第 14 卷第 2 期
1997 年 4 月

控制理论与应用
CONTROL THEORY AND APPLICATIONS

Vol. 14, No. 2
Apr., 1997

Bilinear Adaptive Feedforward Control and Its Application *

SUN Xi
(Department of Automation, Sichuan Union University · Sichuan Chengdu, 610065, PRC)
JIN Yihui and FANG Chongzhi
(Department of Automation, Tsinghua University · Beijing, 100084, PRC)

Abstract: An adaptive feedforward control algorithm for bilinear systems with measurable and bounded disturbances and the proof of its stability are given in this paper. It can eliminate the effect of measurable disturbances and ensures that the system variables are bounded and the generalized tracking error is small, even for some nonminimum phase systems. Simulation studies show that the algorithm is well suited for the pH neutralization control, and can give superior control performance compared with other adaptive control algorithms.

Key words: bilinear systems; adaptive feedforward control; pH neutralization process

1 Introduction

Bilinear systems arise quite naturally from basic principles in engineering, biology, Chemistry, etc[1], and a general nonlinear plant may be approximated by a bilinear system more accurately than by a linear one. For these reasons, the study of bilinear systems has a great attraction. Some contributions have been made on the adaptive control of bilinear systems, such as [2~6].

Adaptive control of bilinear systems with measurable disturbances is very important in both theory and practice, but it is still open. In the presence of measurable disturbances, the adaptive control performance without considering the effects of the disturbances will not be satisfactory; especially in the case of pH neutralization process, in which the flow rate of the acid is varied, the controller will virtually be useless. For the control of pH process, some self-tuning control laws are described in [7,8].

2 Adaptive Control Algorithm

Consider a discrete time bilinear systems described by

$$A(q^{-1})y(t)=q^{-d}B(q^{-1})u(t)+q^{-d}C(q^{-1})y(t)u(t)+q^{-d}D(q^{-1})V(t)+\omega(t)+f. \quad (1)$$

where $y(t)$, $u(t)$, $V(t)$ and $\omega(t)$ denote the output, input, bounded measurable disturbance and the bounded disturbance, respectively. f is the steady state parameter, and d is known time delay.

$$A(q^{-1})=1+\sum_{i=1}^{n_a}a_iq^{-i};\quad B(q^{-1})=\sum_{i=0}^{n_b}b_iq^{-i},\quad b_o\neq 0;$$

* This work was supported by the Doctorate Foundation of the National Eduction Council of China and the Youth Science Research Foundation of CUST.

Manuscript received Apr. 10, 1995, revised Jun. 12, 1996.

$$C(q^{-1}) = 1 + \sum_{i=1}^{n_c} c_i q^{-i}; \quad D(q^{-1}) = \sum_{i=0}^{n_d} d_i q^{-i}.$$

The following assumptions are made about the system (1):

A1) The upper bounds for n_a, n_b, n_c and n_d are known.

A2) $C(q^{-1})$ is a stable polynomial.

The controller is to be chosen to minimize the following cost function

$$J = [P(q^{-1})y(t+d) - R(q^{-1})y^*(t+d) + \lambda Q(q^{-1})\delta(t)u(t)]^2 \tag{2}$$

where $R(q^{-1})$ is a weighting polynomial in q^{-1}, $P(q^{-1})$ and $Q(q^{-1})$ are weighting stable polynomials in q^{-1} and $P(0) = 1$, $Q(0) = 1$. $y^*(t)$ is set point. λ is a positive weighting constant and $\delta(t)$ will be defined later.

Introduce the following ploynomial identity

$$P(q^{-1}) = F(q^{-1})A(q^{-1}) + q^{-d}G(q^{-1}) \tag{3}$$

where $F(q^{-1})$ and $G(q^{-1})$ are polynomials with order $d-1$ and n_g, respectively, and $n_g = \max\{n_p - d, n_a - 1\}$. Multiplying (1) by $F(q^{-1})$ and substituting (3) into (1) gives

$$\begin{aligned}P(q^{-1})y(t+d) =& G(q^{-1})y(t) + F(q^{-1})B(q^{-1})u(t) + F(q^{-1})C(q^{-1})y(t)u(t)\\ &+ F(q^{-1})D(q^{-1})V(t) + \bar{\omega}(t+d) + \bar{f} = \varphi(t)^{\mathrm{T}}\theta + \bar{\omega}(t+d)\end{aligned} \tag{4}$$

where

$$\bar{f} = F(1)f,$$

$$\bar{\omega}(t+d) = \omega(t+d) + f_1\omega(t+d-1) + \cdots + f_{d-1}\omega(t+1),$$

$$\begin{aligned}\varphi(t)^{\mathrm{T}} = [&y(t),\cdots,y(t-n_g),u(t),\cdots,u(t-n_b-d+1),\ y(t)u(t),\cdots,\\ &y(t-n_c-d+1)u(t-n_c-d+1),\ V(t),\cdots,V(t-n_a-d+1),1],\end{aligned}$$

$$\theta^{\mathrm{T}} = [\alpha_0,\cdots,\alpha_{ng},\beta_0,\cdots,\beta_{n_b+d-1},\gamma_0,\cdots,\gamma_{n_c+d-1},\eta_0,\cdots,\eta_{n_d+d-1},\bar{f}].$$

It is clear that $\bar{\omega}(t)$ is still a bounded disturbance. Assuming its upper bound is M, i.e. $|\bar{\omega}(t)| \leqslant M$, $\forall\, t \geqslant 0$.

Substituting (4) into (2), we have

$$J \leqslant 2[\varphi(t)^{\mathrm{T}}\theta - R(q^{-1})y^*(t+d) + \lambda Q(q^{-1})\delta(t)u(t)]^2 + 2M^2.$$

Thus the optimal control law that minimize (2) is given by

$$\varphi(t)^{\mathrm{T}}\theta - R(q^{-1})y^*(t+d) + \lambda Q(q^{-1})\delta(t)u(t) = 0. \tag{5}$$

It is easy to see that $u(t)$ is soluble if and only if $\beta_0 + \gamma_0 y(t) + \lambda\delta(t) \neq 0$. In order to guarantee $u(t)$ soluble and bounded, $\delta(t)$ is chosen as

$$\delta(t) = \begin{cases} 1, & \text{if} \quad \beta_o + \gamma_o y(t) \geqslant 0, \\ -1, & \text{if} \quad \beta_o + \gamma_o y(t) < 0 \end{cases} \tag{6}$$

and $\delta(t-i) = \delta(t)$, $i = 1,2,\cdots$.

The adaptive control algorithm is obtained by using a recursive least squares estimator with a dead zone

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \frac{\bar{a}(t)P(t-2)\varphi(t-d)}{1+\varphi(t-d)^{\mathrm{T}}P(t-2)\varphi(t-d)}e(t), \tag{7}$$

$$e(t) = P(q^{-1})y(t) - \varphi(t-d)^{\mathrm{T}}\hat{\theta}(t-1), \tag{8}$$

$$P(t-1) = P(t-2) - \frac{\bar{a}(t)P(t-2)\varphi(t-d)\varphi(t-d)^{\mathrm{T}}P(t-2)}{1+\varphi(t-d)^{\mathrm{T}}P(t-2)\varphi(t-d)}, \tag{9}$$

$$\bar{a}(t)=\begin{cases}\mu, & \text{if } |e(t)|\geqslant 2M,\\ 0, & \text{if } |e(t)|<2M\end{cases} \tag{10}$$

where $0<\mu<\frac{3}{4}$.

$$\varphi(t)^{\mathrm{T}}\hat{\theta}(t)-R(q^{-1})y^{*}(t+d)+\lambda Q(q^{-1})\hat{\delta}(t)u(t)=0, \tag{11}$$

$$\hat{\delta}(t)=\begin{cases}1, & \text{if } \hat{\beta}_o(t)+\hat{\gamma}_o(t)y(t)\geqslant 0,\\ -1, & \text{if } \hat{\beta}_o(t)+\hat{\gamma}_o(t)y(t)<0.\end{cases} \tag{12}$$

3 Analysis of Adaptive Control Algorithm

Define

$$\tilde{\theta}(t)=\theta-\hat{\theta}(t),$$

$$\varepsilon(t)=\hat{\theta}(t-1)-\hat{\theta}(t-d), \tag{13}$$

$$\Phi(t)=P(q^{-1})y(t)-R(q^{-1})y^{*}(t)+\lambda Q(q^{-1})\hat{\delta}(t-d)u(t-d). \tag{14}$$

Lemma 1 The parameter estimation (7)~(10) has the following properties:

I) $$\lim_{t\to\infty}\frac{\bar{a}(t)\,e(t)^2}{1+\varphi(t-d)^{\mathrm{T}}P(t-2)\varphi(t-d)}=0. \tag{15}$$

II) $$\lim_{t\to\infty}\|\varepsilon(t)\|=0. \tag{16}$$

Proof From (4) and (8) yields

$$e(t)=\varphi(t-d)^{\mathrm{T}}\tilde{\theta}(t-1)+\bar{\omega}(t). \tag{17}$$

Multiplying (9) by $P(t-2)^{-1}$ from the right gives

$$P(t-1)P(t-2)^{-1}=I-\frac{\bar{a}(t)P(t-2)\varphi(t-d)\varphi(t-d)^{\mathrm{T}}}{1+\varphi(t-d)^{\mathrm{T}}P(t-2)\varphi(t-d)}. \tag{18}$$

Using (7), (17) and (18), we have

$$\tilde{\theta}(t)=P(t-1)P(t-2)^{-1}\tilde{\theta}(t-1)-\frac{\bar{a}(t)P(t-2)\varphi(t-d)\bar{\omega}(t)}{1+\varphi(t-d)^{\mathrm{T}}P(t-2)\varphi(t-d)}. \tag{19}$$

Now, we define

$$V(t)=\tilde{\theta}(t)^{\mathrm{T}}P(t-1)^{-1}\tilde{\theta}(t). \tag{20}$$

Thus using the similar argument adopted by Gu and Wang[9], the conclusion (I) and (II) can be proved. The details are given in the Appendix A.

Lemma 2 Let the assumption A1) and A2) hold for the system (1). Further, asuming that a positive constant K_0 and $0<K_1<\infty$.

$$|P(q^{-1})y(t+d)-K_o y(t)|\leqslant K_1\max_{0\leqslant\tau\leqslant t+d}|\Phi(\tau)|+K_1|y(t)|+K_1, \tag{21}$$

if the control algorithm (7)~(12) is applied to the system (1), then

$$[|u(t)|+|y(t)|+|y(t)u(t)|]\leqslant K_2+K_2\max_{0\leqslant\tau\leqslant t+d}|\Phi(\tau)|. \tag{22}$$

Proof Using the similar methods in [10], we can obtain (22). Because of the limitation of pages. the details are omitted here.

Since K_1 may be large constant, the assumption (21) is not a strict constraint on the system output; and the condition can be hold for first order bilinear system[10].

Lemma 3 Let $s_1(t)$, $s_2(t)$ and $s_3(t)$ be non-negative real scalar sequence and $\lim_{t\to\infty}s_3(t)=0$, if

$$s_1(t)\leqslant K_4+K_5\max_{0\leqslant\tau\leqslant t+d}[s_2(\tau)+s_1(\tau)s_3(\tau)], \tag{23}$$

where $K_4 > 0, K_5 > 0$, then there exist constant K_6, K_7, Such that

$$s_1(t) \leqslant K_6 + K_7 \max_{0\leqslant\tau\leqslant t} |s_2(\tau)|. \tag{24}$$

Proof See the Reference of Tsiligiannis and Svoronos[11].

Theorem 1 Under the conditions of Lemma 2, the control algorithm applied to system (1) leads to

Ⅰ) The closed loop system BIBO stable.

Ⅱ) There exists time $T < \infty$ such that

$$|P(q^{-1})y(t+d) - R(q^{-1})y^*(t+d) + \lambda Q(q^{-1})\hat{\delta}(t)u(t)| < 2M, \quad \forall \ t \geqslant T.$$

Proof From (8), (11) and (14), we have

$$\Phi(t) = e(t) + \varphi(t-d)^T(\hat{\theta}(t-1) - \hat{\theta}(t-d)). \tag{25}$$

Using (22) and the definition of $\varphi(t)$ gives

$$\| \varphi(t-d) \| \leqslant K_8 + K_8 \max_{0\leqslant\tau\leqslant t} |\varnothing(\tau)|, \tag{26}$$

where $0 \leqslant K_8 < \infty$.

Then from (13), (25) and (26) it follows that

$$\| \varphi(t-d) \| \leqslant K_8 + K_8 \max_{0\leqslant\tau\leqslant t}(|e(\tau)| + \| \varepsilon(\tau) \| \cdot \| \varphi(\tau-d) \|).$$

Now using (16) and Lemma 3, we obtain

$$\| \varphi(t-d) \| \leqslant K_9 + K_9 \max_{0\leqslant\tau\leqslant t} |e(\tau)| \tag{27}$$

where $0 \leqslant K_9 < \infty$.

From (27) we know that Theorem 1-Ⅰ) is true if $e(t)$ is bounded.

Defining $H_2 = \{t: |e(t)| \geqslant 2M, t \geqslant 0\}$.

If H_2 is a finite set, then $e(t)$ must be bounded. Suppose H_2 is an infinite set, we take any subsequence $\{t_k: |e(t_k)| = \max_{0\leqslant\tau\leqslant t_k} |e(\tau)|, k = 1,2,\cdots,\infty\}$ from H_2.

Along the subsequence $\{t_k\}$, from (27) it follows that for sufficiently large K

$$\begin{aligned}
&\left| \frac{\bar{a}(t_k)^{1/2} e(t_k)}{[1 + \varphi(t_k - d)^T P(t_k - 2)\varphi(t_k - d)]^{1/2}} \right| \\
&\geqslant \frac{\mu^{1/2} |e(t_k)|}{[1 + \lambda_{\max}(P(-1)) \| \varphi(t_k - d) \|^2]^{1/2}} \\
&\geqslant \frac{\mu^{1/2} |e(t_k)|}{[1 + \lambda_{\max}(P(-1))(K_9 + K_9 |e(t_k)|)^2]^{1/2}} \\
&\geqslant \frac{\mu^{1/2}}{[\frac{1}{4M^2} + \lambda_{\max}(P(-1)(K_9 + K_9/2M)^2]^{1/2}} > 0
\end{aligned}$$

where $\lambda_{\max}(P(-1))$ denotes the maximum eigenvalue of matrix $P(-1)$. But this contradicts (15) of Lemma 1, and hence the assumption that H_2 is an infinite set is false and $e(t)$ is bounded, thus from (27) the boundedness of $\varphi(t)$ follow, i.e. the conclusion Ⅰ) of Theorem 1 is true.

From the definition of $\bar{a}(t)$ in (10), there exist $T > 0$ such that $\bar{a}(t) = 0, \ \forall \ t \geqslant T$, since H_2 is a finite set. Now from (7) yields for $t \geqslant T$

$$\hat{\theta}(t) = \hat{\theta}(t-1) = \theta_0 \quad \text{(constant vector)}$$

and using (25) and the definition on H_2, we have

$$|\Phi(t)| \leqslant |e(t)| + \| \varphi(t-d) \| \cdot \| \hat{\theta}(t-1) - \hat{\theta}(t-d) \| < 2M, \quad \forall t \geqslant T$$

which completes the proof.

4 Simulation of pH Control

A typical pH neutralization process described by Buchholt and Kummel[7], Goodwin and Sin[8] is as follows:

$$\bar{y}(t+T) = [1 - \frac{T}{V}\bar{F}(t)]\bar{y}(t) - \frac{\bar{b}T}{V}\bar{u}(t) - \frac{T}{V}\bar{y}(t)\bar{u}(t) + \frac{aT}{V}\bar{F}(t) + \omega(t+T) \quad (28)$$

where V is volume of stirred tank, $\bar{F}(t)$ is flow rate of strong acid, a is concentration of strong acid, $\bar{u}(t)$ is flow rate of the base, $\bar{b}$ is concentration of the base. $\omega(t+T)$ represents the combined effect of measurement, actuator, and modelling error. T is sampling period. $\bar{y}(t) = [H^+] - [OH^-]$ is the distance from neutrality, it can be determined by

$$\bar{y}(t) = 10^{-p(t)} - 10^{p(t)}K_w, \quad (29)$$

where $p(t)$ is the pH value, $K_w \approx 10^{-14}$ is water equilibrium constant.

The following parameter values are adopted for simulations:

$$\bar{F}(t) = 0.1 \sim 0.25 \text{ l/min}, \quad \bar{u}(t) = 0 \sim 0.2 \text{ l/min}$$

$$a = 10^{-3} \text{ mol/l}, \quad \bar{b}(t) = 10^{-3} \text{ mol/l}, \quad V = 2 \text{ l}, \quad T = 1 \text{ min}.$$

The control model in pH is taken in following simulations, pH value $y(t)$ is obtained from (29). The control model in $\bar{y}(t)$ as in Goodwin and Sin[8] may be difficult to implement in practical engineering, because very small disturbances such as meaurement errors, computation residuals have a great effect on the value of $\bar{y}(t)$.

In simulations, $\varphi(t)$ and $\hat{\theta}(0)$ are chosen as follows:

1) The proposed bilinear adaptive feedforward control algorithm

$$\varphi(t)^T = [u(t), y(t)u(t), y(t), y(t)V(t), V(t), 1], \quad \hat{\theta}(0)^T = [1,0,0,0,0,0].$$

2) Bilinear adaptive control algorithm of [6]

$$\varphi(t)^T = [u(t), y(t)u(t), y(t), 1], \quad \hat{\theta}(0)^T = [1,0,0,0].$$

3) First order linear feedforward control algorithm employing a least squares estimator

$$\varphi(t)^T = [u(t), y(t), y(t)V(t), V(t), 1], \quad \hat{\theta}(0)^T = [19.3, 0.09, 0, 0, 4.2].$$

4) Second order linear adaptive feedforward control algorithm employing a least squares estimator

$$\varphi(t)^T = [u(t), u(t-1), y(t), y(t-1), y(t)V(t), V(t), 1],$$

$$\theta(0)^T = [1,0,0,0,0,0,0,0].$$

The set value of pH is 7. In order to compare the proposed the algorithm with other algorithms, let the acid flow $\bar{F}(t)$ change from 0.1 to 0.125 l/min, i.e.

$$\bar{F}(t) = \begin{cases} 0.1 \text{ l/min}, & \text{if } t < 50, \\ 0.125 \text{ l/min}, & \text{if } t \geqslant 50. \end{cases}$$

The pH responses of above algorithms are shown in Figure 1 and 2, respectively.

If the acid flow $\bar{F}(t)$ is varied in a slow sine wave, i.e.

$$\bar{F}(t) = \begin{cases} 0.1125 \text{ l/min}, & \text{if } t < 50, \\ 0.1125 + 0.0125\sin\left(\frac{t\pi}{25}\right) \text{ l/min}, & \text{if } t \geqslant 50. \end{cases}$$

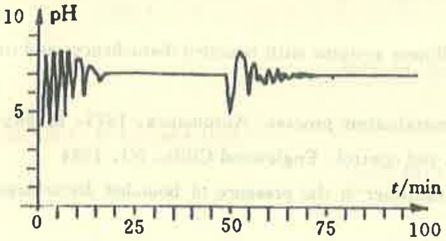

Fig. 1 Output of proposed bilinear adaptive feedforward control algorithm

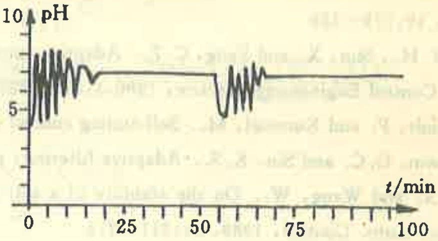

Fig. 2 Output of bilinear adaptive control algorithm

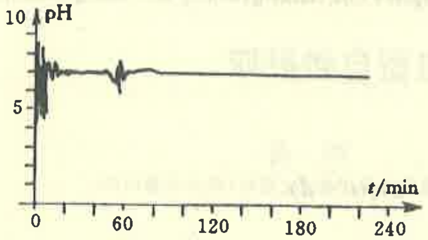

Fig. 3 Output of the proposed bilinear adaptive feedforward control algorithm

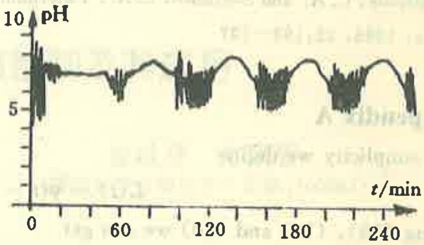

Fig. 4 Output of bilinear adaptive control algorithm

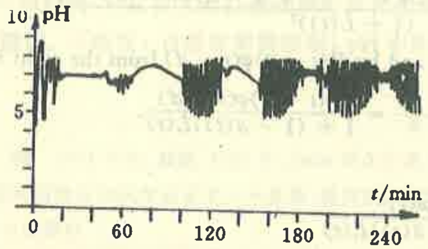

Fig. 5 Output of first order linear adaptive feedforward control algorithm

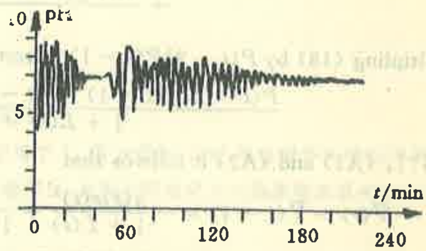

Fig. 6 Output of second order linear adaptive feedforward control algorithm

Then the pH responses are demonstrated in Fig. 3～Fig. 6.

From Fig. 1～Fig. 6, it is clear that the proposed control algorithm gives far superior control to either bilinear adaptive control or linear adaptive feedforwrd control algorithms. Especially in the case of varying acid flow rate, the other adaptive control algorithms have bad performances, whereas the bilinear adaptive feedforward control algorithm works very well.

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Appendix A

For simplicity we define

$$L(t) = \varphi(t-d)^{\mathrm{T}}P(t-2)\varphi(t-d).$$

Using (18), (19) and (20) we can get

$$\begin{aligned}\bar{V}(t) - \bar{V}(t-1) = &-\frac{\bar{a}(t)\varphi(t-d)^{\mathrm{T}}\tilde{\theta}(t-1)[\varphi(t-d)^{\mathrm{T}}\tilde{\theta}(t-1)+2\bar{\omega}(t)]}{1+L(t)} \\ &+\frac{\bar{a}(t)^2\varphi(t-d)^{\mathrm{T}}P(t-2)P(t-1)^{-1}P(t-2)\varphi(t-d)\bar{\omega}(t)^2}{(1+L(t))^2}.\end{aligned} \tag{A1}$$

Multipling (18) by $P(t-2)P(t-1)^{-1}$ from the left and by $P(t-2)\varphi(t-d)$ from the right we obtain

$$\frac{P(t-2)P(t-1)^{-1}P(t-2)\varphi(t-d)}{1+L(t)} = \frac{P(t-2)\varphi(t-d)}{1+(1-\bar{a}(t))L(t)}. \tag{A2}$$

From (17), (A1) and (A2) it follwos that

$$\begin{aligned}\bar{V}(t) - \bar{V}(t-1) &= -\frac{\bar{a}(t)e(t)^2}{1+L(t)} + \frac{\bar{a}(t)\bar{\omega}(t)^2}{1+(1-\bar{a}(t))L(t)} \\ &= -\frac{3\bar{a}(t)\left[1+(1-\frac{4}{3}\bar{a}(t))L(t)\right]e(t)^2}{4(1+L(t))(1+(1-\bar{a}(t))L(t))} - \frac{\bar{a}(t)\left[\frac{e(t)^2}{4}-\bar{\omega}(t)^2\right]}{1+(1-\bar{a}(t))L(t)}.\end{aligned} \tag{A3}$$

Thus from (10) and (A3) yields

$$\bar{V}(t) - \bar{V}(t-1) \leqslant 0$$

and noting that $\bar{V}(t)$ is a nonnegative, nonincreasing sequence, it converges, from (A3) gives

$$\sum_{t=1}^{\infty}\frac{\bar{a}(t)\left[1+(1-\frac{4}{3}\bar{a}(t))L(t)\right]e(t)^2}{(1+L(t))(1+(1-\bar{a}(t))L(t))} < \infty.$$

Using the properties of convergence series, we have

$$\lim_{t\to\infty}\frac{\bar{a}(t)\left[1+(1-\frac{4}{3}\bar{a}(t))L(t)\right]e(t)^2}{(1+L(t))(1+(1-\bar{a}(t))L(t))} = 0.$$

Let us take $\mu^* \in \left(\mu, \frac{3}{4}\right)$, then

$$1 \geqslant \frac{1+\left(1-\frac{4}{3}\bar{a}(t)\right)L(t)}{1+(1-\bar{a}(t))L(t)} \geqslant \frac{1/L(t)+1-\frac{4}{3}\mu^*}{1/L(t)+1} \geqslant 1-\frac{4}{3}\mu^* = C_0.$$

It follows that

$$\lim_{t\to\infty}\frac{\bar{a}(t)e(t)^2}{1+\varphi(t-d)^{\mathrm{T}}P(t-2)\varphi(t-d)} = 0. \tag{A4}$$

This establishes Lemma 1- I).

Using (7) and noting $\bar{a}(t) < 1$, we have

$$\| \hat{\theta}(t) - \hat{\theta}(t-1) \| = \frac{\bar{a}(t)^2 \varphi(t-d)^{\mathrm{T}} P(t-2)^2 \varphi(t-d) e(t)^2}{(1+L(t))^2}$$

$$\leqslant \frac{\bar{a}(t)^2 L(t) e(t)^2 \lambda_{\max}(P(-1))}{(1+L(t))^2} \leqslant \frac{\bar{a}(t) e(t)^2 \lambda_{\max}(P(-1))}{1+L(t)}$$

and from (A4) it follows that

$$\lim_{t\to\infty} \| \hat{\theta}(t) - \hat{\theta}(t-1) \| = 0$$

then

$$\lim_{t\to\infty} \| \hat{\theta}(t) - \hat{\theta}(t-d) \| \leqslant \sum_{i=1}^{d-1} \lim_{t\to\infty} \| \hat{\theta}(t-i) - \hat{\theta}(t-i-1) \| = 0.$$

This completes the proof of Lemma 1.

双线性自适应前馈控制及其应用

孙 西 金以慧 方崇智
(四川联合大学(西区)自动化系·成都,610065) (清华大学自动化系·北京,100084)

摘要:本文对存在可测干扰和有界扰动的双线性系统提出了一种自适应前馈控制算法,证明了算法的稳定性.本算法可以消除可测干扰的影响,确保输入和输出有界并使广义跟踪误差最小.仿真表明本方法很适合于 pH 中和控制,且比其它的自适应算法具有更好的性能.

关键词:双线性;自适应前馈控制;pH 中和过程

本文作者简介

孙 西 1961 年生.教授.1983 年、1986 年在东南大学分别获得学士、硕士学位,1992 年在清华大学获得博士学位.在国内外学术刊物及会议发表论文三十多篇,曾完成多项纵、横向科研项目.目前主要研究方向是非线性系统的自适应控制理论,工业过程控制.

金以慧 见本刊 1997 年第 2 期第 151 页.

方崇智 1919 年生.重庆中央大学机械工程系工学士,英国伦敦大学玛丽皇后学院哲学博士.北京清华大学自动化系教授,博士生导师.学术方向为自动控制理论及应用,尤其是工业过程的建模、控制与优化.