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Robust Stability of Linear Interval Systems with Time-Delay

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Abstract: In this paper, some results for robust stability of linear interval time-delay systems are derived and some previous criteria for stability of linear time-delay systems are further developed, the stability degree are also discussed. Finally, the robust stability of large-scale linear interval time-delay system is studied.

Key words: interval matrix; robust stability; time-delay; large-scale system

1 Introduction

Since time-delay is frequently encountered in various engineering systems, the study of time-delay systems has provided considerable interests among researchers from both home and abroad. Many methods to check the stability of time-delay systems were obtained by Mori and Kokame[1], Hmamed[2], Wang et al.[3], additionally, since the robust stability is useful in designing control systems, the robust stability of interval systems is also studied by many authors. Many results to test the robust stability of uncertain time-delay systems were proposed by Wang and Lin[4], Liu Y. Q. and Tang G. Y.[5], Shyu and Yan[6].

In this paper, we will study robustly stability of system

$$\begin{cases} \dot{x}(t) = G[B,C]x(t) + G[D,E]x(t-h), \quad t > 0, \\ x(t) = \varphi(t), \quad t \in [-h, 0]. \end{cases} \tag{1.1}$$

2 Robust Stability of Linear Interval Time-Delay System

Consider the linear time-delay system described by the following interval differential-difference equations:

$$\begin{cases} \dot{x}(t) = G[B,C]x(t) + G[D,E]x(t-h), t > 0, \\ x(t) = \varphi(t), \quad t \in [-h, 0]. \end{cases} \tag{2.1}$$

Where $x \in \mathbb{R}^n$ is a state vector, matrices $G[B,C], G[D,E]$ are $n \times n$ interval matrices, $h > 0$ is a constant, $\varphi(t)$ is a continuous vector-valued initial function, $A = (a_{ij})_{n\times n}, B = (b_{ij})_{n\times n}, C = (c_{ij})_{n\times n}, D = (d_{ij})_{n\times n}, E = (e_{ij})_{n\times n}$ are matrices.

Let $A_0 = \frac{1}{2}(B+C), K = \frac{1}{2}(C-B), M = (m_{ij})_{n\times n}, m_{ij} = \max\{|d_{ij}|, |e_{ij}|\}; i,j = 1, 2, \cdots, n$; for any $A \in G[B,C], A_1 \in G[D,E]$, consider system

$$\dot{x}(t) = A_0 x(t) + (A - A_0)x(t) + A_1 x(t-h). \tag{2.2}$$

The system (2.1) is said to be robustly stable if for any $A \in G[B,C], A_1 \in G[D,E]$, system (2.2) is asymptotic stable.

Definition 2.1 The system (2.1) is said to have stability degree $\gamma > 0$, if there exists $k > 0$ (depending on initial conditions), such that any solution of (2.2) satisfies

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$$\|x(t)\| \leqslant k\|x(t_0)\| \exp[-\gamma(t-t_0)] \text{ for all } t, t_0 \in \mathbb{R}^+ \text{ and } t > t_0.$$

In this paper, $\|x\|$ is an Euclidean norm of vector x, $\|x\| = \sqrt{(x^T x)}$; $\|A\|$ is a matrix norm of matrix A, $\|A\| = \sqrt{\lambda_{max}(A^T A)}$.

Suppose interval matrix $G[B,C]$ is stable, then A_0 is stable, so there exists a positive definite matrix P, such that $A_0 P + PA_0 = -2I$.

Theorem 2.1 If the condition $\|PK\| + \alpha\|P\|\|M\| < 1$ is satisfied, then systems (2.1) is robustly stable.

Proof Let $V(x(t)) = x^T(t)Px(t)$, then

$$\begin{aligned}\dot V(x(t))|_{(2.2)} = & -2x^T(t)x(t) + x^T(t)[(A-A_0)^T P + P(A-A_0)]x(t) \\ & + x^T(t-h)A_1^T Px(t) + x^T(t)PA_1 x(t-h) \\ \leqslant & -2\|x(t)\|^2 + 2\|PK\|\|x(t)\|^2 + 2\|P\|\|M\|\|x(t)\|\|x(t-h)\|.\end{aligned}$$

Assume $V(x(t-h)) < q^2 V(x(t))$, then $\|x(t-h)\| < q\alpha\|x(t)\|$.

Where $\alpha = \left[\sqrt{\dfrac{\lambda_{max}(P)}{\lambda_{min}(P)}}\right]^{\frac{1}{2}}$, so we have;

$$\dot V(x(t))|_{(2.2)} \leqslant -(2 - 2\|PK\| - 2q\alpha\|P\|\|M\|)\|x(t)\|^2.$$

Therefore, if $\|PK\| + \alpha\|P\|\|M\| < 1$, then a sufficient small $q > 1$ exists such that $\dot V(x(t))|_{(2.2)} < 0$. Thus according to (Hale 1977, Theorem p. 127)system (2.2) is stable. the theorem follows.

Theorem 2.2 If there exists $\beta > 0$, such that constant matrix

$$\begin{bmatrix} 2 - 2\|PK\| - \beta & -\|P\|\|M\| \\ -\|P\|\|M\| & \beta \end{bmatrix}$$

is positive definite, then systems (2.1)is robustly stable.

Proof Suppose $\beta > 0$, let $V(x(t)) = x^T(t)Px(t) + \beta\int_{t-h}^{t} x^T(s)x(s)ds$, then

$$\begin{aligned}\dot V(x(t))|_{(2.2)} \leqslant & -2\|x(t)\|^2 + 2\|PK\|\|x(t)\|^2 + 2\|P\|\|M\|\|x(t)\|\|x(t-h)\| \\ & + \beta(x^T(t)x(t) - x^T(t-h)x(t-h)) \\ = & -(\|x(t)\|, \|x(t-h)\|)\begin{bmatrix} 2 - 2\|PK\| - \beta & -\|P\|\|M\| \\ -\|P\|\|M\| & \beta \end{bmatrix} \\ & \cdot \begin{pmatrix} \|x(t)\| \\ \|x(t-h)\| \end{pmatrix}.\end{aligned}$$

If matrix $\begin{bmatrix} 2 - 2\|PK\| - \beta & -\|P\|\|M\| \\ -\|P\|\|M\| & \beta \end{bmatrix}$ is positive definite, then $\dot V(x(t))|_{(2.2)} < 0$.

The theorem is proved.

Corollary 2.3 If the condition $\|PK\| + \|P\|\|M\| < 1$ is satisfied. then systems (2.1) is robustly stable.

Proof Let $\beta = 1 - \|PK\|$, if the condition of corollary is satisfied, then $\beta > 0$ and matrix $\begin{bmatrix} 2 - 2\|PK\| - \beta & -\|P\|\|M\| \\ -\|P\|\|M\| & \beta \end{bmatrix}$ is asymptotic stable, the corollary follows.

It is obvious that the result of Theorem 2.2 contains the result of Theorem 2.1.

Consider constant linear system

$$\dot{x}(t) = A_0x(t) + A_1x(t-h). \tag{2.3}$$

We have:

Corollary 2.4 If the condition $\|PA_1\| < 1$ is satisfied, then the system (2.1) is robustly stable.

Condition of this corollary is less restriction than those given by Hmamed[2], Shyu and Yan[6].

Theorem 2.5 If the condition $\|\bar{P}K\| + e^{\gamma h}\|\bar{P}\|\,\|M\| < 1$ is satisfied, then system (2.1) have stability degree γ.

Proof Let $z(t) = e^{\gamma t}x(t)$, the system (2.2) can be transform into

$$\dot{z}(t) = (A_0 + \gamma I)z(t) + (A - A_0)z(t) + e^{\gamma h}A_1x(t-h). \tag{2.4}$$

Let $V(z(t)) = z^{\mathrm{T}}(t)\bar{P}z(t) + (1 - \|\bar{P}K\|)\int_{t-h}^{t} z^{\mathrm{T}}(s)z(s)\mathrm{d}s$, then

$$\dot{V}(z(t))\,|_{(2.4)} \leqslant -(\|z(t)\|, \|z(t-h)\|)\begin{bmatrix} 1 - \|\bar{P}K\| & -e^{\gamma h}\|\bar{P}\|\,\|M\| \\ -e^{\gamma h}\|\bar{P}\|\,\|M\| & 1 - \|\bar{P}\|\,\|K\| \end{bmatrix}\begin{pmatrix} \|z(t)\| \\ \|z(t-h)\| \end{pmatrix}.$$

If the condition of the theorem is satisfied, the $\dot{V}(z(t))\,|_{(2.4)} < 0$. this complete proof of the theorem.

Since the system discussed by Liu Y. Q. and Tang G. Y. in work[5] is a special case of system (2.1), so some main results are contained in Theorem 2.2 and Theorem 2.5.

3 Robust Stability High Dimension System

Suppose interval time-delay system can be decomposed as following defferential-difference equations:

$$\begin{cases} \dot{x}^{(i)}(t) = G[B_{ii}, C_{ii}]x^{(i)}(t) + G[D_{ii}, E_{ii}]x^{(i)}(t-h) + \sum\limits_{j=1 j\neq i}^{r} [G[B_{ij}, C_{ij}]x^{(j)}(t) \\ \qquad + G[D_{ij}, E_{ij}]x^{(j)}(t-h)], \quad t > 0; \quad i, j = 1, 2, \cdots, r, \\ x^{(i)}(t) = \varphi^{(i)}(t), \quad t \in [-h, 0]. \end{cases} \tag{3.1}$$

Where: $G[B_{ij}, C_{ij}], G[D_{ij}, E_{ij}]$ are $n_i \times n_j$ interval matrices, $\sum\limits_{i=1}^{r} n_i = n$, $A_{ij} = (a_{kl}^{(ij)})_{n_i \times n_j}$, $B_{ij} = (b_{kl}^{(ij)})_{n_i \times n_j}$, $C_{ij} = (c_{kl}^{(ij)})_{n_i \times n_j}$, $D_{ij} = (d_{kl}^{(ij)})_{n_i \times n_j}$, $E_{ij} = (e_{kl}^{(ij)})_{n_i \times n_j}$.

Let $A_{ij}^0 = \frac{1}{2}(B_{ij} + C_{ij})$, $K_{ij} = \frac{1}{2}(B_{ij} - C_{ij})$, $M_{ij} = (m_{kl}^{(ij)})_{n_i \times n_j}$, $\overline{M}_{ij} = (\overline{m}_{kj}^{(ij)})_{n_i \times n_j}$, $m_{kl}^{(ij)} = \max\{|b_{kl}^{(ij)}|, |c_{kl}^{(ij)}|\}$; $\overline{m}_{kl}^{(ij)} = \max\{|d_{kl}^{(ij)}|, |e_{kl}^{(ij)}|\}$; $i, j = 1, 2, \cdots, n$; for any $A_{ij} \in G[B_{ij}, C_{ij}]$, $A_{ij}^1 \in G[D_{ij}, E_{ij}]$, consider the following system

$$\dot{x}^{(i)}(t) = A_{ii}^0x^{(i)}(t) + (A_{ii} - A_{ii}^0)x^{(i)}(t) + A_{ii}^1x^{(i)}(t-h) + \sum_{j=1, j\neq i}^{r} (A_{ij}x^{(j)}(t) + A_{ij}^1x^{(j)}(t-h)). \tag{3.2}$$

Denote: $z_i(t) = (\|x^{(i)}(t)\|, \|x^{(i)}(t-h)\|)^{\mathrm{T}}$,

$$U = \begin{bmatrix} -\lambda_{\min}(W_{11}) & \|W_{12}\| & \cdots & \|W_{1r}\| \\ \|W_{21}\| & -\lambda_{\min}(W_{22}) & \cdots & \|W_{2r}\| \\ \vdots & \vdots & \vdots & \vdots \\ \|W_{r1}\| & \|W_{r2}\| & \cdots & -\lambda_{\min}(W_{rr}) \end{bmatrix},$$

$$W_{ii} = \begin{bmatrix} 1 - \|P_iK_{ii}\| & -\|P_i\|\,\|M_{ii}\| \\ -\|P_i\|\,\|M_{ii}\| & 1 - \|P_i\|\,\|K_{ii}\| \end{bmatrix};$$

$$W_{ij}=\begin{bmatrix}\|P_i\|\|M_{ij}\|+\|P_j\|\|M_{ji}\| & \|P_i\|\|\overline{M}_{ij}\|\\ \|P_j\|\|\overline{M}_{ji}\| & 0\end{bmatrix},\quad (i\neq j),$$

since $W_{ij}=W_{ij}^{\mathrm{T}}$, so U is a symmetric matrix. Assume A_{ii}^0 is asymptotic stable, then there exists positive definite matrix P_i, such that: $A^0{}_{ii}^{\mathrm{T}}P_i+P_iA_{ii}^0=-2I_{n_i}$, then we have following theorem:

Theorem 3.1 Suppose following conditions

1) $\|P_iK_{ii}\|+\|P_i\|\|\overline{M}_{ii}\|<1;\quad i=1,2,\cdots,r;$

2) matrix U is negative definite

are satisfied, then system (3.1) is robustly stable.

Proof Since $\|P_iK_{ii}\|+\|P_i\|\|\overline{M}_{ii}\|<1$; so $\beta_i=1-\|P_iK_{ii}\|>0$. Let

$$V(x(t))=\sum_{i=1}^{r}V_i(x(t)),V_i(x(t))=x^{(i)^{\mathrm{T}}}(t)P_ix^{(i)}(t)+\beta_i\int_{t-h}^{t}x^{(i)^{\mathrm{T}}}(s)x^{(i)}(s)ds,$$

then

$$\begin{aligned}
&\dot V(x(t))|_{(3.2)}\\
&=\sum_{i=1}^{r}[A_{ii}^0x^{(i)}(t)+(A_{ii}-A_{ii}^0)x^{(i)}(t)+A_{ii}^1x^{(i)}(t-h)]P_ix^{(i)}(t)\\
&\quad+\sum_{i=1}^{r}x^{(i)^{\mathrm{T}}}(t)P_i[A_{ii}^0x^{(i)}(t)+(A_{ii}-A_{ii}^0)x^{(i)}(t)+A_{ii}^1x^{(i)}(t-h)]\\
&\quad+\sum_{i=1}^{r}\sum_{j=1,j\neq i}^{i}[A_{ij}x^{(j)}(t)+A_{ij}^1x^{(j)}(t-h)]^{\mathrm{T}}P_ix^{(i)}(t)\\
&\quad+\sum_{i=1}^{r}\sum_{j=1,j\neq i}^{i}x^{(i)^{\mathrm{T}}}(t)P_i[A_{ij}x^{(j)}(t)+A_{ij}^1x^{(j)}(t-h)]\\
&\quad+\sum_{i=1}^{r}[\beta_ix^{(i)^{\mathrm{T}}}(t)x^{(i)}(t)-\beta_ix^{(i)^{\mathrm{T}}}(t-h)x^{(i)}(t-h)]\\
&\leqslant\sum_{i=1}^{r}[-2\|x^{(i)}(t)\|^2+2\|P_iK_{ii}\|\|x^{(i)}(t)\|^2+2\|P_i\|\|M_{ii}\|\|x^{(i)}(t)\|\|x^{(i)}(t-h)\|\\
&\quad+\beta_ix^{(i)^{\mathrm{T}}}(t)x^{(i)}(t)-\beta_ix^{(i)^{\mathrm{T}}}(t-h)x^{(i)}(t-h)]\\
&\quad+2\sum_{i=1}^{r}\sum_{j=1,j\neq i}^{i}\|P_i\|(\|M_{ij}\|\|x^{(i)}(t)\|\|x^{(j)}(t)\|+\|\overline{M}_{ij}\|\|x^{(i)}(t)\|\|x^{(j)}(t-h)\|]\\
&=-\sum_{i=1}^{j}(\|x^{(i)}(t)\|,\|x^{(i)}(t-h)\|)\begin{bmatrix}1-\|P_iK_{ii}\| & -\|P_i\|\|M_{ii}\|\\ -\|P_i\|\|M_{ii}\| & 1-\|P_iK_{ii}\|\end{bmatrix}\begin{bmatrix}\|x^{(i)}(t)\|\\ \|x^{(i)}(t-h)\|\end{bmatrix}\\
&\quad+\sum_{i=1}^{r}\sum_{j=1,j\neq i}^{r}(\|x^{(i)}(t)\|,\|x^{(i)}(t-h)\|)\begin{bmatrix}\|P_i\|\|M_{ij}\|+\|P_j\|\|M_{ji}\| & \|P_i\|\|\overline{M}_{ij}\|\\ \|P_i\|\|\overline{M}_{ji}\| & 0\end{bmatrix}\\
&\quad\cdot\begin{bmatrix}\|x^{(j)}(t)\|\\ \|x^{(j)}(t-h)\|\end{bmatrix}\\
&\leqslant(\|z_1(t)\|,\|z_2(t)\|\cdots,\|z_r(t)\|)U(\|z_1(t)\|,\|z_2(t)\|\cdots,\|z_r(t)\|)^{\mathrm{T}}.
\end{aligned}$$

If the condition of the theorem is satisfied, then $\dot V(x(t))|_{(3.2)}<0$, the theorem is proved.

Since the system which discussed by Liu Y. Q. and Tang G. Y. is a special case of system (3.1), so Theorem 3.1 contains relevant results of work[5].

4 Example

Consider the time-delay system

$$\dot{X}(t) = G[B,C]X(t) + G[D,E]X(t-h)$$

where

$$G[B,C] = \begin{pmatrix} [-2.4, -1.8] & [0.8, 1.6] \\ [-2.2, -1.6] & [-0.8, 0.2] \end{pmatrix};$$

$$G[D,E] = \begin{pmatrix} [-0.15, 0.14] & [-0.1, 0.14] \\ [-0.12, 0.14] & [-0.16, 0.16] \end{pmatrix}.$$

Calculate

$$A_0 = \begin{pmatrix} -2.1 & 1.2 \\ -1.9 & -0.3 \end{pmatrix}; K = \begin{pmatrix} 0.3 & 0.4 \\ 0.3 & 0.5 \end{pmatrix}; M = \begin{pmatrix} 0.15 & 0.14 \\ 0.14 & 0.16 \end{pmatrix}; P \approx \begin{pmatrix} 0.9464 & -0.5198 \\ -0.5198 & 1.2543 \end{pmatrix};$$

$\|P\| \approx 1.6424$; $\|PK\| \approx 0.5018$; $\|M\| \approx 0.2951$; $\|P\|\|M\| \approx 0.4847$.

Let $\beta = 1 - \|PK\| \approx 0.4982$; since matrix

$$\begin{pmatrix} 0.4982 & -0.4847 \\ -0.4847 & 0.4982 \end{pmatrix}$$

is postive definite, according to our result in theorem 2.2, we can declare that this system is robustly stable.

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具有时滞的线性区间系统的鲁棒稳定性

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摘要:本文给出了线性区间时滞系统鲁棒稳定性的一些结果,这些结果推广和改进了前人关于线性时滞系统鲁棒稳定性的相关结论,同时还讨论了线性区间时滞系统的稳定度,最后讨论了线性区间时滞大系统的鲁棒稳定性.

关键词:区间矩阵;鲁棒稳定性;时滞;大系统

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