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Studies on Multiobjective State-Feedback Control for Linear Continuous Systems with Variance Constraints*

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Abstract: This paper consists of two parts. In the first part, the problem of controller design for linear continuous systems with prespecified H_∞ norm, circular pole and steady-state variance constraints is considered. Furthermore, in the second part, the problem of performance robust controller design for linear continuous uncertain systems with variance and circular pole constraints is studied. Effective, algebraic, modified Riccati equation approaches are developed to solve the addressed problems. Numerical examples are provided to show the usefulness and applicability of the present approaches.

Key words: linear continuous stochastic systems; H_∞ control; robust control; constrained variance design; regional pole placement

1 Introduction

The problem of constrained variance design has received much attention in the past decade, since the performance requirements of many engineering control systems are naturally described in terms of the acceptable variance value of the system states. Covariance control theory[1~8] provides a more direct methodology to achieve the individual variance constraint than the LQG control theory. Although the design of variance-constrained control systems is well studied, only few contributions have been given to exploiting the extra freedom that it leaves to the designer. For example, much of the variance-constrained control literature focuses on the steady-state behaviors and robustness, but the transient properties such as regional pole assignment and disturbance rejection property such as H_∞ norm constraints are seldom considered. To this end, this paper will introduce purely algebraic approaches which deal with the problem of controller design with multiple performance objectives including steady-state variance constraints, robustness to parameter uncertainty, regional pole assignment and H_∞ norm constraint on the disturbance transfer matrix.

In the present paper, two different problems are considered respectively, The first is how to design state feedback controller which can achieve a specified state covariance upper bound, such that the H_∞ norm constraint and the steady-state variance constraint can be simultaneously satisfied, and such that the closed-loop poles lie within a specified circular re-

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gion. The aim of the second problem is to find the robust controller, for linear continuous system with unstructured parameter uncertainties, such that the closed-loop system simultaneously satisfies the prespecified circular pole and individual variance constraints. Algebraic approaches are developed to solve the multiobjective design problem addressed in the present paper.

2 Problem Formulation and Preliminaries

Consider the following linear stochastic continuous certain and uncertain systems respectively described by

$$\dot{x}(t) = Ax(t) + Bu(t) + Dw(t), y(t) = Cx(t) \tag{2.1}$$

and

$$\dot{x}(t) = (A + \Delta A)x(t) + Bu(t) + Dw(t) \tag{2.2}$$

where $x(t) \in \mathbb{R}^{n_x}, u(t) \in \mathbb{R}^{n_u}, w(t) \in \mathbb{R}^{n_w}, y(t) \in \mathbb{R}^{n_y}$ and A, B, D, C are constant matrices with appropriate dimensions and $DD^T > 0$. $w(t)$ is a zero mean white noise process with covariance I, and $w(t)$ and $x(0)$ are uncorrelated. The pairs (A, B) and (A, D) are assumed to be controllable. $\Delta A(\cdot)$ represents the system matrix uncertainty which is of the structure $\Delta A = MFN$, where M, N are known real constant matrices with appropriate dimensions, and F, whose elements are Lebesgue measurable, is an unknown matrix function bounded as $FF^T \leqslant I$.

When a state feedback control law $u(t) = Gx(t)$ is applied to systems (2.1) and (2.2), the closed-loop systems are respectively obtained as

$$\dot{x}(t) = A_c x(t) + Dw(t), A_c = A + BG, y(t) = Cx(t) \tag{2.3}$$

and

$$\dot{x}(t) = (A_c + \Delta A)x(t) + Dw(t), A_c = A + BG. \tag{2.4}$$

Furthermore, for the system (2.1), the closed-loop transfer function $H(s)$ from noise input $w(t)$ to ouput $y(t)$ may be written as $H(s) = C(sI - A_c)^{-1}D$. If the closed-loop systems (2.3) and (2.4) are asymptotically stable, then the steady-state covariances of systems (2.3) and (2.4), which are defined as $X = \lim_{t\to\infty} \mathrm{E}[x(t)x^T(t)]$. exist and satisfy the following continuous Lyapunov equations respectively

$$A_c X + XA_c^T + DD^T = 0, \tag{2.5}$$

$$(A_c + \Delta A)X + X(A_c + \Delta A)^T + DD^T = 0. \tag{2.6}$$

We further consider a circular region $D(q, r)$ in the left half complex plane with the center at $-q + j0 (q > 0)$ and the radius $r (r < q)$ for the continuous systems. Now, We are in a position to formulate the problems under study as follows.

1) H_∞ norm, circular pole and variance-constrained controller design problem (denoted as problem A): For the certain system (2.1), determine the state-feedback gain, G, such that: A1) The H_∞ norm of the disturbance transfer matrix $H(s)$ from $w(t)$ to $y(t)$ meets the constraint $\| H(s) \|_\infty \leqslant v$, where $\| H(s) \|_\infty = \sup_{\omega\in\mathbb{R}} \sigma_{\max}[H(j\omega)]$ and $\sigma_{\max}[\cdot]$ denotes the largest singular value of $[\cdot]$; and v is a given positive constant; A2) The closed-loop poles are constrained to lie within the circular region $D(q, r)$, i.e., $\sigma(A_c) \subset D(q, r)$; A3) The individual

state variance constraints are statisfied, i. e., $[X]_{ii} \leqslant \sigma_i^2, i = 1,2,\cdots,n_x$, where $[X]_{ii}$ is the ith diagonal element of X, and $\sigma_i (i = 1,2,\cdots,n_x)$ denotes the root-mean-squared value constraint for the variance of system state.

2) Robust circular pole and variance-constrained controller design problem (denoted as problem B): For the uncertain system (2. 2), seek the controller G such that: B1) The closed-loop poles are situated within the specified circle (i. e. $\sigma(A_c + \Delta A) \subset D(q,r)$) for all admissible uncertainties; B2) The same as requirement A3).

3 Solution to the Problem A

Theorem 3. 1 Given a constant $v > 0$ and circular region $D(q,r)$. Then the requirements A1) and A2) are satisfied if the following matrix equation

$$A_c Q A_c^T + (q^2 - r^2)Q + q(A_c Q + Q A_c^T + v^{-2} Q C^T C Q + D D^T) = 0 \tag{3.1}$$

has a positive definite solution Q. Furthermore, in this case, the steady-state covariance X exists and satisfies $X \leqslant Q$. (For the proof see Appendix A.)

Remark 3. 1 By using Theorem 3. 1, we can assign a desired value to the positive definite matrix Q, such that this matrix Q meets $[Q]_{ii} \leqslant \sigma_i^2, i = 1,2,\cdots,n_x$, and find the set of feedback controller G which satisfies (3. 1) for the specified Q. If such a controller exists and can be obtained, then from Theorem 3. 1, we will have $[X]_{ii} \leqslant [Q]_{ii} \leqslant \sigma_i^2, i = 1,2,\cdots,n_x$, and $\| H(s) \|_\infty \leqslant v$ and $\sigma(A_c) \subset D(q,r)$. Therefore, the design task will be accomplished, and the problem A can be converted to such an auxiliart "Q-matrix assignment" problem.

To make the problem more tractable, we give the following definition and some lemmas which are useful in the proof of main theorems.

Definition 3. 1 Given a desired positive constant v and a desired circular region $D(q,r)$. Let Q be a prespecified positive definite matrix which meets $[Q]_{ii} \leqslant \sigma_i^2, i = 1,2,\cdots,n_x$. Then Q is called a v-D- assignable matrix if there exists a set of controller G such that the equation (3. 1) has the positive definite solution Q.

Lemma 3. 1[5] Let $M \in \mathbb{R}^{n\times n}$ and $N \in \mathbb{R}^{n\times p} (m \leqslant p)$. There exists a matrix V which simultaneously satisfies $N = MV, VV^T = I$ if and only if $MM^T = NN^T$. In this case, a general solution for V can be expressed as

$$V = V_M \begin{bmatrix} I & 0 \\ 0 & U \end{bmatrix} V_N^T, \quad U \in \mathbb{R}^{(n-r_M)\times(p-r_M)}, \quad UU^T = I,$$

where V_M and V_N come from the singular value decomposition of M and N respectively,

$$M = U_M \begin{bmatrix} Z_M & 0 \\ 0 & 0 \end{bmatrix} V_M^T, \quad N = U_N \begin{bmatrix} Z_M & 0 \\ 0 & 0 \end{bmatrix} V_N^T,$$

and $$r_M = \text{rank}(M), \quad U_M = U_N, \quad Z_M = Z_N.$$

Lemma 3. 2[13] Given the circular region $D(q,r)$. The poles of matrix A are located within $D(q,r)$ if and only if there exists a positive definite solution Q satisfying

$$AQA^T + (q^2 - r^2)Q + q(AQ + QA^T) + P = 0$$

where P is an arbitrary positive definite matrix.

Theorem 3. 2 A specified positive definite matrix Q satisfying $[Q]_{ii} \leqslant \sigma_i^2, (i = 1,2,\cdots,$

n_x) is v-D- assignable, if and only if

$$r^2Q - q(v^{-2}QC^TCQ + DD^T) \geqslant 0, \tag{3.2}$$

$$(I - BB^+)[r^2Q - q(v^{-2}QC^TCQ + DD^T) - (A + qI)Q(A + qI)^T](I - BB^+) = 0. \tag{3.3}$$

Proof We can rearrange (3.1) as follows:

$$(A_cQ^{1/2} + qQ^{1/2})(A_cQ^{1/2} + qQ^{1/2})^T = r^2Q - q(v^{-2}QC^TCQ + DD^T). \tag{3.4}$$

Consider (3.4), since its left-hand side is positive semidefinite, Q is required to meet (3.2). To prove (3.3), we first define $W = r^2Q - q(v^{-2}QC^TCQ + DD^T)$, and take the square root of W: $W = TT^T$. From Lemma 3.1, (3.4) is equivalent to $A_cQ^{1/2} + qQ^{1/2} = TV$ or

$$BG = TVQ^{-1/2} - qI - A \tag{3.5}$$

where V is some orthogonal matrix with dimension n_x. Thus, it follows from [9] that, there exists an orthogonal matrix V such that (3.5) or (3.4) has a solution for G if and only if there exists an orthogonal matrix V such that $(I - BB^+)(TVQ^{-1/2} - qI - A) = 0$ or

$$(I - BB^+)TV = (I - BB^+)(A + qI)Q^{1/2} \tag{3.6}$$

holds. It is now clear that the given $Q > 0$ satisfying $[Q]_{ii} \leqslant \sigma_i^2 (i = 1, 2, \cdots, n_x)$ and (3.2) is v-D- assignable if and only if there exists an orthogonal matrix V satisfying (3.6), or equivalently, from Lemma 3.1, if and only if

$$\begin{aligned}&[(I - BB^+)T][(I - BB^+)T]^T \\ &= [(I - BB^+)(A + qI)Q^{-1/2}][(I - BB^+)(A + qI)Q^{-1/2}]^T.\end{aligned} \tag{3.7}$$

It is not difficult to see that the equation (3.7) is just (3.3). This proves Theorem 3.2.

Now, we will parametrize the feedback gain guaranteeing the mixed circular pole, H_∞ norm and variance constraints. We first take the following singular value decompositions:

$$M = (I - BB^+)T = U_M\begin{bmatrix} Z_M & 0 \\ 0 & 0 \end{bmatrix}V_M^T, \tag{3.8}$$

$$N = (I - BB^+)(A + qI)Q^{1/2} = U_N\begin{bmatrix} Z_N & 0 \\ 0 & 0 \end{bmatrix}V_N^T. \tag{3.9}$$

It follows Theorem 3.1 and [9] that, if the given positive definite matrix Q is v-D- assignable, then a general solution of (3.5) is

$$G = B^+ (TVQ^{-1/2} - qI - A) + (I + B^+ B)Z \tag{3.10}$$

where $Z \in \mathbb{R}^{n_u \times n_x}$ is arbitrary and V is any orthogonal matrix satisfying $MV = N$, i.e., equation (3.6). By using Lemma 3.1, the orthogonal matrix V satisfying $MV = N$ can be expressed as

$$V = V_M\begin{bmatrix} I & 0 \\ 0 & U \end{bmatrix}V_N, \quad U \in \mathbb{R}^{(n_x - r_M)\times(n_x - r_M)} \tag{3.11}$$

where matrix U is arbitrary orthogonal. Finally, substituting (3.11) into (3.10) yields the following theorem.

Theorem 3.3 Assume that the given positive definite matrix Q satisfying $[Q]_{ii} \leqslant \sigma_i^2 (i = 1, 2, \cdots, n_x)$ is v-D -assignable, then the set of all controllers that assign this Q is parametrized as

$$G = B^+ \left(TV_M\begin{bmatrix} I & 0 \\ 0 & U \end{bmatrix}V_N^T Q^{-1/2} - qI - A\right) + Z - B^+ BZ, \qquad (3.12)$$

where $TT^T = r^2Q - q(v^{-2}QC^TCQ + DD^T)$, $Z \in \mathbb{R}^{n_u \times n_x}$ is arbitrary $U \in \mathbb{R}^{(n_x - r_M)\times(n_x - r_M)}$ is arbitrary orthogonal, M, N, V_M, V_N are defined in (3.8) and (3.9), and $r_M = \text{rank}M$.

To this end, the following result is easily accessible.

Theorem 3.4 Given the desired constant v, the desired circular region $D(q,r)$ and the individual state variance constraints $\sigma_i^2 (i = 1,2,\cdots,n_x)$. Assume that a specified positive definite matrix Q is v-D- assignable, i.e., this Q meets $[Q]_{ii} \leqslant \sigma_i^2 (i = 1,2,\cdots,n_x)$ and (3.2)(3.3). Then the solution of the H_∞ norm, circular pole and variance constrained-design problems can be obtained from (3.12).

4 Solution to the Problem B

In this section, the problem of robust circular pole and variance-constrained controller design for uncertain systms will be studied using the similar approach proposed in previous section, but the major results are quite different. The following lemmas play an important role in the proof of main theorems.

Lemma 4.1[10] For arbitrary positive constant $\varepsilon_1 > 0$ and arbitrary positive definite matrix P, we have

$$(A_c + \Delta A)P + P(A_c + \Delta A)^T \leqslant \varepsilon_1 MM^T + \varepsilon_1^{-1} PN^TNP + A_cP + PA_c^T. \qquad (4.1)$$

Lemma 4.2[11] Suppose that there exist positive scalar $\varepsilon_2 > 0$ and positive definite matrix $P > 0$ such that $\varepsilon_2 NPN^T < I$. Then

$$(A_c + \Delta A)P(A_c + \Delta A)^T \leqslant A_cPA_c^T + A_cPN^T(\varepsilon_2^{-1}I - NPN^T)^{-1}NPA_c^T + \varepsilon_2^{-1}MM^T. \qquad (4.2)$$

With the lemmas provided above, we present the following theorem which plays a main key for solving the Problem B.

Theorem 4.1 Consider the uncertain closed-loop system (2.2). Let the desired circular region $D(q,r)$ and the controller G be given. If there exist positive scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$ and positive definite matrix P satisfying

$$q(A_cP + PA_c^T) + A_cRA_c^T + H = 0 \qquad (4.3)$$

where

$$R = P + PN^T(\varepsilon_2^{-1}I - NPN^T)^{-1}NP,$$
$$H = (q^2 - r^2)P + q\varepsilon_1^{-1}PN^TNP + (\varepsilon_2^{-1} + q\varepsilon_1)MM^T + qDD^T,$$

then the requirement (B1) is satisfied. Furthermore, in this case, the steady-state covariance X exists and meets $X \leqslant P$. (For the proof see Appendix B.)

Definition 4.1 Let the positive definite matrix $P > 0$ satisfy $[P]_{ii} \leqslant \sigma_i^2 (i = 1,2,\cdots,n_x)$. If there exists a feedback gain G meeting (4.3) for the prespecified matrix $P > 0$, this positive definite matrix P is called a RD-assignable matrix.

Remark 4.1 By using Theorem 4.1, we can assign a desired value to the positive definite matrix P such that $[P]_{ii} \leqslant \sigma_i^2 (i = 1,2,\cdots,n_x)$. If this matrix P is RD-assignable, then from Theorem 4.1, we will have $[X]_{ii} \leqslant [P]_{ii} \leqslant \sigma_i^2 (i = 1,2,\cdots,n_x)$ and $\sigma(A_c + \Delta A) \subset D(q,$

r) ,and therefore the problem B will be solved. Such an auxiliary " P- matrix assignment" problem consists of two parts:1) find the conditon for the existence of RD-assignable matrix $P > 0$,and 2) find the set of all controllers achieving the RD-assignable matrix $P > 0$.

Now,in the key Theorem 4.1,the equation (4.3) can be rewritten as follows;

$$(A_cR^{1/2} + qPR^{-1/2})(A_cR^{1/2} + qPR^{-1/2})^{\mathrm{T}} = q^2PR^{-1}P - H. \tag{4.4}$$

Note that the important equation (4.4) is very similar to (3.4),we can solve Problem B in a manner dealing with Problem A. In this case,we will simply summarize main results of this section without delailed proofs in order to aviod duplicating previous section.

Theorem 4.2 A specified positive definite matrix P satisfying $[P]_{ii} \leqslant \sigma_i^2 (i = 1,2,\cdots,n_x)$ is RD-assignable if and only if there exist scalar parameters $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ such that

$$\varepsilon_2 NPN^{\mathrm{T}} < I, \quad q^2PR^{-1}P - H \geqslant 0, \tag{4.5}$$

$$(I - BB^+)(qPA^{\mathrm{T}} + qAP + ARA^{\mathrm{T}} + H)(I - BB^+) = 0, \tag{4.6}$$

where R,H are defined in Thorem 4.1.

Theorem 4.3 Supposed that the given positive definite matrix P satisfying $[P]_{ii} \leqslant \sigma_i^2 (i = 1,2,\cdots,n_x)$ is RD-assignable,then all controllers that assign this P can be expressed as

$$G = B^+ \left(JV_K\begin{bmatrix} I & 0 \\ 0 & U \end{bmatrix}V_L^{\mathrm{T}}R^{-1/2} - qPR^{-1} - A\right) + (I - B^+ B)Z \tag{4.7}$$

where $TT^{\mathrm{T}} = q^2PR^{-1}P - H, U \in \mathbb{R}^{(n_x - r_K)\times(n_x - r_K)}$ is arbitrary orthogonal, $r_K \stackrel{\triangle}{=} \mathrm{rank}K, Z \in \mathbb{R}^{n_u \times n_x}$ is arbitrary, K,L,V_K,V_L are defined in following singular value decompositions:

$$K = (I - BB^+)J = U_K\begin{bmatrix} Z_K & 0 \\ 0 & 0 \end{bmatrix}V_K^{\mathrm{T}},$$

$$L = (I - BB^+)(qPR^{-1} + A)R^{1/2} = U_L\begin{bmatrix} Z_L & 0 \\ 0 & 0 \end{bmatrix}V_L^{\mathrm{T}}.$$

Theorem 4.4 Given the desired circular region $D(q,r)$ and the individual state variance constraints $\sigma_i^2 (i = 1,2,\cdots,n_x)$. Assume that a specified positive definite matrix P satisfying $[P]_{ii} \leqslant \sigma_i^2 (i = 1,2,\cdots,n_x)$ is RD-assignable. Then the solution to the problem of mixed robust control design with circular pole and variance constraints can be obtained from(4.7).

5 Discussion on Numerical Algorithms and Examples

In the design of practical control systems,we are usually required to construct a v-D-assignable matrix Q satisfying variance constraints and (3.2) from the assignability condition (3.3) in the case of solving Problem A,or construct a RD-assignable matrix P satisfying variance constraints and (4.5) from the assignability condition (4.6) in the case of solving Problem B,and then obtain the desired controllers from (3.12) and (4.7) immediately. Notice that the equations(3.3)and (4.6) are actually generalized algebraic Riccati equations which also appeared in Chang and Chung[6] and Skelton and Iwasaki[7] with a similar form,we can solve them by using the same method proposed in [6,7]which is suitable for relatively lower order models. For the relatively higher order model, a possible approach to solving the nonlinear programming problems (3.2)(3.3) and (4.5)(4.6) is to exploit the iterative numerical search method[8,12].

The numerical examples are provided in Appendix C to show the usefulness and applicability of the present approaches.

6 Conclusions

This paper has shown how to design variance-constrained state-feedback control systems which satisfy multiple performance requirements such as transient property, H_∞ disturbance attenuation behaviour and robustness. Based on the singular value decomposition technique and the generalized inverse theory, sufficient conditions for the existence of desired controllers and the set of solutions have been introduced. Though the results developed in the present paper are restricted to linear continuous systms with constant gain state feedback, it is not difficult to extend the theory to discrete-time systems and the dynamic output feedback. This results will appear at later date.

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Appendix A

Proof of Theorem 3. 1.

The proof of the conclusion $\sigma(A_c) \subset D(q,r)$ follows from Lemma 3. 2 immediately. Next, (3. 1) can be rewritten as follows

$$A_cQ + QA_c^{\mathrm{T}} + v^{-2}QC^{\mathrm{T}}CQ + DD^{\mathrm{T}} + \Sigma = 0 \tag{A1}$$

where $\Sigma = q^{-1}[A_cQA_c^{\mathrm{T}} + (q^2 - r^2)Q]$. Since $\Sigma > 0$, the proof of $\| H(s) \|_\infty \leqslant v$ can be completed by a standard manipulation of(A1): for detail see Lemma 1 of[14]. Finally, subtract (2. 5) from (A1) to obtain

$$A_c(Q-X)+(Q-X)A_c^{\mathrm{T}}+v^{-2}QC^{\mathrm{T}}CQ+\Sigma=0. \tag{A2}$$

Because A_c is asymptotically stable and $v^{-2}QC^{\mathrm{T}}CQ+\Sigma>0$, (A2) is then equivalent to

$$Q-X=\int_0^\infty e^{A_ct}[v^{-2}QC^{\mathrm{T}}CQ+\Sigma]e^{A_c^{\mathrm{T}}t}dt\geqslant 0 \tag{A3}$$

and $X\leqslant Q$ follows immediately. This proves Theorem 3. 1.

Appendix B

Proof of Theorem 4. 1.

From Lemma 4. 1 and Lemma 4. 2, we have the the following inequalities

$$0\leqslant\Psi_1:=\varepsilon_1MM^{\mathrm{T}}+\varepsilon_1^{-1}PN^{\mathrm{T}}NP+A_cP+PA_c^{\mathrm{T}}-(A+\Delta A)P-P(A+\Delta A)^{\mathrm{T}}, \tag{B1}$$

$$0\leqslant\Psi_2:=A_cPA_c^{\mathrm{T}}+A_cPN^{\mathrm{T}}(\varepsilon_2^{-1}I-NPN^{\mathrm{T}})^{-1}NPA_c^{\mathrm{T}}+\varepsilon_2^{-1}MM^{\mathrm{T}}$$
$$-(A_c+\Delta A)P(A+\Delta A)^{\mathrm{T}}. \tag{B2}$$

By using (B1) and (B2), (4. 3) can be rewritten as

$$(A_c+\Delta A)P(A_c+\Delta A)^{\mathrm{T}}+(q^2-r^2)P$$
$$+q[(A_c+\Delta A)P+P(A_c+\Delta A)^{\mathrm{T}}]+q\Psi_1+\Psi_2+qDD^{\mathrm{T}}=0. \tag{B3}$$

Since $q\Psi_1+\Psi_2+qDD^{\mathrm{T}}>0$, it follows from Lemma 3. 2 that $\sigma(A_c+\Delta A)\subset D(q,r)$. Now, we define

$$\Psi_3:=q^{-1}[(A_c+\Delta A)P(A_c+\Delta A)^{\mathrm{T}}(q^2-r^2)p+q\Psi_1+\Psi_2]\geqslant 0$$

and (B3) can be rearranged as

$$(A_c+\Delta A)P+P(A_c+\Delta A)^{\mathrm{T}}+DD^{\mathrm{T}}\backslash\%+\Psi_3=0. \tag{B4}$$

Subtract (2. 6) from (B4) to obtain

$$(A_c+\Delta A)(P-X)+(P-X)(A_c+\Delta A)^{\mathrm{T}}+\Psi_3=0 \tag{B5}$$

which is equivalent to

$$P-X=\int_0^\infty e^{(A_c+\Delta A)t}\Psi_3e^{(A_c+\Delta A)^{\mathrm{T}}t}dt\geqslant 0 \tag{B6}$$

and the inequality $X\leqslant P$ follows directly. The proof of Theorem 4. 1 is completed.

Appendix C

Illustrative Examples.

Example 1 Consider linear stochastic continuous system (2. 1) with parameters

$$A=\begin{bmatrix}-4&0&0\\0&-3&1\\0&0&1\end{bmatrix},\quad B=\begin{bmatrix}0&0\\1&0\\0&1\end{bmatrix},\quad D=\begin{bmatrix}0.2\\0\\0\end{bmatrix},\quad C=I.$$

The performance requirements on Problem A are as follows:

$$\|H(s)\|_\infty\leqslant 0.8,\quad [X]_{11}\leqslant 0.6,\quad [X]_{22}\leqslant 1.3,\quad [X]_{33}\leqslant 6.1,\quad D(q,r)=D(3,2).$$

It is assumed that the positive definite matrix Q is of the form $Q=[q_{ii}](i,j=1,2,3)$, the condition (3. 3) means that $4q_{11}-3(0.8^{-2}q_{11}^2+0.04)-q_{11}=0$, therefore $q_{11}=0.5971$. Furthermore, subject to the prescribed performance requirements and (3. 2), we can choose an appropriate v-D- assignable matrix Q and obtain the related controller from (3. 12) as:

$$Q=\begin{bmatrix}0.5971&0.0083&0.0042\\0.0083&1.2473&0.0125\\0.0042&0.0125&5.7832\end{bmatrix},\quad G=\begin{bmatrix}0.0115&-4.9982&-0.0182\\-0.0028&-0.1258&-1.0006\end{bmatrix}.$$

Finally, we can easily obtain $\|H(s)\|_\infty=0.6125$, $[X]_{11}=0.005$, $[X]_{22}=0.00012$, $[X]_{33}=0.00009$, and colsed-loop poles -4.00001, -4.99881, -1.00018. It is clear that these results satisfy fhe respecified constraints.

Example 2 Consider the linear uncertain system (2. 2). The parametersA, B, D are the same as those in example 1, and the uncertainty is assumed to be $\Delta A=MFN=(0.5I_3)(\sin\alpha I_3)(0.5I_3)$, and the performance requirements on Problem B are defined as follows

$$[X]_{11} \leqslant 1.025, \quad [X]_{22} \leqslant 4.327, \quad [X]_{33} \leqslant 0.918, \quad \sigma(A_c + \Delta A) \subset D(q,r) = D(4,3.6).$$

Suppose that the positive definite matrix P has the form $P = [p_{ij}](i,j = 1,2,3)$. Substituting P into equation (4.6), and considering the inequality constraints (4.5) and the desired performance requirements, we can construct a RD-assignable matrix P and the related scalar parameters $\varepsilon_1, \varepsilon_2$ as

$$P = \begin{bmatrix} 0.9804 & -1.8966 & 0.00255 \\ -1.8966 & 3.9045 & 0.0123 \\ 0.00255 & 0.0123 & 0.8365 \end{bmatrix}, \quad \varepsilon_1 = 1.1624, \quad \varepsilon_2 = 0.25.$$

Using the results provided in previous section, we can easily get

$$G = \begin{bmatrix} 0.0126 & -5.0184 & -0.0116 \\ -0.0167 & -0.2076 & -0.9936 \end{bmatrix}.$$

It is not difficult to test that the prespecified requirement constraints of example 2 are satisfied.

方差约束下线性连续系统的多指标随机控制

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摘要：本文由两部分组成，第一部分考虑具有给定 H_∞ 范数、圆形区域极点及稳态方差约束的线性连续系统的控制器设计问题，第二部分则进一步研究方差及圆形区域极点约束下不确定线性连续系统的性能鲁棒控制器设计问题．文中提出有效的代数黎卡提方程方法来解决如上问题，并用数值算例说明所提方法的直接性与可行性．

关键词：线性连续随机系统；H_∞ 控制；鲁棒控制；约束方差设计；区域极点配置

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王子栋 1966 年生．1986 年于苏州大学数学系获理学学士学位，1988 年于上海中国纺织大学应用数学专业获理学硕士学位．1994 年于南京理工大学自动控制系获工学博士学位，同年晋升为副教授．1996 年获德国洪堡博士后研究基金．目前主要研究方向为随机控制与估计，H_∞ 控制与估计，鲁棒控制，容错控制，模型降阶简化，数据融合理论等．

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