Structure and Interconnected Robust Stabilization of the Multigroup Time-Delays Uncertain Nonlinear Interconnected Large Scale Control Systems *

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Abstract: Some new concepts of structure for nonlinear constant interconnected large scale control systems with multigroup time-delays are formulated. Based on the equivalent method of robust stabilization given by Liu Yongqing[1], the interconnected robust stabilization of the multigroup time-delays and parameters of structure perturbation uncertain nonlinear interconnected large scale control systems with the time-delay control vector function is proposed. Meanwhile the estimated formulas of bounds for delay and perturbed parameter and nonlinear terms are given.

Key words: multigroup time-delays; large scale nonlinear control systems; interconnected robust stabilization

1 The Structure and Interconnected Robust Stabilization

Consider the nonlinear uncertain interconnected large scale control systems:

$$\dot{X}(t)=A^{(0)}X(t)+B^{(0)}U(t)+\sum_{r=1}^{N_1}A^{(r)}E_1^{(r)}X(t)+\sum_{s=1}^{N_2}B^{(s)}E_2^{(s)}X(t-\tau^{(s)})$$

$$+\sum_{d=1}^{N_3}B^{(d)}E_3^{(d)}U(t)+\sum_{f=1}^{N_4}B^{(f)}E_3^{(f)}U(t-\tau^{(f)})+F(X(t),E_1^{(r)}X(t),$$

$$E_2^{(s)}X(t-\tau^{(s)}),U(t),E_3^{(d)}U(t),E_4^{(f)}U(t-\tau^{(f)}))=F_1(\cdot), \tag{1.1}$$

$$\overline{Y}_i(t)=C^{(0)}X(t)+\sum_{g=1}^{N_5}C^{(g)}E_4^{(g)}X(t)+\sum_{h=1}^{N_6}C^{(h)}E_6^{(h)}X(t-\tau^{(h)})=F_2(\cdot), \qquad (1.1)'$$

where

$A^{(0)}=(a_{ij}^{(0)})_{n\times n}$, $A^{(r)}=(a_{ij}^{(r)})_{n\times n}$, $A^{(s)}=(a_{ij}^{(s)})_{n\times n}$, $B^{(0)}=(b_{ij}^{(0)})_{n\times m}$, $B^{(d)}=(b_{ij}^{(d)})_{n\times m}$, $B^{(f)}=(b_{ij}^{(f)})_{n\times m}$, $C^{(g)}=(c_{ij}^{(g)})_{p\times n}$, $C^{(h)}=(c_{ij}^{(h)})_{p\times n}$, $E_1^{(r)}=(e_{1ij}^{(r)})_{n\times n}$, $E_2^{(s)}=(e_{2ij}^{(s)})_{n\times n}$, $E_3^{(d)}=(e_{3ij}^{(d)})_{m\times m}$, $E_4^{(f)}=(e_{4ij}^{(f)})_{m\times m}$, $E_5^{(g)}=(e_{5ij}^{(g)})_{p\times n}$, $E_6^{(h)}=(e_{6ij}^{(h)})_{p\times n}$, $E_\alpha=(e_{\alpha ij})$, $(\alpha=1,2,3,4,5,6)$ which is generated by the fundamental interconnection matrix $\overline{E}_\alpha=(\bar{e}_{\alpha ij})$ (denoted by $E_\alpha\in\overline{E}_\alpha$), i.e., $\bar{e}_{\alpha ij}=0$ implies $e_{\alpha ij}=0$, $\bar{e}_{\alpha ij}=1$ where $e_{\alpha ij}$ is either 0 or 1.

$(r=1,\cdots,N_1;s=1,\cdots,N_2;\ d=1,\cdots,N_3;f=1,\cdots,N_4;\ g=1,\cdots,N_5;h=0,\cdots,N_6)$,

$X(t)=(x_1(t),\cdots,x_n(t))^{\mathrm{T}}$, $U(t)=(u_1(t),\cdots,u_m(t))^{\mathrm{T}}$, and $Y(t)=(y_1(t),\cdots,y_p(t))^{\mathrm{T}}$, constant matrices $A^{(r)}$, $A^{(s)}$, $B^{(d)}$, $C^{(f)}$, $B^{(f)}$, $C^{(g)}$, $C^{(h)}$ are not known precisely but satisfy

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$$\begin{cases} P_1^{(r)} \leqslant A^{(r)} \leqslant Q_1^{(r)},\ P_2^{(s)} \leqslant A^{(s)} \leqslant Q_2^{(s)},\ P_3^{(d)} \leqslant B^{(d)} \leqslant Q_3^{(d)},\ P_4^{(f)} \leqslant C^{(f)} \leqslant Q_4^{(f)}, \\ P_4^{(f)} \leqslant B^{(f)} \leqslant Q_4^{(f)},\ P_5^{(g)} \leqslant C^{(g)} \leqslant Q_5^{(g)},\ P_6^{(h)} \leqslant C^{(h)} \leqslant Q_6^{(h)}, \\ r=0,\cdots,N_1;\ s=1,\cdots,N_2;\ d=0,\cdots,N_3;\ f=0,\cdots,N_4;\ g=0,\cdots,N_5;\ h=1,\cdots,N_6 \end{cases} \tag{1.2}$$

where $P_\beta^{(\alpha)}$, $Q_\beta^{(\alpha)}$ $(\alpha=r,s,d,f,g,h;\ \beta=1,2,3,4,5,6)$ are the upper and lower bounds of the intervals.

Let
$$\begin{cases} A = \frac{1}{2}(P_1^{(0)} + Q_1^{(0)}), \quad B = \frac{1}{2}(P_3^{(0)} + Q_3^{(0)}), \\ C = \frac{1}{2}(P_5^{(0)} + Q_5^{(0)}). \end{cases} \tag{1.3}$$

If the structure perturbation interconnected term, the delays perturbation interconnected term and nonlinear terms in multidelays system (1. 1) are not considered, then system (1. 1) becomes a linear constant control system without delays and structure perturbation parameters

$$\dot{X}(t) = AX(t) + BU(t), \tag{1.4}$$
$$\overline{Y}(t) = CX(t). \tag{1.4$'$}$$

Where $A=(a_{ij})_{n\times n}$, $B=(b_{ij})_{n\times m}$, $C=(c_{ij})_{p\times n}$ are constant matrices, system (1. 1), (1. 1)$'$ can be written as the following W connection subsystems

$$\begin{aligned} \dot{X}_i(t) = {} & A_{ii}X_i(t) + B_{ii}U_i(t) + \sum_{j=1,j\neq i}^{W} A_{ij}X_j(t) + \sum_{j=1,j\neq i}^{W} B_{ij}U_j(t) \\ & + \sum_{j=1,j\neq i}^{W}(A_{ij}^{(0)} - A_{ij})X_j(t) + \sum_{j=1,j\neq i}^{W}(B_{ij}^{(0)} - B_{ij})U_j(t) + \sum_{r=1}^{N_1}\sum_{j=1}^{W} A_{ij}^{(r)}E_{1ij}^{(r)}X_j(t) \\ & + \sum_{s=1}^{N_2}\sum_{j=1}^{W} A_{ij}^{(s)}E_{2ij}^{(s)}X_j(t) + \sum_{s=1}^{N_2}\sum_{j=1}^{W} A_{ij}^{(s)}E_{2ij}^{(s)}[X_j(t-\tau^{(s)}) - X_j(t)] \\ & + \sum_{d=1}^{N_3}\sum_{j=1}^{W} B_{ij}^{(d)}E_{3ij}^{(d)}U_j(t) + \sum_{f=1}^{N_4}\sum_{j=1}^{W} B_{ij}^{(f)}E_{4ij}^{(f)}U_j(t) + \sum_{f=1}^{N_4}\sum_{j=1}^{W} B_{ij}^{(f)}E_{4ij}^{(f)} \\ & * [U_j(t-\tau^{(f)}) - U_j(t)] + F_i(X(t), E_1^{(r)}X(t), E_2^{(s)}X(t-\tau^{(s)}), \\ & U(t), E_3^{(d)}U(t), E_4^{(f)}U(t-\tau^{(f)})), \end{aligned} \tag{1.5}$$

$$\begin{aligned} Y_i(t) = {} & C_{ii}X_i(t) + \sum_{j=1,j\neq i}^{W} C_{ij}X_j(t) + \sum_{j=1}^{W}(C_{ij}^{(0)} - C_{ij})X_j(t) \\ & + \sum_{g=1}^{N_5}\sum_{j=1}^{W} C_{ij}^{(g)}E_{5ij}^{(g)}X_j(t) + \sum_{h=1}^{N_6}\sum_{j=1}^{W} C_{ij}^{(h)}E_{6ij}^{(h)}X_j(t-\tau^{(h)})), \quad (i=1,\cdots,W). \end{aligned} \tag{1.5$'$}$$

Definition 1 For a group of delays $\tau_{ij}^{(s)} \geqslant 0, z_{ij}^{(f)} \geqslant 0$ $(s=1,2,\cdots,N_2; f=1,2,\cdots,N_4)$ and mutually connected matrices $E_\beta^{(\alpha)} \in \overline{E}_\beta^{(\alpha)}$ $(\alpha=r,s,d,f,g,h;\ \beta=1,2,3,4,5,6)$, call the system (1. 1) interconnected stabilization, if the trivial solution of closed-loop system of nonlinear interconnected large scale control system with multigroup time-delays defined by (1. 1) is asymptotically stable.

Corresponding to (1. 4), (1. 4)$'$, the isolated subsystems are

$$\dot{X}_i(t) = A_{ii}X_i(t) + B_{ii}U_i(t), \tag{1.6}$$
$$Y_i(t) = C_{ii}X_i(t), \quad (i=1,\cdots,W) \tag{1.6$'$}$$

of $x(t)$ and $u(t)$, $F(\cdot)$ is a continuous function of $x(t)$ and $u(t)$ satisfying

$$|F_i(x_1(t),\cdots,x_n(t),e_{1i1}^{(r)}x_1(t),\cdots,e_{1in}^{(r)}x_n(t),e_{2i1}^{(s)}x_1(t-\tau_{i1}^{(s)}),\cdots,e_{2in}^{(s)}x_n(t-\tau_{in}^{(s)}),$$
$$u_1(t),\cdots,u_n(t),e_{3i1}^{(d)}u_1(t),\cdots,e_{3im}^{(d)}u_m(t),e_{4i1}^{(f)}u_1(t),\cdots,e_{4im}^{(f)}u_m(t))|$$
$$\leqslant R_{i1}\{\sum_{j=1}^{n}|x_j(t_K)|+\sum_{r=1}^{N_1}\sum_{j=1}^{n}|e_{1ij}^{(r)}||x_j(t)|+\sum_{s=1}^{N_2}\sum_{j=1}^{n}|e_{2ij}^{(s)}||x_j(t-\tau_{ij}^{(s)})|$$
$$+\sum_{j=1}^{m}|u_j(t)|+\sum_{d=1}^{N_3}\sum_{j=1}^{m}|e_{3ij}^{(d)}||u_j(t)|+\sum_{f=1}^{N_4}\sum_{j=1}^{m}|e_{4ij}^{(f)}||u_j(t-\tau_{ij}^{(f)})|\}, \tag{1.7}$$
$$R_1=\max(R_{i1},i=n_1+\cdots+n_{v-1}+1,\cdots,n_1+\cdots+n_v,v=1,\cdots,W).$$

Let

$$E_1=\max\left\{\begin{array}{l}|a_{ij}|,\begin{array}{c}i=1,\cdots,n_1\\ j=n_1+1,\cdots,n\end{array};\begin{array}{c}i=n_1+1,\cdots,n_1+n_2\\ j=1,\cdots,n_1\\ i=n_1+n_2+1,\cdots,n\end{array};\cdots;\begin{array}{c}i=n-n_W+1,\cdots,n\\ j=1,\cdots,n-n_W\end{array}\\ \frac{1}{2}|q_{1ij}^{(0)}-p_{1ij}^{(0)}|,\ |p_{1ij}^{(r)}|,\ |q_{1ij}^{(r)}|,\ |p_{2ij}^{(s)}|,\ |q_{2ij}^{(s)}|,\\ i,j=1,\cdots,n;\ r=1,\cdots,N_1;\ s=1,\cdots,N_2\end{array}\right\},$$

$$E_2=\max\left\{\begin{array}{l}|b_{ij}|,\begin{array}{c}i=1,\cdots,n_1\\ j=m_1+1,\cdots,m\end{array};\begin{array}{c}i=n_1+1,\cdots,n_1+n_2\\ j=1,\cdots,m_1\\ i=m_1+m_2+1,\cdots,m\end{array};\cdots;\begin{array}{c}i=n-n_W+1,\cdots,n\\ j=1,\cdots,m-m_W\end{array}\\ \frac{1}{2}|q_{3ij}^{(0)}-p_{3ij}^{(0)}|,\ |p_{3ij}^{(d)}|,\ |q_{3ij}^{(d)}|,\ |p_{4ij}^{(f)}|,\ |q_{4ij}^{(f)}|\\ i=1,\cdots,n;\ j=1,\cdots,m;\ d=1,\cdots,N_3;\ f=1,\cdots,N_4\end{array}\right\},$$

$$a_1=\max\left\{\begin{array}{l}\frac{1}{2}|p_{1ij}^{(0)}+q_{1ij}^{(0)}|,\ |p_{1ij}^{(r)}|,\ |q_{1ij}^{(r)}|,\ |p_{2ij}^{(s)}|,\ |q_{2ij}^{(s)}|\\ r=1,\cdots,N_1;\ s=1,\cdots,N_2;\ i,j=1,\cdots,n\end{array}\right\},$$

$$b_1=\max\left\{\begin{array}{l}\frac{1}{2}|p_{3ij}^{(0)}+q_{3ij}^{(0)}|,\ |p_{3ij}^{(d)}|,\ |q_{3ij}^{(d)}|,\ |p_{4ij}^{(f)}|,\ |q_{4ij}^{(f)}|\\ i=1,\cdots,n;\ j=1,\cdots,m;\ d=1,\cdots,N_3;\ f=1,\cdots,N_4\end{array}\right\},$$

$$p_1=\max\{|p_{ij}^{(v)}|;\quad i,j=n_1+\cdots+n_{v-1}+1,\cdots,n_1+\cdots+n_v,\quad v=1,\cdots,W)\},$$

$$K_1=\max\left\{|K_{ij}^{(v)}|;\quad v=1,\cdots,W,\quad\begin{array}{c}i=n_1+\cdots+n_{v-1}+1,\cdots,n_1+\cdots+n_v\\ j=n_1+\cdots+n_{v-1}+1,\cdots,n_1+\cdots+n_v\end{array}\right\},$$

$$\tau=\max\{|\tau_{ij}^{(s)}|,|z_{ia}^{(f)}|;i,j=1,\cdots,n;\alpha=1,\backslash:,m;s=1,\cdots,N_2;f=1,\cdots,N_4\},\quad D_1=\max\{E_1,E_2\},$$
$$\beta_1=\min\{[\beta_{1i},\quad i=1,\cdots,W,]\},\quad\beta_2=\max\{[\beta_{2i},\quad i=1,\cdots,W]\}. \tag{1.8}$$

For system (1.1), the parts of system without delays, perturbation structure parameter and nonlinear terms are with the same form as (1.4). For the other parts, if the perturbation a_2, b_2 and the delay τ and nonlinear term are very small, then it works as the perturbation term. In this case, we take the optimal negative feedback vector function of the system (1.6) as the suboptimal negative feedback vector function of system (1.1). Now, we take the sum of Lyapunov V-function of system (1.4)

$$V(X(t))=\sum_{j=1}^{W}V_i(X_i(t)) \tag{1.9}$$

as the positive definite symmetric quadratic form Lyapunov V-function of the multigroup time-delays nonlinear constant large scale control subsystem (1.5) (1.5)′, we can obtain following theorem.

Theorem 1 Suppose $(A_{ii}, B_{ii})(i = 1,\cdots,W)$ is controllable, $(A_{ii}, C_{ii})(i = 1,\cdots,W)$ is observable. Eq. (1.7) is satisfied with nonlinear terms, if there exists constants $\Delta_1 > 0$ and $\Delta_2 > 0$, $\Delta_3 > 0$, such that

$$D_1 < \Delta_1, \quad \tau < \Delta_2, \quad R_1 < \Delta_3$$

the asymptotical stabilization of the trivial solution of closed-loop system of uncertain linear large scale control system (1.4) without delays and structure perturbation parameters implies the interconnected asymptotical stabilization of the multigroup time-delays uncertain nonlinear interconnected large scale control system (1.1) with the time delay control vector functions for any $E_\beta^{(\alpha)} \in \overline{E}_\beta^{(\alpha)}(\alpha = r,s,d,f,g,h;\ \beta = 1,2,3,4,5,6)$. Where, Δ_1, Δ_2, Δ_3 are defined as

$$\Delta_1=\min\left\{\beta_{3v}/[4p_1(1+K_1m)(\sum_{j=1,j\neq i}^{W} n_j^2+n_v(n-n_v))+Gn_v(n+\frac{4\beta_{2v}}{\beta_{1v}}n_v)],v=1,\cdots,W\right\}, \quad (1.10)$$

$$\Delta_2 = \min\left\{[\beta_{3v}/[4p_1nn_vH(n + \frac{4\beta_{2v}}{\beta_{1v}}n_v)],v = 1,\cdots,W\right\}, \quad (1.11)$$

$$\Delta_3=\min\left\{\beta_{3v}/[4p_1(1+N_1+(N_2+K_1mN_4)n+K_1m(1+N_3))(n_vn+n_v^2\frac{4\beta_{2v}}{\beta_{1v}})],v=1,\cdots,W\right\}, \quad (1.12)$$

$$G = [1 + N_1 + N_2n + K_1m(1 + N_3 + N_4)],$$
$$H = (a_1N_2 + b_1K_1mN_4)[a_1(1 + N_1) + b_1K_1m(1 + N_3)],$$

where $\beta_{1v} = \lambda_{\min}(P)$, $\beta_{2v} = \lambda_{\max}(P)$, $\beta_{3v} = \lambda_{\max}(C^TC + PBR^{-1}B^TP)$, p is the unique symmetrically positive definite solution of Riccati matrix algebra nonlinear equation $A^TP + PA - PBR^{-1}B^TP + C^TC = 0$.

The proof of Theorem 1 is omitted here.

Theorem 2 If the trivial solution of closed-loop system of system (1.4) is asymptotically stable and there exists $\Delta_4 > 0$, $\Delta_5 > 0$, $\Delta_6 > 0$ such that

$$D_1 < \Delta_4, \quad \tau < \Delta_5, \quad R_1 < \Delta_6$$

then for any $E_\beta^{(\alpha)} \in \overline{E}_\beta^{(\alpha)}(\alpha = r,s,d,f,g,h;\ \beta = 1,2,3,4,5,6)$, the trivial solution of closed-loop system of (1.1) is robust interconnected asymptotically stable, i.e. (1.1) is interconnected robust stabilization.

The proof of this theorem is similar to Theorem 1, we omit it.

2 Conclusion

This paper at the first time considered the problem of the structure and interconnected robust stabilization for the multigroup time-delays uncertain nonlinear large scale control systems with the time delay control vector functions. The estimated formulas of bounds are given for delay, perturbed parameter and nonlinear terms.

If (1.1) is a constant or time-varying neutral nonlinear interconnected control large scale systems, the structure and interconnected robust stabilization similar to that of Theorem 1 may be obtained by "robust stabilization equivalent method".

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多组多滞后区间非线性关联控制大系统的结构与关联鲁棒镇定

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摘要：建立了多组多滞后区间系数定常非线性关联控制大系统的结构新概念.采用鲁棒镇定等价法,给出了具有扰动结构参数的、带有时滞控制向量函数的、多组多滞后区间常系数、非线性关联控制大系统的结构与关联鲁棒镇定,同时给出了扰动参数与滞后非线性项界限的估计公式.

关键词：多组多滞后；非线性控制大系统；结构与关联鲁棒镇定

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