). To obtain the result for $\|e_k\|_\lambda$ we can use eqn. (1b) with $\|\Delta u_k\|_\lambda$ and $\|\Delta x_k\|_\lambda$ being bounded, namely, an asymptotic bound is $b_e = (c+k_h)b_x + (d+k_h)b_u$ where $d = \sup_{t\in[0,T]}\|D(t)\|$. It has also demonstrated that the actual output converges to the desired output as b_{x0} tends to zero, even in the presence of periodic disturbances.

4 ILCS with Asymptotically Repetitive Initial Conditions and Disturbance Constraints

In this section, we restrict our consideration to a simpler form of updating law by changing (2c) to the following form

$$u_{ff,k+1}(t) = u_k(t) + \sum_{i=1}^{N} Q_i(t)e_{k-i+1}(t). \tag{14}$$

A4) For all k and $t\in[0,T]$, the disturbances $w_k(\cdot)$ and $v_k(\cdot)$ and the initial setting $x_k(0)$ are assumed to satisfy that

$$\|w_{k+1}(t) - w_k(t)\| \leqslant b_w,\quad \|v_{k+1}(t) - v_k(t)\| \leqslant b_v,\quad \|x_{k+1}(0) - x_k(0)\| \leqslant b_{x0}.$$

Theorem 2 Suppose that iterative learning control system (1), (2a), (2b), (14) satisfies Assumptions A1), A2), A4), and the learning gains $Q_i(t)$, $i = 0,1,2,\cdots,N$ are chosen such that

$$\bar{\beta}_0 > 0,\quad \sum_{i=1}^{N}\bar{\beta}_i/\bar{\beta}_0 < 1, \tag{15}$$

where $\bar{\beta}_0 = \inf_{t\in[0,T]}\{\beta_0(t)\}$, $\beta_0(t) = \|[I + D(t)Q_0(t)]^{-1}\|^{-1} - k_h\|Q_0(t)\|$, $\bar{\beta}_i = \sup_{t\in[0,T]}\{\beta_i(t)\}$, $\beta_1(t) = \|I - D(t)Q_1(t)\| + k_h\|Q_1(t)\|$, $\beta_i(t) = \|D(t)Q_i(t)\| + k_h\|Q_i(t)\|$, $i = 2,3,\cdots,N$, then the output $y_k(t)$ converges to a neighborhood of the desired output $y_d(t)$ for $t\in[0,T]$ as $k\to\infty$. Furthermore, the output error $e_k(t)$ converges uniformly to zero for $t\in[0,T]$ as $k\to\infty$ if $\lim_{k\to\infty}\delta x_k(0) = 0$, $\lim_{k\to\infty}\delta w_k(t) = 0$, $\lim_{k\to\infty}\delta v_k(t) = 0$.

Proof The output error at the $(k+1)$th iteration can be written as

$$\begin{aligned} e_{k+1}(t) &= e_k(t) - \{y_{k+1}(t) - y_k(t)\} \\ &= e_k(t) - C(t)\delta x_k(t) - D(t)\delta u_k(t) - \delta h_k(t) - \delta v_k(t) \\ &= [I + D(t)Q_0(t)]^{-1}\{[I - D(t)Q_1(t)]e_k(t) \end{aligned}$$

$$- \sum_{i=2}^{N} D(t)Q_i(t)e_j(t) - C(t)\delta x_k(t) - \delta h_k(t) - \delta v_k(t)\}$$

where $j = k - i + 1$. Computing the norm of both sides yields

$$\beta_0(t) \| e_{k+1}(t) \| \leqslant \sum_{i=1}^{N} \beta_i(t) \| e_j(t) \| + (\| C(t) \| + k_h) \| \delta x_k(t) \| + \| \delta v_k(t) \| \tag{16}$$

and further taking the λ- norm of(16) yields

$$\overline{\beta}_0 \| e_{k+1} \|_\lambda \leqslant \sum_{i=1}^{N} \overline{\beta}_i \| e_j \|_\lambda + (c + k_h) \| \delta x_k \|_\lambda + \| \delta v_k \|_\lambda, \tag{17}$$

where $c = \sup_{t\in[0,T]} \| C(t) \|$, Through similar argument to eqn. (11), we obtain as $\lambda > k_f$,

$$\| \delta x_k \|_\lambda \leqslant \| \delta x_k(0) \| + \lambda_f(k_f \| \delta u_k \|_\lambda + \| \delta w_k \|_\lambda), \tag{18}$$

where $\lambda_f = \dfrac{1 - e^{(k_f - \lambda)T}}{\lambda - k_f}$. Taking the λ- norm of eqn. (2a) gives

$$\| \delta u_k \|_\lambda \leqslant \sum_{i=0}^{N} q_i \| e_j \|_\lambda, \tag{19}$$

where $q_i = \sup_{t\in[0,T]} \| Q_i(t) \|$. Substituting (19) into (18), we have

$$\| \delta x_k \|_\lambda \leqslant \| \delta x_k(0) \| + \lambda_f k_f \sum_{i=0}^{N} q_i \| e_j \|_\lambda + \lambda_f \| \delta w_k \|_\lambda. \tag{20}$$

Substituting (20) into (17), we have

$$\eta_0 \| e_{k+1} \|_\lambda \leqslant \sum_{i=1}^{N} \eta_i \| e_j \|_\lambda + \zeta_{k+1}, \tag{21}$$

where $\eta_0 = \overline{\beta}_0 - q_0\lambda_f k_f(c + k_h)$, $\eta_i = \overline{\beta}_i + q_i\lambda_f k_f(c + k_h)$, $\zeta_{k+1} = (c + k_h) \| \delta x_k(0) \| + \lambda_f(c + k_h) \| \delta w_k \|_\lambda + \| \delta v_k \|_\lambda$. Due to the condition (15), it is possible to choose λ large enough such that $\sum_{i=1}^{N} \eta_i / \eta_0 < 1$. Thus, by Lemma 1, the output error converges to a neighborhood of radius as

$$\limsup_{k\to\infty} \| e_k \|_\lambda \leqslant \frac{(c + k_h)b_{x0} + \lambda_f(c + k_h)b_w + b_v}{\eta_0 - \sum_{i=1}^{N} \eta_i}. \tag{22}$$

Note that $\zeta_{k+1} = 0$ as $\delta x_k(0) = 0$, $\delta w_k(t) = 0$, and $\delta v_k(t) = 0$, $\forall\, k$. Thus, the output error converges to zero. By Lemma 1, we further obtain that if $\lim_{k\to\infty} \delta x_k(0) = 0$, $\lim_{k\to\infty} \delta w_k(t) = 0$, and $\lim_{k\to\infty} \delta v_k(t) = 0$, the output error also converges to zero. i. e., $\lim_{k\to\infty} e_k(t) = 0$ for $t \in [0, T]$. This completes the proof.

Remark 2 In Theorem 2, the convergence only requires that $x_k(0)$, $w_k(t)$, and $v_k(t)$ are asymptotically invariant during iterations. It is a milder condition for convergence than that required by previous literature. Note that the size of the error bound in eqn. (22) can be adjusted by the learning gain of the feedback output error. From the above theorem, there exist $M > 0$ and a positive integer K such that $\| e_k \|_\lambda \leqslant M / \{\eta_0 - \sum_{i=1}^{N} \eta_i\}$ as $k > K$. For the given $\varepsilon > 0$, if the learning gain $Q_0(t)$ is chosen so that $\eta_0 > M/\delta + \sum_{i=1}^{N} \eta_i$, $\delta = \varepsilon e^{-\lambda T}$, then $\| e_k(t) \|$

$< \varepsilon$ for each $t \in [0,T]$.

5 Simulation

In this section a numerical example is presented to illustrate the performance of iterative learning control scheme described above. Consider the following nonlinear ILCS

$$\dot{x}_1(t) = x_2(t), \tag{23a1}$$

$$\dot{x}_2(t) = -x_1(t) - 2x_2(t) + u(t) + 0.5\sin(x_1(t)u(t)), \tag{23a2}$$

$$y(t) = x_2(t) + u(t), \tag{23b}$$

$$u_{k+1}(t) = u_k(t) + q_0 e_{k+1}(t) + q_1 e_k(t), \tag{23c}$$

where $t \in [0,1]$. Suppose the desired output trajectory is given by setting $u_d(t) = \sin(6.28t)$, and $x_d^1(0) = x_d^2(0) = 0.0$. Let the initial input $u_0(t) = 0.0$. The tolerance root mean square of the output error is assumed to be rms=0.005.

We have conducted simulation for two cases by choosing $q_0 = 0.4, q_1 = 0.8$: one is for the initial state bias $x_k^1(0) = 0.0, x_k^2(0) = 0.5$. It requires four iterations such that the output converges to the desired trajectory with the given accuracy. The other is the case for $x_k^1(0) = 0.0, x_k^2(0) = 0.5 + 0.5\exp(-k)$. In this case, the six iterations are sufficient to generate a trajectory to recovery the desired.

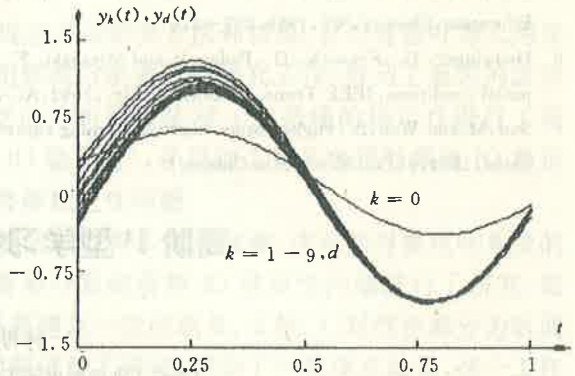

Fig. 1 Plant output histories with $x_k^1(0) = 0$, $x_k^2(0) = 0.5\cos(k)$

To compare the performance of the proposed controller, the simulations are also performed for the UBB initial condition and output disturbance. Let $q_0 = 0.6, q_1 = 0.9$. Fig. 1 shows the iteration histories of the output with the initial condition $x_k^1(0) = 0.0, x_k^2(0) = 0.5\cos(k)$. Fig. 2 shows the output trajectories when a unit disturbance is introduced in the output of the ILCS in the interval $[0.5, 0.6]$.

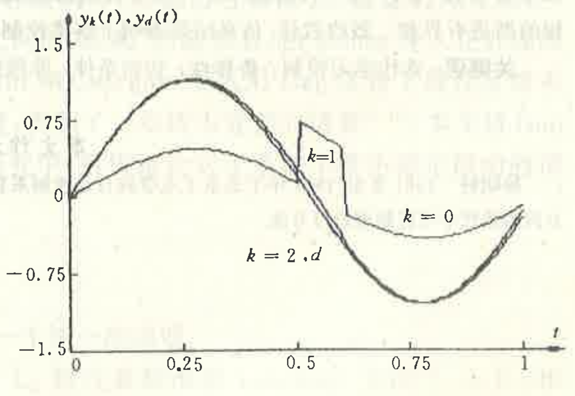

Fig. 2 Ouput recovery histories from the disturbance

6 Conclusion

In the presence of periodic disturbances of the system state and output, and uncertain but bounded initial conditions, it has been proved that the asymptotic boundedness of the control input, state, and output of a class general nonlinear systems, and shown that the bounds are only dependent on the bound on the initial state errors but independent of the periodic disturbances. Under an assumption on boundedness of the disturbances and the initail conditions between two successive trials a new asymptotic bound of the output error has been obtained in this paper. It deserves to note that the convergence can be guaranteed as the initial conditions and disturbances are asymptotically invariant during iterations.

References

1 Ahn, H. S. and Choi, C. H.. Iterative learning controller for linear systems with a periodic disturbance. Electronics Letters, 1990, 26(18): 1542—1544

2 Sugie, T. and Ono, T.. An iterative learning control law for dynamical systems. Automatica, 1991, 27(4): 729—732

3 Bien, Z. and Huh, K. M.. Higher-order iterative learning control algorithm. IEE Proc. D, 1989, 136(3): 105—112

4 Kuc, T. Y., Lee, J. S. and Nam, K.. An iterative learning control theory for a class of nonlinear dynamic systems. Automatica, 1992, 28(6): 1215—1221

5 Arimoto, S. Kawamura, S. and Miyazaki, F.. Convergence, stability, and robustness of learning control schemes for robot manipulators. In Jamshidi, M. J. Luh, L. Y, and Shahinpoor, M. (eds), Recent Trends in Robotics: Modelling, Control, and Education, Elsevier, NY, 1986, 307—316

6 Heinzinger, G., Fenwick, D., Paden, B. and Miyazaki, F.. Stability of learning control with disturbances and uncertain initial conditions. IEEE Trans. Automat. Contr., 1992, AC-37(1): 110—114

7 Sun, M. and Wan, B.. Higher-order iterative learning control algorithm for nonlinear dynamical systems. Control and Decision, 1994, 9(3): 195—199 (in Chinese)

高阶 P 型学习控制的鲁棒性

孙明轩

(西安工业学院电子工程系·西安,710032)

摘要:通过线性化处理,本文将开闭环配合的高阶 P 型迭代学习控制律的适用范围推广到更一般的非线性动态系统.对于 UBB 初始条件和周期干扰,以及渐进重复初始条件和干扰的情形,文中分别证明了学习过程的渐进有界和一致收敛性.仿真结果表明了这类控制器的鲁棒性能.

关键词:迭代学习控制;鲁棒性;初始条件;非线性系统

本文作者简介

孙明轩 1961 年生.1987 年于北京工业学院自动控制系获工学硕士学位.现为西安工业学院电子工程系副教授.研究方向为迭代学习控制理论与方法.