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A Note on the Decoupling Controller Design for Unity Feedback System

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Abstract: In order to have a solution for the decoupling controller, doubly-coprime factorization technique is employed for investigating the constraints on the assigned diagonal closed-loop transfer matrix for the unity feedback system with a non-square, rational and proper plant. When the constraints are satisfied, computation formulas, employing existing algorithms for time domain analysis, for the controller are derived.

Key words: decoupling controller; non-square plant; doubly coprime factorization; internal stabilization; time domain analysis; right inverse; singular value decomposition

1 Introduction

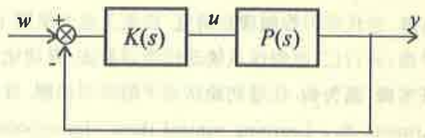

Fig. 1 Unity feedback closed-loop system

The design of a rational, proper decoupling controller K for a unity feedback closed-loop system with a given rational, proper plant P shown in the figure has been studied by many investigators[1~4].

The solvability of this problem depends on the appropriate assignment of the ideal diagonal closed-loop transfer matrix $G_c = PK(I+PK)^{-1}$ from input W to output Y. For a square and invertible plant P, Lin and Hsieh[5] has recently put forward the constraints on the assigned diagonal closed-loop transfer matrix, and has given an algorithm for the computation of the controller. We generalized Lin's results to the case of non-square plants, and found that in this case the constraints on the diagonal closed-loop transfer matrix are almost in parallel to that of Lin. For the sake of investigation, we started our approach from the doubly-coprime factorization; however, we found eventually that this factorization is actually not necessary. The key point of the computation of the controller K lies in the inversion of a non-square numerical matrix. By referring to the work of some other authors[6~8], we modified the inversion methods for time domian analysis by employing the singular value decomposition.

2 Constraints on the Closed-Loop Transfer Matrix

Let P: $[A,B,C,D]$ be an $p \times m (p \leqslant m)$ rational and proper transfer matrix, not necessary stable. The doubly-coprime factorization of P is[5]

$$P = NM^{-1} = \widetilde{M}^{-1}\widetilde{N}, \tag{1}$$

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$$\begin{bmatrix} M & Y \\ N & X \end{bmatrix} = \begin{bmatrix} \tilde{X} & -\tilde{Y} \\ -\tilde{N} & \tilde{M} \end{bmatrix}^{-1}. \tag{2}$$

where*

$$N:[A-BF,B,C-DF,D], \quad M:[A-BF,B,-F,I], \tag{3}$$

$$\tilde{N}:[A-HC,B-HD,C,D], \quad \tilde{M}:[A-HC,-H,C,I], \tag{4}$$

$$Y:[A-BF,H,-F,0], \quad X:[A-BF,H,C-DF,I], \tag{5}$$

$$\tilde{Y}:[A-HC,H,-F,0], \quad \tilde{X}:[A-HC,B-HD,F,I]. \tag{6}$$

F and H are numerical matrices which are chosen such that matrix $A-BF$ and matrix $A-HC$ are stable. Please note that transfer matrices $\tilde{M}$ and M are biproper, and so are $\tilde{M}^{-1}$ and M^{-1}.

It is well known that the set of internally stabilizing controllers is

$$K=-(Y-MQ)(X-NQ)^{-1} \tag{7}$$

or

$$K=-(\tilde{X}-Q\tilde{N})^{-1}(\tilde{Y}-Q\tilde{M})^{-1}.$$

Where Q is any rational proper and stable transfer matrix. By properly choosing Q, we can make the stabilizing controller K also a decoupling one. However, this route will lead to a two-sided model matching problem, requiring both left and right inversion of matrix; therefore we will leave this line of approach and proceed in the following to determine separately the factors $(Y-MQ)$ and $(X-NQ)$ of K in (7). It will turn out at a later stage that this appoach needs only a right inversion of a numerical matrix and the explicit computation of $\tilde{N}, X, \tilde{X}, Y$ and $\tilde{Y}$ are actually not required.

Premultiplying the left side of (2) by $\begin{bmatrix} I & -Q \\ 0 & I \end{bmatrix}^{-1}$ and premultiplying the right hand side of (2) by $\begin{bmatrix} -I & -Q \\ 0 & I \end{bmatrix}^{-1} = \begin{bmatrix} I & Q \\ 0 & I \end{bmatrix}$ yield

$$\begin{bmatrix} M & Y-MQ \\ N & X-NQ \end{bmatrix}^{-1} = \begin{bmatrix} \tilde{X}-Q\tilde{N} & -(\tilde{Y}-Q\tilde{M}) \\ -\tilde{N} & \tilde{M} \end{bmatrix}. \tag{8}$$

Denoting $U=Y-MQ, \quad V=X-NQ, \quad \tilde{V}=\tilde{X}-Q\tilde{N}, \quad \tilde{U}=\tilde{Y}-Q\tilde{M},$ (9)

(8)can be written as

$$\begin{bmatrix} \tilde{V} & -\tilde{U} \\ -\tilde{N} & \tilde{M} \end{bmatrix} \begin{bmatrix} M & U \\ N & V \end{bmatrix} = I. \tag{10}$$

and the set of stabilizing controllers of (7) can be expressed as

$$K=-UV^{-1} \tag{11}$$

where U and V are proper and stable transfer matrices.

Form (10) we find that $-\tilde{N}U+\tilde{M}V=I$ or

$$(I+PK)^{-1}=V\tilde{M}. \tag{12}$$

By making use of (12), the closed-loop transfer matrix can be written as

$$G_c=PK(I+PK)^{-1}=I-(I+PK)^{-1}=I-V\tilde{M} \tag{13}$$

or

$$G_c=PK(I+PK)^{-1}=-NM^{-1}UV^{-1}(I+PK)^{-1}=-NM^{-1}U\tilde{M}. \tag{14}$$

(13)gives the relation between V and G_c as follows:

$$V=(I-G_c)\tilde{M}^{-1}. \tag{15}$$

* For stable P, we have: $F=H=0$, $N=\tilde{N}=P$, $M=\tilde{M}=I$, $X=\tilde{X}=I$, $Y=\tilde{Y}=0$.

To determine the relation between U and G_c, let us denote N^{-1} as the right inverse of N, i.e. $NN^{-1}=I$, and write $G_c=NN^{-1}G_c$, we find from (14) that $N(N^{-1}G_c+M^{-1}U\widetilde{M})=0$. Since N is of full row rank and U is to be determined, we can select the solution $N^{-1}G_c+M^{-1}U\widetilde{M}=0$. This gives the following relation between U and G_c

$$U=-MN^{-1}G_c\widetilde{M}^{-1}. \tag{16}$$

(15) and (16) are the basic equations for determining the constraints on G_c.

Let us denote the assigned diagonal closed-loop transfer matrix by

$$G_c=\mathrm{diag}\left\{\frac{\beta_1}{\alpha_1}\cdots\frac{\beta_p}{\alpha_p}\right\}, \tag{17}$$

the constraints on β_i and α_i are determined such that V ad U are stable and proper transfer matrices as required by the stabilizing controller.

1) We find from (15) that

$$V=(I-G_c)\widetilde{M}^{-1}=\mathrm{diag}\left\{\frac{\alpha_j-\beta_j}{\alpha_j}\right\}\widetilde{M}^{-1}. \tag{18}$$

For V to be stable, the right half plane (rhp) poles in the jth row elements of $\widetilde{M}^{-1}$ must be canceled by the rhp zeros of $(\alpha_j-\beta_j)$. Thus, $(\alpha_j-\beta_j)$ must have as its zero polynomial the least common multiplier (lcm) of the rhp pole polynomials in the jth row of $\widetilde{M}^{-1}$.

The properness of V. is obvious, since both $\mathrm{diag}\{(\alpha_j-\beta_j)/\alpha_j\}$ and $\widetilde{M}^{-1}$ are biproper.

2) It is seen from (16) that

$$U=-MN^{-1}G_c\widetilde{M}^{-1}=-MN^{-1}\mathrm{diag}\left\{\frac{\beta_1}{\alpha_1}\cdots\frac{\beta_p}{\alpha_p}\right\}\widetilde{M}^{-1}. \tag{19}$$

For U to be stable, the rhp poles in the jth column elements of N^{-1} must be canceled by the rhp zeros of β_j. Thus, β_j should have as its zero polynomial the lcm of the rhp pole polynomials in the jth column elements of N^{-1}.

Because both M and $\widetilde{M}$ in (19) are bioproper as we mentioned earlier, for U to be proper, the pole-zero excess of the jth diagonal element in G_c must be greater than the zero-poles excess in the jth column elements in N^{-1}. Specifically, let $N^{-1}=\{n_{ij}/d_{ij}\}$ and $\Delta_j=\max\{\deg(n_{ij})-\deg(d_{ij})\}$, then it is necessary that

$$\deg(\alpha_j)\geqslant\deg(\beta_j)+\Delta_j. \tag{20}$$

One might suspect that the cancellation of the rhp poles in the jth column elements of N^{-1} is not sufficient to assure the stability of U because of the existence of $\widetilde{M}^{-1}$ in (19) which might be unstable. However, we will show by contradiction that this suspicion is unnecessary. For this purpose, let us consider the relation $-\widetilde{N}U+\widetilde{M}V=I$ obtained from (10) and write it as $V=\widetilde{M}^{-1}(I+\widetilde{N}U)$. Now, assume that U in (19) have rhp poles, these rhp poles must come from $\widetilde{M}^{-1}$, because the rhp poles in N^{-1} have been canceled by β_j. Note that rhp poles of $\widetilde{M}^{-1}$ can not be canceled by $\widetilde{N}$ since $\widetilde{M}^{-1}$ and $\widetilde{N}$ are coprime. As a result, these rhp poles will appear in $\widetilde{N}U$ in the above expression for V, thus making V unstable. However, the stability of V has been assured by the cancellation of the rhp poles in the jth row elements of $\widetilde{M}^{-1}$ by the rhp zeros of $(\alpha_j-\beta_j)$. We thus see that this assumption of the instability of U causes a contradiction.

3 The Inversion of N: $[A-BF, B, C-DF, D]$

The key problem in our approach is the determination of the right inverse N^{-1} of the nonsquare $p \times m$ $(p \leqslant m)$ matrix N: $[A-BF, B, C-DF, D]$ defined in (3). This inversion can be carried out by making use of the existing software package for time domain analysis. By referring to [4], [5] and [6], we give a discussion on this problem by considering various cases of dimension and rank.

Case a If $p \leqslant m$ and rank $D=p$, i. e. D is of full row rank, then the right inverse of N is

$$N^{-1}: [A-BD^{-1}C, BD^{-1}, F-D^{-1}C, D^{-1}] \tag{21}$$

or in the transfer matrix form

$$N^{-1}(s)=(F-D^{-1}C)(sI-A+BD^{-1}C)^{-1}BD^{-1}+D^{-1} \tag{22}$$

where D^{-1} is the right inverse of D such that $DD^{-1}=I$. We will show in the following that

$$N(s)N^{-1}(s)=I,$$

$$\begin{aligned} N(s)N^{-1}(s)&=[(C-DF)(sI-A+BF)^{-1}B+D][(F-D^{-1}C)(sI-A+BD^{-1}C)^{-1}BD^{-1}+D^{-1}] \\ &=I+(DF-C)(sI-A+BD^{-1}C)^{-1}BD^{-1}+(C-DF)(sI-A+BF)^{-1} \\ &\quad \cdot (BF-BD^{-1}C)(sI-A+BD^{-1}C)^{-1}BD^{-1}+(C-DF)(sI-A+BF)^{-1}BD^{-1}. \end{aligned}$$

If we write the last term of the above equation as

$$\begin{aligned} &(C-DF)(sI-A+BF)^{-1}BD^{-1} \\ &=(C-DF)(sI-A+BF)^{-1}(sI-A+BD^{-1}C)(sI-A+BD^{-1}C)^{-1}BD^{-1}. \end{aligned}$$

then

$$N(s)N^{-1}(s)=I+(DF-C)(sI-A+BD^{-1}C)^{-1}BD^{-1}+(C-DF)(sI-A+BD^{-1}C)^{-1}BD^{-1}=I.$$

When $p=m$ and rank $D=p$, D is square and invertible, then D^{-1} is the inverse of D in ordinary sense. When $p<m$, we can find D^{-1} by employing the singular value decomposition of D:

$$D=\Phi[\Sigma \quad 0]\Gamma^{T}, \tag{23}$$

where $\Sigma=\text{diang}(\sigma_1, \cdots, \sigma_p)$ and $\sigma_1, \cdots \sigma_p$ are singular values of D. Square matrices Φ and Γ are respectively the left and right singular matrix of D. Both of them are unitary, i. e. $\Phi^T\Phi=\Phi\Phi^T=I$ and $\Gamma^T\Gamma=\Gamma\Gamma^T=I$. Since Γ is unitary, we have from (23) that

$$D\Gamma=[\Phi\Sigma \quad 0]. \tag{24}$$

If we denote $\Gamma=[\Gamma_1 \quad \Gamma_2]$, where Γ_1 is of dimension $m\times p$ and Γ_2 is of dimension $m\times(m-p)$, (24) can be written as

$$D\Gamma_1\Sigma^{-1}\Phi^T=I, \tag{25}$$

$$D\Gamma_2=0. \tag{26}$$

Postmultiplying both sides of (26) by an arbitrary $(m-p)\times p$ matrix Z and summing up the product with (25) yield

$$\text{D}(\Gamma_1\Sigma^{-1}\Phi^T+\Gamma_2 Z)=I. \tag{27}$$

We thus find the right inverse D^{-1} as

$$D^{-1}=\Gamma_1\Sigma^{-1}\Phi^T+\Gamma_2 Z. \tag{28}$$

The set of poles of $N^{-1}(s)$ consists of two parts: the first part are zeros of $N(s)$ which are fixed; the remaining part depends on Z. When $N(s)$ has no rhp zero, $N^{-1}(s)$ can be made stable by properly choosing Z.

Case b If $p \leqslant m$ and rank $D=q<p$, i. e. D is not of full row rank, we can find a $p\times p$ full rank matrix T such that

$$TD=\begin{bmatrix} D_q \\ 0_{p-q} \end{bmatrix} \tag{29}$$

where the $q\times m$ matrix D_q is of full row rank, i. e. rank $D_q=q(q<p)$, and the zero matrix 0_{p-q} is of dimension $(p-q)\times m$. This procedure can be carried out also by employing the singular value decomposition* of D

$$D=\Phi\begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix}\Gamma^{T} \tag{30}$$

where $\Sigma=\text{diag}(\sigma_1,\cdots\sigma_q)$, $\sigma_1,\cdots,\sigma_q$ are singular values of D. Both matrix Φ and matrix Γ are unitary. Noting that $\Phi^T\Phi=I$ and decomposing

$$\Gamma=[\Gamma_1 \quad \Gamma_2] \tag{31}$$

where the dimensions of Γ_1 and Γ_2 are respectively $m\times q$ and $m\times(m-q)$. We have

$$\Phi^T D=\begin{bmatrix} \Sigma\Gamma_1^T \\ 0_{p-q} \end{bmatrix} \tag{32}$$

from (30). Comparing (32) with (29) we find that

$$T=\Phi^T, \tag{33}$$

$$D_q=\Sigma\Gamma_1^T. \tag{34}$$

Note that the product $\Sigma\Gamma_1^T$ is of full row rank since both the $q\times q$ matrix Σ and the $q\times m$ matrix Γ_1^T are of full row rank.

Let the state equation and the output equation of $N:[A-BF,B,C-DF,D]$ be $\dot{\xi}(t)=(A-BF)\cdot\xi(t)+B\eta(t)$ and $\zeta(t)=(C-DF)\xi(t)+D\eta(t)$ respectively. Premultiplying both sides of the output equation with the unitary matrix T yields

$$T\zeta(t)=T(C-DF)\xi(t)+TD\eta(t). \tag{35}$$

Denoting
$$T\zeta(t)=\begin{bmatrix} z_q(t) \\ z_{p-q}(t) \end{bmatrix},\quad T(C-DF)=\begin{bmatrix} C_q \\ C_{p-q} \end{bmatrix}. \tag{36}$$

(35) can be written as
$$\begin{bmatrix} z_q(t) \\ z_{p-q}(t) \end{bmatrix}=\begin{bmatrix} C_q \\ C_{p-q} \end{bmatrix}\xi(t)+\begin{bmatrix} D_q \\ O_{p-q} \end{bmatrix}\eta(t). \tag{37}$$

Replacing $z_{p-q}(t)$ in (37) by $\dot{z}_{q-p}(t)$ results in

$$\begin{bmatrix} z_q(t) \\ \dot{z}_{p-q}(t) \end{bmatrix}=\begin{bmatrix} C_q \\ C_{p-q}(A-BF) \end{bmatrix}\xi(t)+\begin{bmatrix} D_q \\ C_{p-q}B \end{bmatrix}\eta(t) \tag{38}$$

If
$$\text{rank}\begin{bmatrix} D_q \\ C_{p-q}B \end{bmatrix}=p \tag{39}$$

we define
$$N_1:\left[(A-BF),B,\begin{bmatrix} C_q \\ C_{p-q}(A-BF) \end{bmatrix},\begin{bmatrix} D_q \\ C_{p-q}B \end{bmatrix}\right] \tag{40}$$

and find the right inverse N_1^{-1} for N_1 using the method mentioned in Case a.

In order to make use of (22) for N_1^{-1}, N_1 in (40) is written as

$$N_1:[A-BF,B,C_1-D_1F,D_1]$$

where
$$C_1=\begin{bmatrix} C_q+D_qF \\ C_{p-q}A \end{bmatrix},\quad D_1=\begin{bmatrix} D_q \\ C_{p-q}B \end{bmatrix}.$$

To find the relation between N^{-1} and N_1^{-1}, we first investigate the relation between N and N_1 as

* If $D=0$, a null matrix, then $\Sigma=0$, $\Phi=I$, $\Gamma=I$.

follows. Form(40) we have the transfer matrix

$$N_1(s)=\begin{bmatrix} C_q \\ C_{p-q}(A-BF) \end{bmatrix}(sI-A+BF)^{-1}B+\begin{bmatrix} D_q \\ C_{p-q}B \end{bmatrix}. \tag{41}$$

Taking Laplace transform on both sides of (38) yields

$$\begin{bmatrix} I_q & 0 \\ 0 & sI_{p-q} \end{bmatrix}\begin{bmatrix} z_p(s) \\ z_{p-q}(s) \end{bmatrix}=\begin{bmatrix} C_q \\ C_{p-q}(A-BF) \end{bmatrix}\xi(s)\begin{bmatrix} D_q \\ C_{p-q}B \end{bmatrix}\eta(s). \tag{42}$$

Since $\xi(s)=(sI-A+BF)^{-1}B\eta(s)$, we find from(42)

$$\begin{bmatrix} I_q & 0 \\ 0 & sI_{p-q} \end{bmatrix}T\zeta(s)=\left\{\begin{bmatrix} C_q \\ C_{p-q}(A-BF) \end{bmatrix}(sI-A+BF)^{-1}B+\begin{bmatrix} D_q \\ C_{p-q}B \end{bmatrix}\right\}\eta(s)=N_1(s)\eta(s) \tag{43}$$

by(41). Recalling that $\zeta(s)=N(s)\eta(s)$, we find from (43) that

$$\begin{bmatrix} I_q & 0 \\ 0 & sI_{p-q} \end{bmatrix}TN(s)=N_1(s). \tag{44}$$

Noticing the following manipulations starting from (44)

$$\begin{bmatrix} I_q & 0 \\ 0 & sI_{p-q} \end{bmatrix}TN(s)N_1^{-1}(s)=I, \qquad N(s)N_1^{-1}(s)=T^{-1}\begin{bmatrix} I_q & 0 \\ 0 & sI_{p-q} \end{bmatrix}^{-1},$$

$$N(s)N_1^{-1}(s)\begin{bmatrix} I_q & 0 \\ 0 & sI_{p-q} \end{bmatrix}T=I=N(s)N^{-1}(s)$$

we have

$$N^{-1}(s)=N_1^{-1}(s)\begin{bmatrix} I_q & 0 \\ 0 & sI_{p-q} \end{bmatrix}T. \tag{45}$$

If the rank condition in (39) is not satisfied, iteration is required. The algorithm goes back to (29) and restarts with N replaced by N_1. The iteration stops when a new N_1 with a D-term of full row rank is found.

The strategy of this technique is as follows. When $N(s)$ does not have a D-term with the row rank equal to p, we first rearrange the rows of $N(s)$ so that the upper part of $N(s)$ will have a D-term of full row rank. Then, we multiply each row in the lower part of $N(s)$ by a factor s to raise the order of the numerator polynomials in the lower part elements of $N(s)$. If a full row rank D-term is associated with the lower part of $N(s)$, the combination of the upper part and the new lower part will result in a matrix $N_1(s)$ which has a D-term with a row rank equal to p. This $N_1(s)$ is inverted and its right inverse $N_1^{-1}(s)$ is substituted into (45) for obtaining the transfer matrix $N^{-1}(s)$ required by (19).

4 The Computation of Controller

If the constraints on β_i and α_i are satisfied by the assigned diagonal closed-loop transfer matrix diag$\{\beta_i/\alpha_i\}$, a proper controller K can be determined from (11). By (18) and (19), it follows that

$$K=-UV^{-1}=MN^{-1}\mathrm{diag}\{\beta_i/\alpha_i-\beta_i\}. \tag{46}$$

Example

$$P:[A,B,C,D]=\left\{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 1 & -2 \end{bmatrix},\begin{bmatrix} -1 & 0 & 1 \\ 2 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix},\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix},\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right\},$$

$$P(s)=\frac{1}{(s+1)(s+2)(s-1)}\begin{bmatrix} -2s^2+8 & -s^2+2s+3 & 2s^2+2s-4 \\ s^2+3s+2 & 2s+2 & 0 \end{bmatrix}$$

with poles: $-1,-2,1$. Let

$$F=\begin{bmatrix}4 & 6 & 2\\ -4 & -6 & -2\\ 4 & 6 & 2\end{bmatrix},\quad H=\begin{bmatrix}0 & 0\\ 0 & 0\\ 4 & 3\end{bmatrix}.$$

then $$A-BF=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -2 & -5 & -4\end{bmatrix},\quad A-HC=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -2 & -2 & -9\end{bmatrix}$$

have eignenvalues $\{-1\quad -1\quad -2\}$ and $\{-0.1007+0.466i\quad -0.1007-0.466i\quad -8.7985\}$ respectively.

$$\widetilde{M}^{-1}:[A,-H,-C,I]=\left\{\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 2 & 1 & -2\end{bmatrix},\begin{bmatrix}0 & 0\\ 0 & 0\\ -4 & -3\end{bmatrix},\begin{bmatrix}-1 & 0 & -1\\ 0 & -1 & -1\end{bmatrix},\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\right\},$$

$$\widetilde{M}^{-1}(s)=\frac{1}{(s+1)(s+2)(s-1)}\begin{bmatrix}s^3+6s^2-s+2 & 3s^2+3\\ 4s(s+1) & (s+1)(s^2+4s-2)\end{bmatrix}.$$

As $\widetilde{M}^{-1}(s)$ has a rhp pole polynomial $(s-1)$ in each row, $(\alpha_i-\beta_i)$ must have $(s-1)$ as its zero polynomial.

$$N:[A-BF,B,C-DF,D]=\left\{\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -2 & -5 & -4\end{bmatrix},\begin{bmatrix}-1 & 0 & 1\\ 2 & 1 & -1\\ -1 & 1 & 1\end{bmatrix},\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 1\end{bmatrix},\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\right\},$$

$$N(s)=\frac{1}{s^3+4s^2+5s+2}\begin{bmatrix}-2s^2-10s-2 & -s^2-4s+1 & 2s^2+6s+4\\ s^2-s-2 & -2s-2 & 0\end{bmatrix}.$$

Since rank $D=q=0$, $T=I_{2\times2}$, $D_q=C_q=0$, $C_{p-q}=C$,

$$N_1:[A-BF,B,C(A-BF),CB]$$

$$=\left\{\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -2 & -5 & -4\end{bmatrix},\begin{bmatrix}-1 & 0 & 1\\ 2 & 1 & -1\\ -1 & 1 & 1\end{bmatrix},\begin{bmatrix}-2 & -4 & -4\\ -2 & -5 & -3\end{bmatrix},\begin{bmatrix}-2 & -1 & 2\\ 1 & 0 & 0\end{bmatrix}\right\},$$

$$N(s)=\frac{1}{s^3+4s^2+5s+2}\begin{bmatrix}-2s^3-10s^2-2s & -s^3-4s^2+s & 2s^3+6s^2+4s\\ s^3-s^2-2s & -2s^2-2s & 0\end{bmatrix}$$

Because rank $CB=2$, N_1^{-1} can be found by the method mentioned in Case a. We first find the singular value devomposition

$$CB=\Phi[\Sigma\quad 0][\Gamma_1\quad \Gamma_2]^{\mathrm T}$$

$$=\begin{bmatrix}-0.9732 & -0.2298\\ 0.2298 & -0.9732\end{bmatrix}\begin{bmatrix}3.0777 & 0 & 0\\ 0 & 0.7265 & 0\end{bmatrix}\begin{bmatrix}0.7071 & -0.7071 & 0\\ 0.3162 & -0.3162 & -0.8944\\ -0.6253 & -0.6253 & -0.4472\end{bmatrix}^{\mathrm T}$$

then compute the right inverse of CB and the right inverse of $N_1(s)$:

$$(CB)^{-1}=\Gamma_1\Sigma^{-1}\Phi^{\mathrm T}+\Gamma_2 Z=\begin{bmatrix}0 & 1\\ -0.2 & -0.4\\ 0.4 & 0.8\end{bmatrix}\ (Z=0),$$

$$N_1^{-1}:[A-B(CB)^{-1}CA,B(CB)^{-1},-(CB)^{-1}CA+F,(CB)^{-1}],$$

$$N_1^{-1}(s):=\frac{1}{s^2+0.6s}\begin{bmatrix}-0.4s-0.4 & s^2+4.8s+4.6\\ -0.2s^2+0.2s+0.4 & -0.4s^2-4.4s-5.2\\ 0.4s^2+0.4s & 0.8s^2+4.4s+3.6\end{bmatrix}.$$

By using (45), we find

$$N^{-1}(s)=sN_1^{-1}(s)=\frac{1}{s+0.6}\begin{bmatrix} -0.4s-0.4 & s^2+4.8s+4.6 \\ -0.2s^2+0.2s+0.4 & -0.4s^2-4.4s-5.2 \\ 0.4s^2+0.4s & 0.8s^2+4.4s+3.6 \end{bmatrix}.$$

Since there is no rhp pole in $N^{-1}(s)$, we let

$$\beta_1(s)=b_1, \quad \beta_2(s)=b_2$$

and choose $\alpha_1(s)=(s+1)(s+0.5), \quad \alpha_2(s)=(s+2)(s+0.5).$

Since both $\alpha_1-\beta_1$ and $\alpha_2-\beta_2$ should have a root at $s=1$, we find $b_1=3$, $b_2=4.5$, We thus have assigned the closed-loop transfer matrix

$$G_c=\begin{bmatrix} \dfrac{3}{(s+1)(s+0.5)} & 0 \\ 0 & \dfrac{4.5}{(s+2)(s+0.5)} \end{bmatrix},$$

$$G_c(I-G_c)^{-1}=\begin{bmatrix} \dfrac{3}{(s-1)(s+2.5)} & 0 \\ 0 & \dfrac{4.5}{(s-1)(s+3.5)} \end{bmatrix}.$$

The controller $K(s)$ is computed in the following:

$$M:[A-BF,B,-F,I], \quad M(s)=\frac{1}{s+1}\begin{bmatrix} s-5 & -4 & 0 \\ 6 & s+5 & 0 \\ -6 & -4 & s+1 \end{bmatrix}$$

$$M(s)N^{-1}(s)=\begin{bmatrix} 0.4\dfrac{(s+1)}{s+0.6} & \dfrac{s^2+0.4s-2.2}{s+0.6} \\ -0.2\dfrac{(s+2)(s+1)}{s+0.6} & -0.4\dfrac{(s+2)(s-2)}{s+0.6} \\ 0.4\dfrac{(s+2)(s+1)}{s+0.6} & 0.8\dfrac{(s+2)(s-2)}{s+0.6} \end{bmatrix},$$

$$K(s)=M(s)N^{-1}(s)G_c(I-G_c)^{-1}$$

$$=\begin{bmatrix} 1.2\dfrac{s+1}{(s+0.6)(s-1)(s+2.5)} & 4.5\dfrac{s^2+0.4s-2.2}{(s+0.6)(s-1)(s+3.5)} \\ -0.6\dfrac{(s+2)(s+1)}{(s+0.6)(s-1)(s+2.5)} & -1.8\dfrac{(s+2)(s-2)}{(s+0.6)(s-1)(s+3.5)} \\ 1.2\dfrac{(s+2)(s+1)}{(s+0.6)(s-1)(s+2.5)} & 3.6\dfrac{(s+2)(s-2)}{(s+0.6)(s-1)(s+3.5)} \end{bmatrix}.$$

5 Conclusions

We have determined the constraints on the assigned diagonal closed-loop transfer matrix of a unity feedback system with a non-square, rational and proper plant, in order to have a rational and proper controller which internally stabilizes and decouples the system. Doubly coprime factorization is employed to provide the basis for the determination of the constraints. This paper also puts forward the computation formulas for the controller, when the constraints on the assigned diagonal transfer matrix are satisfied. The central problem of this computation is the determination of the right inverse of a non-square numerical matrix by employing singular value decomposition. In all the computations, state space models are invariably used; frequency domain models are employed only for analysis. This makes it possible to employ the existing algorithms for time domain analysis, thus providing a convenient way for the practical application of the results obtained.

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单位反馈系统解耦控制器的设计

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摘要:本文使用了双互质因子分解法,研究了单位反馈系统解耦时,对于设定的闭环对角传函矩阵,到底有那些限制条件.文中所论的开环系统是多输入多输出的,其传函阵是正则有理的非方阵.文中给出了计算控制器的方法.这些方法采用的是都是目前流行的时域分析算法.因而可以利用现成的一些软件包,如 MATLAB 来完成.

关键词:解耦;非方阵;双互质因子分解法;内稳定;时域分析;右逆;奇异值分解

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