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Robust Convergence Analysis of Iterative Learning Control Systems *

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Abstract: A feedback-assisted filtered-version learning control algorithm is proposed for trajectory tracking of a general class of nonlinear systems. Without the aid of linearization, the robust convergence of this type of learning control algorithm is estabilished with respect to the existence of asymptotically repetitive initial conditions and periodic uncertainties.

Key words: initial condition; convergence; iterative learning control; nonlinear systems

1 Introduction

The problem of perfect trajectory tracking has received considerable attention in the fields of industrial robot manipulators and servomechanism systems. Among the existing studies, Sugie and Ono[1] noted the crucial role of the direct transmission term of the controlled plant and clarified the necessity of the use of error derivative in the learning control process for the plant without direct transmission term. From a practical point of view, the robust convergence of a learning control algorithm with respect to the existence of uncertainties and disturbances deserves special attention. Specifically, is the perfect tracking performance of a learning control algorithm achieved as some forms of uncertainties and disturbances are introduced? Recently, several researchers have answered this question. Ahn and Choi[2] proposed a learning controller for SISO LTI systems in the presence of a periodic disturbance. Hac[3] examined the properties of learning controllers for LTI systems with respect to the existence of periodic state disturbances and imperfect measurements. Porter and Mohamed[4] considered learning control of partially irregular MIMO LTI plants with initial state shifting. In addition, Choi and Jang[5] studied the learning control for a general class of nonlinear systems in the absence of uncertainties. However, the existing studies can present fragmentary results about such robust convergence problem for nonlinear systems. The objective of this paper is to examine the robust convergence properties of the proposed learning control algorithm for a general class of nonlinear systems in the presence of shifted initial conditions, state uncertainties and output disturbances.

2 Problem Formulation and Preliminaries

Consider a general class of nonlinear systems described by

$$\dot{x}_k(t) = f(x_k(t), u_k(t)) + \eta_k(t), \tag{1a}$$

$$y_k(t) = g(x_k(t), u_k(t)) + \zeta_k(t), \tag{1b}$$

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where k is the iterative index. For all $t \in [0,T]$ and $\forall\ k, x_k(t) \in \mathbb{R}^n$ is the state vector, $u_k(t) \in \mathbb{R}^m$ is the input vector, and $y_k(t) \in \mathbb{R}^m$ is the output vector. The functions $f: \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ and $g: \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^m$ are assumed to be unknown. Here, $\eta_k(t) \in \mathbb{R}^n$, and $\zeta_k(t) \in \mathbb{R}^m$ represent some unstructured uncertainties due to state fluctuations or periodic disturbances.

The problem is now formulated as follows. Given the system (1) with the uncertainties $\eta_k(t)$ and $\zeta_k(t)$, find, using a learning technique, an appropriate input sequence to generate an output trajectory $y_k(t), t \in [0,T]$ within the ε-bound of the desired trajectory $y_d(t), t \in [0,T]$.

For solving the problem, we adopt the feedback-assisted filtered-version update law

$$\dot{v}_k(t) = a(v_k(t)) + B(v_k(t))z_k(t), \tag{2a}$$

$$w_k(t) = c(v_k(t)) + D(v_k(t))z_k(t), \tag{2b}$$

$$z_{k+1}(t) = L_1(y_{k+1}(t),t)e_{k+1}(t), \tag{2c}$$

$$z_k(t) = L_2(y_k(t),t)e_k(t), \tag{2d}$$

$$u_{k+1}(t) = u_k(t) + \Gamma_1(y_{k+1}(t),t)w_{k+1}(t) + \Gamma_2(y_k(t),t)w_k(t), \tag{2e}$$

where, for all $t \in [0,T]$ and $\forall\ k, v_k(t) \in \mathbb{R}^r, w_k(t) \in \mathbb{R}^m$ and $z_k(t) \in \mathbb{R}^m$ are intermediate vectors; $e_k(t) = y_d(t) - y_k(t)$ is the output error. The initial condition $v_k(0)$ is assumed to be zero. Here $a(\cdot), B(\cdot), c(\cdot)$ and $D(\cdot)$ are the given matrices with appropriate dimensions. $L_1(\cdot,\cdot), L_2(\cdot,\cdot)$, and $\Gamma_1(\cdot,\cdot)$ and $\Gamma_2(\cdot,\cdot)$ are the gain matrices which are chosen to be bounded on $\mathbb{R}^m \times [0,T]$.

In order to make the problem more tractable, the learning control system described by (1) and (2) is restricted to have the following properties:

A1) For system (1), let S denote a mapping from $(x_k(0), u_k(t), \eta_k(t), t \in [0,T])$ to $(x_k(t), t \in [0,T])$ and O denote a mapping from $(x_k(0), u_k(t), \zeta_k(t), t \in [0,T])$ to $(y_k(t), t \in [0,T])$. Then, for each $x_k(0)$ with the uncertainties $\eta_k(t)$ and $\zeta_k(t)$, the state map S and output map O are one-to-one.

A2) The functions $f(\cdot,\cdot)$ and $g(\cdot,\cdot)$ are uniformly globally Lipschitz in x and u. That is, $\| h(x_1,u_1) - h(x_2,u_2) \| \leqslant k_h(\| x_1 - x_2 \| + \| u_1 - u_2 \|)$ for some constant $k_h, h \in \{f, g\}$.

A3) The functions $a(\cdot)$ and $c(\cdot)$ satisfy Linear Growth condition in v. That is, $\| h(v) \| \leqslant k_h \| v \|$ for some constant $k_h, h \in \{a, c\}$.

A4) The function $g(\cdot,\cdot)$ is differentiable in u, and for all $x \in \mathbb{R}^n, u^i \in \mathbb{R}^m, i = 1,\cdots,m$, the matrix

$$D_s(x,u^1,\cdots,u^m) = \begin{bmatrix} \frac{\partial}{\partial u} g_1(x,u^1) \\ \vdots \\ \frac{\partial}{\partial u} g_m(x,u^m) \end{bmatrix}$$

is bounded by the norm bound b_{Ds}.

A5) The functions $B(\cdot)$ and $D(\cdot)$ are uniformly bounded on $\mathbb{R}^r$ by the norm bounds b_B and b_D, respectively.

A6) For all k, the initial state $x_k(0)$ of the system is bounded. Moreover, the uncertainties $\eta_k(t)$ and $\zeta_k(t)$ are bounded on $[0,T]\ \forall k$.

To assist the presentation of our results we need the following lemma:

Lemma 1 Let d_k be a sequence of real number which converges to the limit d_∞ as $k \to \infty$. Suppose that a_k is a sequence of real number such that

$$pa_k + qa_{k-1} \leqslant d_k, \quad p > -q \geqslant 0.$$

Then $\limsup_{k\to\infty} a_k \leqslant \dfrac{d_\infty}{p+q}$.

Proof Defining $s_k = a_k - \dfrac{d_\infty}{p+q}$, $t_k = \dfrac{d_k - d_\infty}{p}$, and $\rho = -q/p$ yields $s_k \leqslant \rho s_{k-1} + t_k$. Since the sequence d_k is convergent, then for every $\varepsilon > 0$ there is a K such that, for all $k > K$, $|d_k - d_\infty| < p\varepsilon$. Thus, as $k > K$

$$s_k < \rho s_{k-1} + \varepsilon.$$

It follows from $0 \leqslant \rho < 1$ that

$$\limsup_{k\to\infty} s_k < \frac{\varepsilon}{1-\rho}.$$

Hence, $\limsup_{k\to\infty} s_k \leqslant 0$ due to ε being arbitrary. Now using the definition of s_k one can obtain the lemma.

For brevity, in the sequel, we denote $\Delta(\cdot)_k = (\cdot)_d - (\cdot)_k$, and $\delta(\cdot)_k = (\cdot)_{k+1} - (\cdot)_k$.

3 Input Error Convergence

In this section we prove the uniform convergence of learning control algorithm (2) with $\Gamma_1 = I$ and $\Gamma_2 = I$. Assumption A1) implies that given the desired trajectory $y_d, t \in [0,T]$ for system (1) with initial condition $x_d(0)$ and uncertainties $\eta_d(t)$ and $\zeta_d(t)$, there exists a control input $u_d(t), t \in [0,T]$ such that the following differential equations are satisfied

$$\dot{x}_d(t) = f(x_d(t), u_d(t)) + \eta_d(t), \tag{3a}$$

$$y_d(t) = g(x_d(t), u_d(t)) + \zeta_d(t). \tag{3b}$$

We now state one of the main results as follows:

Theorem 1 Let the system described by (1) satisfy the assumptions (A1)~A6)) and use the update law (2) with $\Gamma_1 = I$ and $\Gamma_2 = I$. Assume that the gain matrices L_1 and L_2 are chosen such that

$$\beta/\alpha < 1,$$

where $\alpha = 1/\sup_{t\in[0,T]} \| [I + D(v(t))L_1(y(t),t)D_s(x(t),u^1(t),\cdots,u^m(t))]^{-1} \|$, and $\beta = \sup_{t\in[0,T]} \| I - D(v(t))L_2(y(t),t)D_s(x(t),u^1(t),\cdots,u^m(t)) \|$. Then the input error $u_d(t) - u_k(t)$ converges to zero uniformly for all $t \in [0,T]$ as $k \to \infty$ whenever $\lim_{k\to\infty} x_k(0) = x_d(0)$, $\lim_{k\to\infty} \eta_k(t) = \eta_d(t)$ and $\lim_{k\to\infty} \zeta_k(t) = \zeta_d(t)$ for all $t \in [0,T]$.

Proof It follows from (1b), (2) and (3b) that

$$\begin{aligned}
\Delta u_{k+1} &= \Delta u_k - w_{k+1} - w_k \\
&= \Delta u_k - c(v_{k+1}) - D(v_{k+1})L_1(y_{k+1},t)e_{k+1} - c(v_k) - D(v_k)L_2(y_k,t)e_k \\
&= \Delta u_k - c(v_{k+1}) - c(v_k) \\
&\quad - D(v_{k+1})L_1(y_{k+1},t)\{g(x_{k+1},u_d) - g(x_{k+1},u_{k+1}) + g(x_d,u_d) - g(x_{k+1},u_d) + \Delta\zeta_{k+1}\}
\end{aligned}$$

$$- D(v_k)L_2(y_k,t)\{g(x_k,u_d) - g(x_k,u_k) + g(x_d,u_d) - g(x_k,u_d) + \Delta\zeta_k\}. \quad (4)$$

Using the mean value theorem, (4) gives

$$\begin{aligned}&[I + D(v_{k+1})L_1(y_{k+1},t)D_{s(k+1)}]\Delta u_{k+1}\\ &= [I - D(v_k)L_2(y_k,t)D_{sk}]\Delta u_k - c(v_{k+1}) - c(v_k)\\ &\quad - D(v_{k+1})L_1(y_{k+1},t)\{g(x_d,u_d) - g(x_{k+1},u_d) + \Delta\zeta_{k+1}\}\\ &\quad - D(v_k)L_2(y_k,t)\{g(x_d,u_d) - g(x_k,u_d) + \Delta\zeta_k\},\end{aligned} \quad (5)$$

where $D_{sk} = D_s(x_k,u^1,\cdots,u^m), u^i \in \{\theta u_d + (1-\theta)u_k\}, 0 \leqslant \theta \leqslant 1$; and $D_{s(k+1)} = D_s(x_{k+1},u^1,\cdots,u^m), u^i \in \{\theta u_d + (1-\theta)u_{k+1}\}, 0 \leqslant \theta \leqslant 1$. Then, taking the norms of (5) yields

$$\begin{aligned}\|\Delta u_{k+1}\| \leqslant\; &\|[I + D(v_{k+1})L_1(y_{k+1},t)D_{s(k+1)}]^{-1}\|\{\|I - D(v_k)L_2(y_k,t)D_{sk}\|\,\|\Delta u_k\|\\ &+ \|c(v_{k+1})\| + \|c(v_k)\|\\ &+ \|D(v_{k+1})\|\,\|L_1(y_{k+1},t)\|(\|g(x_d,u_d) - g(x_{k+1},u_d)\| + \|\Delta\zeta_{k+1}\|)\\ &+ \|D(v_k)\|\,\|L_2(y_k,t)\|(\|g(x_d,u_d) - g(x_k,u_d)\| + \|\Delta\zeta_k\|)\},\end{aligned} \quad (6)$$

and using the bounds, Lipschitz conditions and Linear Growth conditions, we have

$$\begin{aligned}\alpha\|\Delta u_{k+1}\| \leqslant\; &\beta\|\Delta u_k\| + b_1(\|\Delta x_{k+1}\| + \|v_{k+1}\|)\\ &+ b_1(\|\Delta x_k\| + \|v_k\|) + b_2(\|\Delta\zeta_{k+1}\| + \|\Delta\zeta_k\|),\end{aligned} \quad (7)$$

where $b_1 = \max\{b_D b_{L1} k_g, b_D b_{L2} k_g, k_c\}$ and $b_2 = \max\{b_D b_{L1}, b_D b_{L2}\}$. Here, b_{L1} and b_{L2} are the norm bounds for $L_1(\cdot,\cdot)$ and $L_2(\cdot,\cdot)$, respectively. Now, integrating (1a) and (3a) gives

$$\begin{aligned}\|\Delta x_k\| &= \|\Delta x_k(0) + \int_0^t \{f(x_d,u_d) - f(x_k,u_k) + \Delta\eta_k\}d\tau\|\\ &\leqslant \|\Delta x_k(0)\| + \int_0^t (k_f\|\Delta x_k\| + k_f\|\Delta u_k\| + \|\Delta\eta_k\|)d\tau,\end{aligned} \quad (8)$$

and then integrating (2a) gives

$$\begin{aligned}\|v_k\| &= \|\int_0^t \{a(v_k) + B(v_k)L_2(y_k,\tau)e_k\}d\tau\| \leqslant \int_0^t (k_a\|v_k\| + b_B b_{L2}\|e_k\|)d\tau\\ &\leqslant \int_0^t \{k_a\|v_k\| + b_B b_{L2}(k_g\|\Delta x_k\| + b_{Ds}\|\Delta u_k\| + \|\Delta\zeta_k\|)\}d\tau.\end{aligned} \quad (9)$$

Adding (8) and (9), it follows that

$$\begin{aligned}\|\Delta x_k\| + \|v_k\| \leqslant \|\Delta x_k(0)\| + \int_0^t \{&b_3(\|\Delta x_k\| + \|v_k\|) + b_4\|\Delta u_k\|\\ &+ b_5(\|\Delta\eta_k\| + \|\Delta\zeta_k\|)\}d\tau,\end{aligned} \quad (10)$$

where $b_3 = \max\{k_f + b_B b_{L1} k_g, k_f + b_B b_{L2} k_g, k_a\}$, $b_4 = \max\{k_f + b_B b_{L1} b_{Ds}, k_f + b_B b_{L2} b_{Ds}\}$ and $b_5 = \max\{b_B b_{L1}, b_B b_{L2}, 1\}$. By applying Bellman-Gronwall lemma to (10), we obtain the following relation

$$\|\Delta x_k\| + \|v_k\| \leqslant \|\Delta x_k(0)\|e^{b_3 t} + \int_0^t e^{b_3(t-\tau)}\{b_4\|\Delta u_k\| + b_5(\|\Delta\eta_k\| + \|\Delta\zeta_k\|)\}d\tau. \quad (11)$$

combining (7) and (11) gives rise to

$$\begin{aligned}\alpha\|\Delta u_{k+1}\| \leqslant\; &\beta\|\Delta u_k\| + b_1 b_4\int_0^t e^{b_3(t-\tau)}(\|\Delta u_{k+1}\| + \|\Delta u_k\|)d\tau\\ &+ b_1 e^{b_3 t}(\|\Delta x_{k+1}(0)\| + \|\Delta x_k(0)\|)\\ &+ b_1 b_5\int_0^t e^{b_3(t-\tau)}(\|\Delta\eta_{k+1}\| + \|\Delta\eta_k\| + \|\Delta\zeta_{k+1}\| + \|\Delta\zeta_k\|)d\tau\end{aligned}$$

$$+ b_2(\| \Delta\zeta_{k+1} \| + \| \Delta\zeta_k \|). \tag{12}$$

Multipling both sides of (12) by $e^{-\lambda t}(\lambda > b_3), t \in [0,T]$, and applying the definition of the λ-norm, we have

$$\bar{\alpha} \| \Delta u_{k+1} \|_\lambda \leqslant \bar{\beta} \| \Delta u_k \|_\lambda + \varepsilon_{k+1}, \tag{13}$$

where $\bar{\alpha} = \alpha - b_1 b_4 \frac{1 - e^{(b_3-\lambda)T}}{\lambda - b_3}, \bar{\beta} = \beta + b_1 b_4 \frac{1 - e^{(b_3-\lambda)T}}{\lambda - b_3}$, and $\varepsilon_{k+1} = b_1(\| \Delta x_{k+1}(0) \| + \| \Delta x_k(0) \|) + b_1 b_5 \frac{1 - e^{(b_3-\lambda)T}}{\lambda - b_3}(\| \Delta\eta_{k+1} \|_\lambda + \| \Delta\eta_k \|_\lambda) + b_1 b_5 \frac{1 - e^{(B_3-\lambda)T}}{\lambda - b_3}(\| \Delta\zeta_{k+1} \|_\lambda + \| \Delta\zeta_k \|_\lambda) + b_2(\| \Delta\zeta_{k+1} \|_\lambda + \| \Delta\zeta_k \|_\lambda)$.

Since $\beta < \alpha$, we can choose λ large enough such that $\bar{\beta} < \bar{\alpha}$. By Lemma 1, the control input error is thus bounded as

$$\limsup_{k\to\infty} \| \Delta u_k \|_\lambda \leqslant \frac{\lim_{k\to\infty} \varepsilon_k}{\alpha - \beta}. \tag{14}$$

It follows immediatelly that the right-hand term of (14) will tend to zero as $\lim_{k\to\infty} x_k(0) = x_d(0), \lim_{k\to\infty} \eta_k(t) = \eta_d(t)$ and $\lim_{k\to\infty} \zeta_k(t) = \zeta_d(t)$ for all $t \in [0,T]$. Moreover, from the definition of the λ-norm, we see that $\sup_{t\in[0,T]} \| \Delta u_k \| \to 0$ as $k \to \infty$. The proof is complete.

Remark 1 To obtain the convergence result for $\| e_k \|$ we can use the following relations

$$\| \Delta x_k \|_\lambda \leqslant \| \Delta x_k(0) \| + (k_f \| \Delta u_k \|_\lambda + \| \Delta\eta_k \|_\lambda) \frac{1 - e^{(k_f-\lambda)T}}{\lambda - k_f} \tag{15}$$

as $\lambda > k_f$ and

$$\| e_k \|_\lambda \leqslant k_g \| \Delta x_k \|_\lambda + b_{Ds} \| \Delta u_k \|_\lambda + \| \Delta\zeta_k \|_\lambda, \tag{16}$$

with $\| \Delta u_k \|$ being convergent as stated in Theorem 1.

4 Output Error Convergence

In this section, we restrict our consideration to a simpler form of update law by replacing z_k with e_k in (2a), (2b). This learning algorithm has the following convergence properties.

Theorem 2 Let the system described by (1) satisfy the assumptions (A1)~A6)) and use the update law (2) with $L_1 = I$ and $L_2 = I$. Under the condition that the uncertainties are asymptotically repetitive, i. e., for all $t \in [0,T]$, there exist the specified vectors $x^0, \eta^0(t)$ and $\zeta^0(t)$ such that $\lim_{k\to\infty} x_k(0) = x^0, \lim_{k\to\infty} \eta_k(t) = \eta^0(t)$ and $\lim_{k\to\infty} \zeta_k(t) = \zeta^0(t)$, and the gain matrices Γ_1 and Γ_2 are chosen such that

$$\beta/\alpha < 1,$$

where $\alpha = 1/\sup_{t\in[0,T]} \| [I + D_s(x(t), u^1(t), \cdots, u^m(t))\Gamma_1(y(t),t)D(v(t))]^{-1} \|$, and $\beta = \sup_{t\in[0,T]} \| I - D_s(x(t), u^1(t), \cdots, u^m(t))\Gamma_2(y(t),t)D(v(t)) \|$. Then the output error $e_k(t)$ converges to zero uniformly on $[0,T]$ as $k \to \infty$.

Proof It follows from (1b) and (2) that

$$\begin{aligned} e_{k+1} &= e_k - (y_{k+1} - y_k) \\ &= e_k - \{g(x_{k+1}, u_{k+1}) - g(x_k, u_k) + \delta\zeta_k\}. \end{aligned} \tag{17}$$

Using the mean value theorem, (17) gives

$$\begin{aligned}&[I + D_{sk}\Gamma_1(y_{k+1},t)D(v_{k+1})]e_{k+1}\\&= [I - D_{sk}\Gamma_2(y_k,t)D(v_k)]e_k\\&\quad - \{g(x_{k+1},u_{k+1}) - g(x_k,u_{k+1}) + D_{sk}\Gamma_1(y_{k+1},t)c(v_{k+1})\\&\quad + D_{sk}\Gamma_2(y_k,t)c(v_k) + \delta\zeta_k\},\end{aligned} \tag{18}$$

where $D_{sk} = D_s(x_k,u^1,\cdots,u^m), u^i \in \{\theta u_k + (1-\theta)u_{k+1}\}, 0 \leqslant \theta \leqslant 1$. Then, taking the norms of (18) yields

$$\begin{aligned}\|e_{k+1}\| \leqslant &\|[I + D_{sk}\Gamma_1(y_{k+1},t)D(v_{k+1})]^{-1}\|\{\|I - D_{sk}\Gamma_2(y_k,t)D(v_k)\|\|e_k\|\\&+ \|g(x_{k+1},u_{k+1}) - g(x_k,u_{k+1})\| + \|D_{sk}\|\|\Gamma_1(y_{k+1},t)\|\|c(v_{k+1})\|\\&+ \|D_{sk}\|\|\Gamma_2(y_k,t)\|\|c(v_k)\| + \|\delta\zeta_k\|\},\end{aligned} \tag{19}$$

and using the bounds, Lipschitz conditions and Linear Growth conditions, we have

$$\alpha\|e_{k+1}\| \leqslant \beta\|e_k\| + b_1(\|\delta x_k\| + \|v_{k+1}\| + \|v_k\|) + \|\delta\zeta_k\|, \tag{20}$$

where $b_1 = \max\{b_D b_{L1} k_c, b_D b_{L2} k_c, k_g\}$. Now, integrating (1a) gives

$$\begin{aligned}\|\delta x_k\| &= \|\delta x_k(0) + \int_0^t\{f(x_{k+1},u_{k+1}) - f(x_k,u_k) + \delta\eta_k\}\mathrm{d}\tau\|\\&\leqslant \|\delta x_k(0)\| + \int_0^t\{k_f\|\delta x_k\| + k_f b_{\Gamma1}(k_c\|v_{k+1}\| + b_D\|e_{k+1}\|)\\&\quad + k_f b_{\Gamma2}(k_c\|v_k\| + b_D\|e_k\|) + \|\delta\eta_k\|\}\mathrm{d}\tau,\end{aligned} \tag{21}$$

where $b_{\Gamma1}$ and $b_{\Gamma2}$ are the norm bounds for $\Gamma_1(\cdot,\cdot)$ and $\Gamma_2(\cdot,\cdot)$, respectively, and then integrating (2a) gives

$$\|v_k\| \leqslant \int_0^t(k_a\|v_k\| + b_B\|e_k\|)\mathrm{d}\tau. \tag{22}$$

Combining (21) and (22), it follows that

$$\begin{aligned}\|\delta x_k\| + \|v_{k+1}\| + \|v_k\| \leqslant &\|\delta x_k(0)\| + \int_0^t\{b_2(\|\delta x_k\| + \|v_{k+1}\| + \|v_k\|)\\&+ b_3(\|e_{k+1}\| + \|e_k\|) + \|\delta\eta_k\|\}\mathrm{d}\tau,\end{aligned} \tag{23}$$

where $b_2 = \max\{k_f, k_f b_{\Gamma1}k_c + k_a, k_f b_{\Gamma2}k_c + k_a\}$, and $b_3 = \max\{k_f b_{\Gamma1}b_D + b_B, k_f b_{\Gamma2}b_D + b_B\}$. By applying Bellman-Gronwall lemma to (23), we obain

$$\begin{aligned}&\|\delta x_k\| + \|v_{k+1}\| + \|v_k\|\\&\leqslant \|\delta x_k(0)\|\mathrm{e}^{b_2 t} + \int_0^t \mathrm{e}^{b_2(t-\tau)}\{b_3(\|e_{k+1}\| + \|e_k\|) + \|\delta\eta_k\|\}\mathrm{d}\tau.\end{aligned} \tag{24}$$

Substituting (24) into (20) gives rise to

$$\begin{aligned}\alpha\|e_{k+1}\| \leqslant &\beta\|e_k\| + b_1 b_3\int_0^t \mathrm{e}^{b_2(t-\tau)}(\|e_{k+1}\| + \|e_k\|)\mathrm{d}\tau\\&+ b_1\mathrm{e}^{b_2 t}\|\delta x_k(0)\| + b_1\int_0^t \mathrm{e}^{b_2(t-\tau)}\|\delta\eta_k\|\mathrm{d}\tau + \|\delta\zeta_k\|.\end{aligned} \tag{25}$$

Multipling $\mathrm{e}^{-\lambda t}$, with $\lambda > b_2$, to both sides of (25), we have

$$\bar{\alpha}\|e_{k+1}\|_\lambda \leqslant \bar{\beta}\|e_k\|_\lambda + \varepsilon_{k+1}, \tag{26}$$

where $\bar{\alpha} = \alpha - b_1 b_3\dfrac{1 - \mathrm{e}^{(b_2-\lambda)T}}{\lambda - b_2}$, $\bar{\beta} = \beta + b_1 b_3\dfrac{1 - \mathrm{e}^{(b_2-\lambda)T}}{\lambda - b_2}$, and $\varepsilon_{k+1} = b_1\|\delta x_k(0)\| + b_1\dfrac{1 - \mathrm{e}^{(b_2-\lambda)T}}{\lambda - b_2}\|\delta\eta_k\|_\lambda + \|\delta\zeta_k\|_\lambda$.

Since $\beta < \alpha$, we can choose λ large enough such that $\bar{\beta} < \bar{\alpha}$. By Lemma 1, the output error is thus bounded as

$$\limsup_{k\to\infty} \| e_k \|_\lambda \leqslant \frac{\lim\limits_{k\to\infty} \varepsilon_k}{\alpha - \beta} . \tag{27}$$

Now it is easy to obtain the theorem in view of the conditions.

Remark 2 From Theorem 1 and Theorem 2, we see that the convergence of learning algorithm (2) only requires that $x_k(0)$, $\eta_k(t)$, and $\zeta_k(t)$ are asymptotically invariant after sufficient trials. It is a milder convergence condition than that required in the previous literature.

Remark 3 Using the update law which is of the form of linear time-varying filter, Sugie and Ono[1] derived a sufficient condition for the nonlinear systems with the dynamical part $\{a(t,x), B(t), c(t,x), D(t)\}$, and claimed the validity of their results if the direct transmission term D depends on the state variable x but did not give any rigorous proof, which implies that Theorem 2 is thus a natural extension of the previous results.

5 Concluding Remarks

In this paper, the proposed feedback-assisted filtered-version learning control algorithm have been characterized by assuming the asymptotic repeatability of initialization and uncertainties. The robust convergence proofs have been provided for a general class of nonlinear systems. Particularly , the sufficient conditions for uniform convergence can be easily satisfied by choosing the gain matrices in the presented update law.

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迭代学习控制系统的鲁棒收敛性分析

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摘要:本文提出一种开闭环配合的滤波器型迭代学习控制算法,并将这种算法应用于一般非线性动态系统的轨迹跟踪.对于渐近重复初始条件和渐近周期干扰的情形,通过控制误差估计和输出误差估计,文中分别证明了学习过程的一致收敛性.证明中未采用线性化手段.

关键词:初始条件;收敛性;迭代学习控制;非线性系统

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