

## Robust Output Tracking for Mobile Robot \*

LI Qingxiang, HU Yueming, PEI Hailong and ZHOU Qijie

(Department of Automatic Control Engineering, South China University of Technology, Guangzhou, 510641, PRC)

**Abstract:** In this paper, the robust output tracking problem of a kind of uncertain nonholonomic systems is discussed. The reduced dimensional state realization and main properties are given under some assumptions. Then the simplified state space model of 3-wheel mobile robot is given under the conditions of pure rolling and non slipping. Finally, the detailed robust output tracking control law based on the simplified model is presented.

**Key words:** mobile robot; nonholonomic system; output tracking; variable structure control; robustness

### 1 Introduction

The control of many practical systems needs to concern about the contact with the environment, especially the control of mechanical systems, this kind of systems are called constrained systems. They are constrained by a group of kinetic equations as they are modeled by a group of general coordinates. These constraints can be classified into holonomic and nonholonomic depend on whether they can be integrated or not. Holonomic constraints limit only the plants' geometric position, therefore we can solve out some variables from them directly, then reduce the constrained system to a lower order system without constraints, thus their control problems have no more difficulties than systems without constraints and many research work can be found<sup>[1~4]</sup>. But in the case of nonholonomic constraints, we can not reduce the system's order by means of solving the variables using constraints which have to be considered into the kinetic equations of the system, thus the problem is complicated and cannot be resolved effectively by the original method<sup>[3~7]</sup>. Moreover, nonholonomic systems have some new properties such as their input-output linearization can be realized not by smooth state feedback, but by the appropriate output projection, and their asymptotic stability under the Lyapunov meaning can be realized not by the smooth state feedback but by the nonsmooth or time-varying state feedback<sup>[3,6]</sup>, these properties show the new challenges to the control field.

Up to now almost all the works about the nonholonomic systems are focus on the certain models, but in fact there always exist neglected dynamics and perturbation in plant modeling. Therefore it has considerable theoretical and practical meaning to take out the research on the robust control of nonlinear uncertain systems subjected to holonomic or nonholonomic constraints.

In recent years, variable structure control method has been applied to the control problem of many kinds of uncertain systems since it has the properties such as robustness to dis-

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turbance, simple to simulate and realize. There must be of certain research value to introduce variable structure control method into the nonholonomic systems which Lyapunov asymptotic stability can be realized only by the non-smooth or time-varying state feedback. In this paper we first employ the method similar to nonlinear system input-output linearization and present the reduced state realization and some of its properties for constrained systems; then apply the variable structure control method to robust output tracking problem; finally we discuss the robust output tracking control law for the simplified model of three-wheel mobile robot under pure rolling and non slipping conditions.

## 2 Reduced State Realization and Its Properties

The uncertain constrained system expressed by nonlinear differential-algebra equations is as follows<sup>[4,7]</sup>

$$\dot{x} = f(x) + \Delta f(x) + (g(x) + \Delta g(x))v + (h(x) + \Delta h(x))u, \quad (1)$$

$$y = C(x), \quad (2)$$

$$z = J(x) = 0, \quad (3)$$

where  $x \in M$ ,  $M$  is an open subset of  $\mathbb{R}^n$ ,  $u \in \mathbb{R}^r$  and  $v \in \mathbb{R}^m$  are control input and constrained input vector respectively;  $y \in \mathbb{R}^r$  is output variable;  $g(x) = (g_1(x), \dots, g_m(x))$ ;  $h(x) = (h_1(x), \dots, h_r(x))$ ;  $J(x) = (J_1(x), \dots, J_m(x))$ ;  $(J_1, \dots, J_m)$  are constraint functions;  $C(x) = (C_1(x), \dots, C_r(x))^T$ ;  $J(x)$  and  $C(x)$  are smooth mapping on  $M$ ;  $f(x), g_i(x) (i = 1, \dots, m)$  and  $h_i(x) (i = 1, \dots, r)$  are smooth vector fields on  $M$ ; we use  $\Delta f(x), \Delta g(x)$  and  $\Delta h(x)$  to denote the uncertain or unknown perturbation of  $f(x), g(x)$  and  $h(x)$  respectively. The variables in the vectors and mappings will be ignored without confusion in the following discussion.

The above constrained system represents the practical control problem of a number of physical systems which include mechanical systems with classical holonomic or nonholonomic constraints, such as mobile robot and other wheeled mobile instruments and so on<sup>[3,5-7]</sup>.

First we assume that the system has relative degree  $\alpha_i (i = 1, \dots, m)$  and uncertain part does not change system's relative degree<sup>[7]</sup>.

Assuming  $z$  of (3) is the output variable, so its  $i$ th component  $z_i$  satisfies

$$\begin{cases} z_i^{(j)} = L_f^j J_i, & j = 0, 1, \dots, \alpha_i - 1, \\ z_i^{(\alpha_i)} = L_f^{\alpha_i} J_i + (P_i + \Delta P_i)v + (dL_f^{\alpha_i-1} J_i)[\Delta f + (h + \Delta h)u]. \end{cases} \quad (4)$$

From above assumption and constrained condition (3) we obtain

$$v = -(P + \Delta P)^{-1} \{W_1 + W_2[\Delta f + (h + \Delta h)u]\} \quad (5)$$

where

$$W_1 = \begin{bmatrix} L_f^{\alpha_1} J_1 \\ \vdots \\ L_f^{\alpha_m} J_m \end{bmatrix}, \quad W_2 = \begin{bmatrix} dL_f^{\alpha_1-1} J_1 \\ \vdots \\ dL_f^{\alpha_m-1} J_m \end{bmatrix}.$$

Substituting (5) into (1) we have

$$\dot{x} = F(x) + \Delta F(x) + [H(x) + \Delta H(x)]u \quad (6)$$

where

$$\begin{aligned}
 F(x) &= f(x) - gP^{-1}W_1, \quad H(x) = h(x) - gP^{-1}W_2h(x), \\
 \Delta F(x) &= \Delta f - (g + \Delta g)(P + \Delta P)^{-1}W_2\Delta f + [gP^{-1} - (g + \Delta g)(P + \Delta P)^{-1}]W_1, \\
 \Delta H(x) &= \Delta h - (g + \Delta g)(P + \Delta P)^{-1}W_2\Delta h + [gP^{-1} - (g + \Delta g)(P + \Delta P)^{-1}]W_2h.
 \end{aligned}$$

The constrained input  $v$  is no longer in (6). From reference [4] we know that  $N^* = \{x \in M \mid L_f^j J_i = 0, j = 0, 1, \dots, \alpha_i - 1, i = 1, \dots, m\}$  is a  $n - \sum_{i=1}^m \alpha_i$  dimensional submanifold and is the state space of (1)~(3). We assume that (6) and (2) have the relative degree  $\mu_i (i = 1, \dots, r)$  to derive state equations on this manifold.

Under above assumptions the  $i$ th component of output  $y$  satisfies

$$\begin{cases}
 y_i^{(j)} = L_f^j C_i, & j = 0, 1, \dots, \mu_i - 1, \\
 y_i^{(\mu_i)} = L_f^{(\mu_i)} C_i + Q_i u + (dL_f^{\mu_i - 1} C_i)(\Delta F + \Delta H u).
 \end{cases} \quad (7)$$

Employ the following coordinates transformation

$$\begin{bmatrix} \tilde{x} \\ \bar{x} \\ \xi \end{bmatrix} = T(x) = \begin{bmatrix} T_1(x) \\ T_2(x) \\ T_3(x) \end{bmatrix} \triangleq \begin{bmatrix} T_{12}(x) \\ T_3(x) \end{bmatrix} \quad (8)$$

where  $\tilde{x} \in \mathbb{R}^a (a = \sum_{i=1}^m \alpha_i)$ ;  $\bar{x} \in \mathbb{R}^\mu (\mu = \sum_{i=1}^r \mu_i)$ ;  $\xi \in \mathbb{R}^{n-\mu-a}$  are defined as follows respectively

$$\begin{aligned}
 \tilde{x} &= (\tilde{x}_1^1, \dots, \tilde{x}_{\alpha_1}^1, \dots, \tilde{x}_1^m, \dots, \tilde{x}_{\alpha_m}^m)^T; & \tilde{x}_j^i &= L_f^{j-1} J_i, j = 1, \dots, \alpha_i, i = 1, \dots, m; \\
 \bar{x} &= (\bar{x}_1^1, \dots, \bar{x}_{\mu_1}^1, \dots, \bar{x}_1^r, \dots, \bar{x}_{\mu_r}^r)^T; & \bar{x}_j^i &= L_f^{j-1} C_i, j = 1, \dots, \mu_i, i = 1, \dots, r; \\
 \xi &= (\xi_1, \dots, \xi_{n-\mu-a})^T; \\
 \xi_i &= T_{3i}(x) \text{ and } (dT_{3i}(x))^T \in N\{dT_{12}(x)\}, i = 1, \dots, n - \mu - a;
 \end{aligned}$$

where  $N\{\cdot\}$  denotes the null space.  $dT_{12}(x)$  is linear independent therefore the  $T_3(x)$  can be chosen appropriately to guarantee that (8) is a diffeomorphism<sup>[4]</sup>.

By the hypothesis of relative degree and applying above diffeomorphism (8), the  $(n - a)$ -dimensional state realization of constrained system (1)~(3) is

$$\begin{cases}
 \dot{\tilde{x}}_j^i = \tilde{x}_{j+1}^i, & j = 1, \dots, \mu_i - 1, \quad i = 1, \dots, r; \\
 \dot{\tilde{x}}_{\mu_i}^i = L_f^{\mu_i} C_i + Q_i u + (dL_f^{\mu_i - 1} C_i)(\Delta F + \Delta H u), & i = 1, \dots, r; \\
 \dot{\xi} = (dT_3(x))[F + \Delta F + (H + \Delta H)u], \\
 y_i = \bar{x}_1^i, \quad i = 1, \dots, r
 \end{cases} \quad (9)$$

where

$$x = T^{-1} \begin{bmatrix} 0 \\ \bar{x} \\ \xi \end{bmatrix}. \quad (10)$$

Assume the null dynamic system (i. e. the subsystem about  $\xi$  when  $\bar{x} = 0$ .) of (9) is BIBS (Boundary-Input/Boundary-State) stable, we then continue to discuss the robust output tracking problem of mobile robot with uncertain disturbance.

### 3 Robust Output Tracking

Denote  $y_d = (y_{d1}, \dots, y_{dr})^T$  as a reference output trajectory which is sufficient smooth, our objective is to design a control input  $u$  to force the output  $y$  of constrained system (1)~(3) to

track  $y_d$ . We apply variable structure control method. Choose  $r$  of switching functions  $S_i$  as<sup>[7]</sup>

$$S_i = \sum_{j=0}^{\mu_i-2} \beta_{ij} (L_F^j C_i - y_{d_i}^{(j)}) + L_F^{\mu_i-1} C_i - y_{d_i}^{(\mu_i-1)}, \quad i = 1, \dots, r \tag{11}$$

where  $\beta_{ij}$  are positive numbers, let  $e_i = y_i - y_{d_i}$  ( $i = 1, \dots, r$ ) to be the tracking error component, thus from (7) and (11) we know that after system entering sliding modes there is  $S_i = 0$ , i. e.

$$e_i^{(\mu_i-1)} + \sum_{j=0}^{\mu_i-2} \beta_{ij} e_i^{(j)} = 0, \quad i = 1, \dots, r \tag{12}$$

therefore the sliding mode movement is independent with disturbance and the error of output tracking is decoupled. For any  $i$ , appropriate  $\beta_{ij}$  can be chosen to ensure the eigenvalues of (12) have assigned distribution, hence we can guarantee output tracking error to approach zero by factor of assigned exponent.

Then we design proper variable structure control law to guarantee the realization of above sliding mode.

We have

$$\begin{aligned} \dot{S}_i &= \sum_{j=0}^{\mu_i-2} \beta_{ij} e_i^{(j+1)} + e_i^{(\mu_i)} \\ &= \sum_{j=0}^{\mu_i-2} \beta_{ij} e_i^{(j+1)} + L_F^{\mu_i} C_i + (Q_i + \Delta Q_i)u + \Delta R_i - y_{d_i}^{(\mu_i)} \end{aligned} \tag{13}$$

with

$$\Delta Q_i = (dL_F^{\mu_i-1} C_i) \Delta H, \quad \Delta R_i = (dL_F^{\mu_i-1} C_i) \Delta F, \quad i = 1, \dots, r$$

i. e.

$$\dot{S} = (\dot{S}_1, \dots, \dot{S}_r)^T = e_S + W + (Q + \Delta Q)u + \Delta R \tag{14}$$

where

$$e_S = \begin{bmatrix} \sum_{j=0}^{\mu_1-2} \beta_{1j} e_1^{(j+1)} - y_{d_1}^{(\mu_1)} \\ \vdots \\ \sum_{j=0}^{\mu_r-2} \beta_{rj} e_r^{(j+1)} - y_{d_r}^{(\mu_r)} \end{bmatrix}, \quad W = \begin{bmatrix} L_F^{\mu_1} C_1 \\ \vdots \\ L_F^{\mu_r} C_r \end{bmatrix}, \quad \Delta Q = \begin{bmatrix} \Delta Q_1 \\ \vdots \\ \Delta Q_r \end{bmatrix}, \quad \Delta R = \begin{bmatrix} \Delta R_1 \\ \vdots \\ \Delta R_r \end{bmatrix}.$$

Assume there are known constants  $\gamma_0, \gamma_1$  such that

$$\| \Delta Q Q^{-1} \| \leq \gamma_0 < 1, \quad \| \Delta R \| \leq \gamma_1 \tag{15}$$

where  $\| \cdot \|$  denote the norms of following matrix or vector

$$\| (d_{ij})_{p \times q} \| = \sum_{i=1}^p \sum_{j=1}^q |d_{ij}|. \tag{16}$$

Choose variable structure control law as

$$u = -Q^{-1} [e_S + W + k \operatorname{sgn}(S)] \tag{17}$$

where  $k$  satisfies

$$k > (1 - \gamma_0)^{-1}(\gamma_1 + \gamma_0 \|e_s + W\|) \tag{18}$$

then from (14)~(18) we obtain

$$\begin{aligned} S^T \dot{S} &= S^T \{ \Delta R - k \operatorname{sgn}(S) - \Delta Q Q^{-1} (e_s + W + k \operatorname{sgn}(S)) \} \\ &\leq -k \|S\| + \gamma_0 k \|S\| + \|S\| (\gamma_1 + \gamma_0 \|e_s + W\|) \\ &= -(1 - \gamma_0) [k - (1 - \gamma_0)^{-1} (\gamma_1 + \gamma_0 \|e_s + W\|)] \|S\| \end{aligned}$$

thus  $S^T \dot{S} < 0$ .

Hence the condition of arriving sliding mode is satisfied and the tracking error approaches zero exponentially.

The general method of realizing robust output tracking for nonlinear uncertain constrained system is given above, the condition of relative degree is common to nonlinear systems. It is easy to realize input-output linearization without disturbance but it is very difficult for input-state linearization.

### 4 Application on Mobile Robot

Consider the three-wheel mobile robot in reference [5], which motion is completely described by vector of 7 general coordinates as  $q = (x, y, \theta, \beta, \varphi_1, \varphi_2, \varphi_3)^T$ , and is shown in Fig. 1. It employs two motors to provide the torque for the rotation of wheel 2 and 3;  $m_i$  is the mass of wheel  $i$ ;  $M$  is the mass of the trolley (including the loads);  $I_{ri}$  is inertia moment of wheel  $i$  around its axis of rotation,  $I_{pi}$  is inertia moment of wheel  $i$  around the vertical axis passing through its perpendicular axis;  $I_0$  is the inertia moment of the trolley around the vertical axes passing through its center of mass;  $e_1, e_2$  are coordinates of the center of mass of the trolley in the frame attached to the trolley  $\{Q, I_1, I_2\}$ .

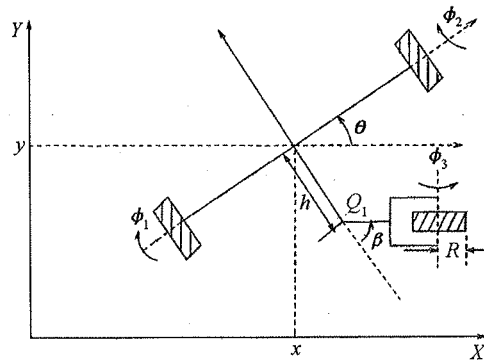


Fig. 1 Three-wheel mobile robot

In the case of pure rolling contact and no slipping between wheels and ground, we obtain 5 independent motion constraint equations

$$z = A(\beta)R(\theta)\dot{q} = 0 \tag{19}$$

with

$$A(\beta) = \begin{bmatrix} -\sin\beta & \cos\beta & -h\sin\beta & 0 & R & 0 & 0 \\ 0 & 1 & h & 0 & 0 & R & 0 \\ 0 & -1 & h & 0 & 0 & 0 & R \\ \cos\beta & \sin\beta & d + h\cos\beta & d & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & I_4 \end{bmatrix}$$

where  $I_n$  denote  $n \times n$  unit matrix, one of the motion constraints is nonholonomic.

Applying Lagrange formalism we obtain the following equivalent dynamic state space model of the mobile robot [5]

$$\begin{cases} P^T M^*(\beta) P \dot{\eta} = P^T f^*(\theta, \beta, \eta) P^T G(\beta) v, \\ \dot{x} = -\eta_1 \sin \theta, \\ \dot{y} = \eta_1 \cos \theta, \\ \dot{\theta} = \eta_2, \\ \dot{\beta} = D_1(\beta) P \eta, \\ \dot{\varphi} = D_2(\beta) P \eta \end{cases} \quad (20)$$

where  $P^T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  is a matrix relative to the null space of constraints;  $M^*(\beta)$  is a  $3 \times 3$  positive symmetric matrix relative to the kinetic energy and constraints;  $\eta = \begin{cases} \eta_1 = -\dot{x} \sin \theta + \dot{y} \cos \theta \\ \eta_2 = \dot{\theta} \end{cases}$  is a coordinate transformation;  $f^*(\theta, \beta, \eta)$  is a  $3 \times 1$  matrix relative to system constraints and states;  $G(\beta)$  is a  $3 \times 1$  matrix relative to input torque and constraints;  $D_1(1 \times 3), D_2(3 \times 3)$  are matrices relative to constraints;  $\varphi = (\varphi_1, \varphi_2, \varphi_3)^T$ .

The system (20) has a triangular structure; the variable  $\xi$  appears only in the first 5 equations. Since our purpose is to control the trajectory of the robot in the plane, i. e. only the variables  $\xi$ , we can restrict the analysis to these first 5 equations. No problem about the internal stability can occur from this reduction because  $\dot{\beta}$  and  $\dot{\varphi}$  being uniformly bounded provided  $\eta$  is bounded. Moreover, it is easy to check that the input matrix  $P^T G(\beta)$  has full rank for the considered configurations, consequently for any  $(u_1, u_2)$  there exists one and only one static state feedback  $v(\theta, \dot{\theta}, \dot{\beta})$  such that the system of equations (20) reduces to:

$$\begin{cases} \dot{\eta}_1 = u_1, & \dot{\eta}_2 = u_2, \\ \dot{x} = -\eta_1 \sin \theta, \\ \dot{y} = \eta_1 \cos \theta, \\ \dot{\theta} = \eta_2 \end{cases} \quad (21)$$

which exactly has the form of certain part of (9).

We can verify that mobile robot satisfies the relative degree condition and  $\alpha_i = 1 (i = 1, \dots, 5), \mu_i = 2 (i = 1, 2)^{[7]}$ , thereby the variable structure control law  $u$  can be designed using the method of above section to control the output  $y$  to track reference trajectory  $y_d$ .

Consider a reference point  $Q_1$  in the mobile robot system as shown in Fig. 1. Define the position of  $Q_1$  as the output of the system, i. e.

$$Y = \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} x + h \sin \theta \\ y - h \cos \theta \end{bmatrix}. \quad (22)$$

Due to (2) and (7) and the condition of relative degree  $\alpha_i = 1 (i = 1, \dots, 5), \mu_i = 2 (i = 1, 2)$ , we design the variable structure control law for system (21) and (22) as

$$u = (u_1, u_2)^T = -Q^{-1} [e_s + W + k \operatorname{sgn}(S)] \quad (23)$$

where

$$Q = \begin{bmatrix} -\sin \theta & h \cos \theta \\ \cos \theta & h \sin \theta \end{bmatrix},$$

$$e_s = \begin{bmatrix} \beta_{10}\dot{e}_1 - \ddot{y}_{d1} \\ \beta_{20}\dot{e}_2 - \ddot{y}_{d2} \end{bmatrix} = \begin{bmatrix} \beta_{10}(\dot{y}_1 - \dot{y}_{d1}) - \ddot{y}_{d1} \\ \beta_{20}(\dot{y}_2 - \dot{y}_{d2}) - \ddot{y}_{d2} \end{bmatrix}$$

$$= \begin{bmatrix} \beta_{10}(\dot{x} + h\cos\theta \cdot \dot{\theta} - \dot{y}_{d1}) - \ddot{y}_{d1} \\ \beta_{20}(\dot{y} + h\sin\theta \cdot \dot{\theta} - \dot{y}_{d2}) - \ddot{y}_{d2} \end{bmatrix};$$

$\beta_{i0}, i = 1, 2$  are parameters to be assigned

$$W = \begin{bmatrix} L_F^2 C_1 \\ L_F^2 C_2 \end{bmatrix} = \begin{bmatrix} -\eta_1 \dot{\theta} \cos\theta - h\dot{\theta}^2 \sin\theta \\ -\eta_2 \dot{\theta} \sin\theta + h\dot{\theta}^2 \cos\theta \end{bmatrix};$$

$$S = \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} = \begin{bmatrix} \beta_{10}(C_1 - y_{d1}) + L_F C_1 - \dot{y}_{d1} \\ \beta_{20}(C_2 - y_{d2}) + L_F C_2 - \dot{y}_{d2} \end{bmatrix}$$

$$= \begin{bmatrix} \beta_{10}(x + h\sin\theta - y_{d1}) + \dot{x} + h\cos\theta \cdot \dot{\theta} - \dot{y}_{d1} \\ \beta_{20}(y + h\cos\theta - y_{d2}) + \dot{y} + h\sin\theta \cdot \dot{\theta} - \dot{y}_{d2} \end{bmatrix};$$

$k$  satisfies equation (18)

$$k > (1 - \gamma_0)^{-1}(\gamma_1 + \gamma_0 \| e_s + W \|);$$

and

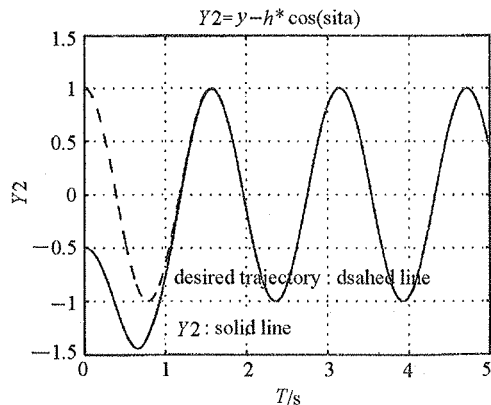
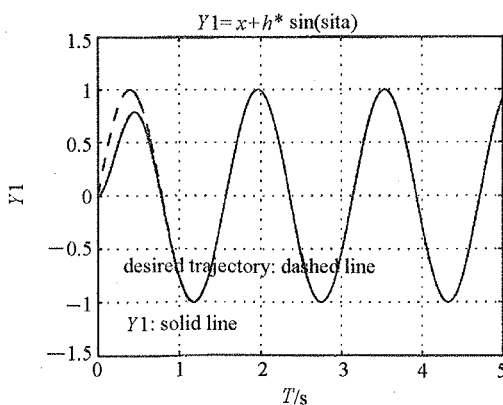
$$\| \Delta Q Q^{-1} \| \leq \gamma_0 < 1, \quad \| \Delta R \| \leq \gamma_1.$$

Consider the following system with the uncertain parameters  $\Delta$ , such that (21) as

$$\begin{cases} \dot{\eta}_1 = u_1, & \dot{\eta}_2 = u_2, \\ \dot{x} = -\eta_1 \sin\theta + \Delta, \\ \dot{y} = \eta_1 \cos\theta + \Delta, \\ \dot{\theta} = \eta_2 + \Delta. \end{cases} \tag{24}$$

Let the standard output  $y_d = \begin{pmatrix} y_{d1} \\ y_{d2} \end{pmatrix} = \begin{pmatrix} \sin 4t \\ \cos 4t \end{pmatrix}$ , applying the control law of (23) follow

the steps discussed above, we simulate under three cases as choosing  $\Delta$  varying in the range of  $(0, 0.3), (0, 0.5), (0, 1)$ , respectively, the simulate results are shown in Fig. 2. We can see from the figures, the system output track the desired trajectory well as the system has uncertain parameters, the tracking error increases as the disturbance increases but remain relative small, our robust control method is effective.



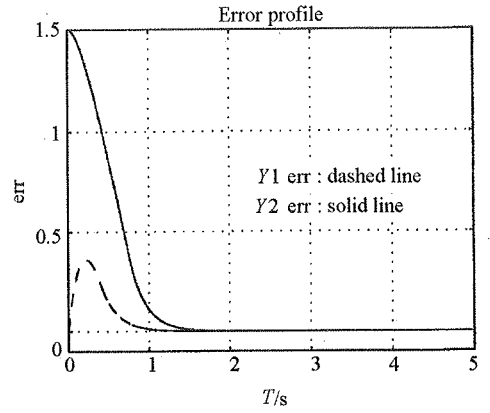
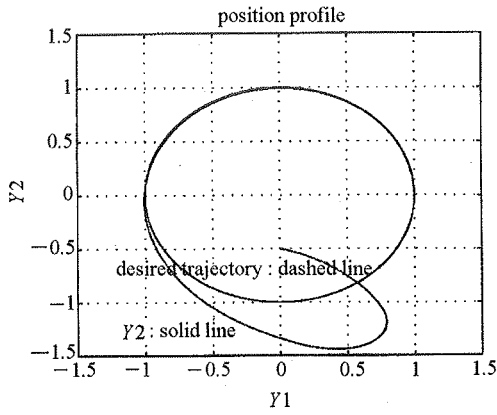


Fig. 2(a) no  $\Delta$

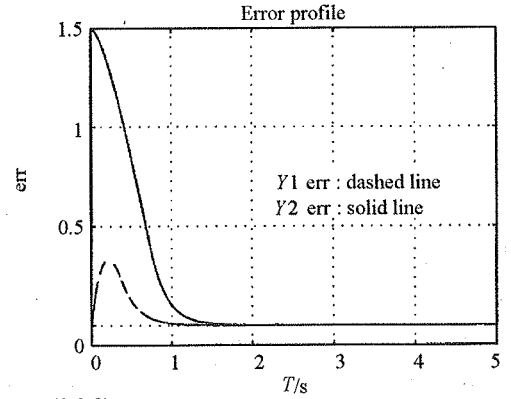
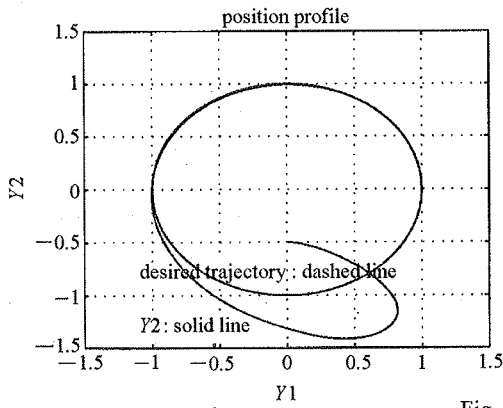
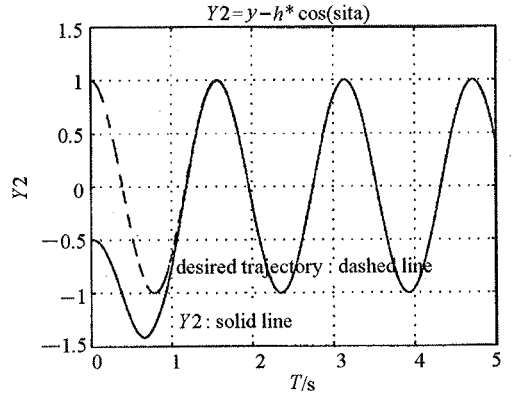
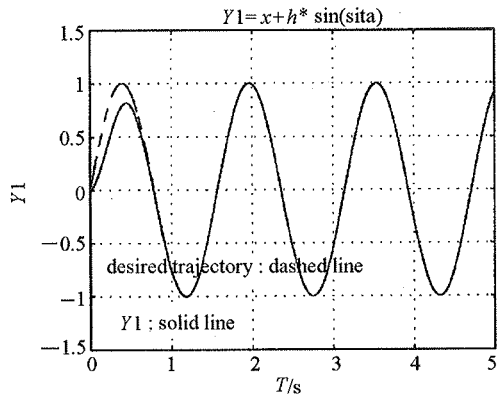
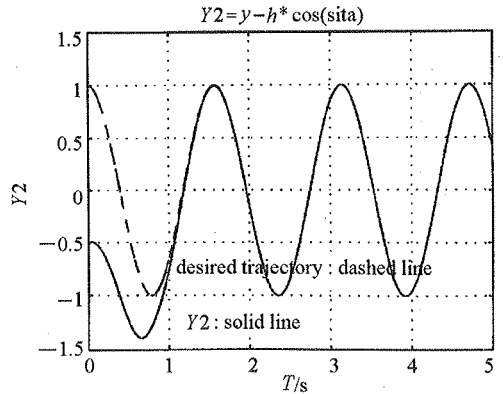
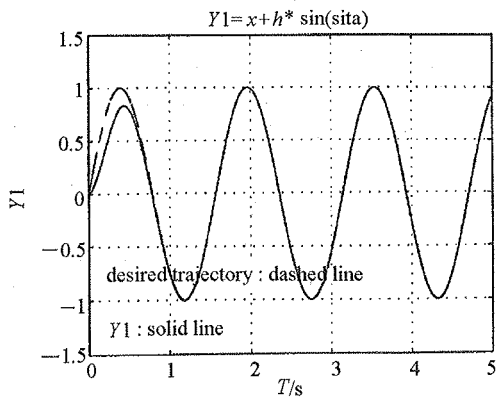


Fig. 2(b)  $\Delta \epsilon (0,0.3)$





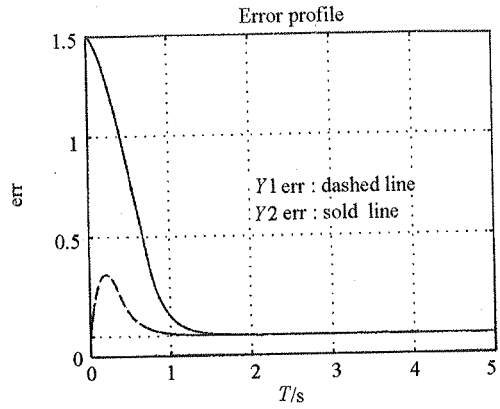
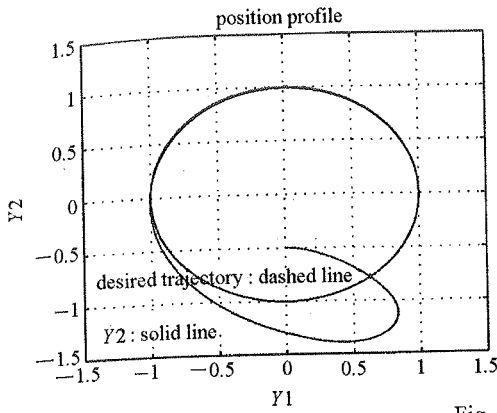


Fig. 2(c)  $\Delta \epsilon \in (0,0.5)$

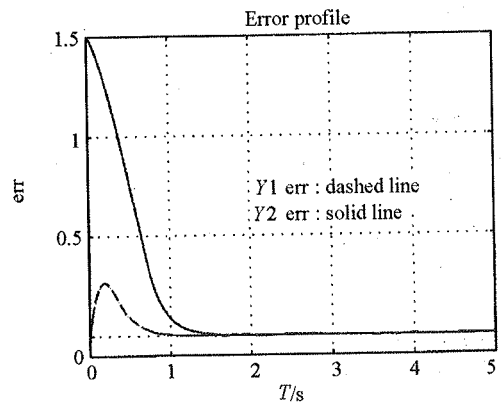
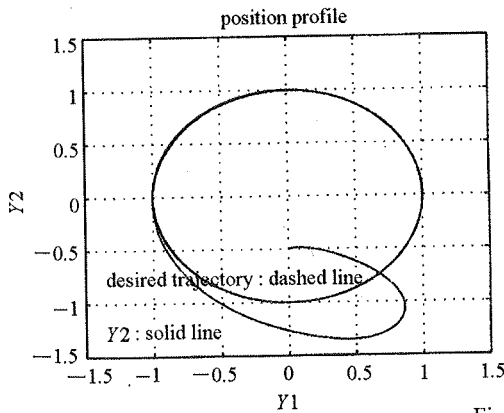
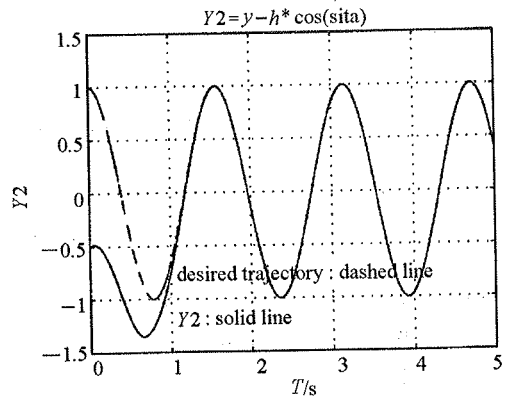
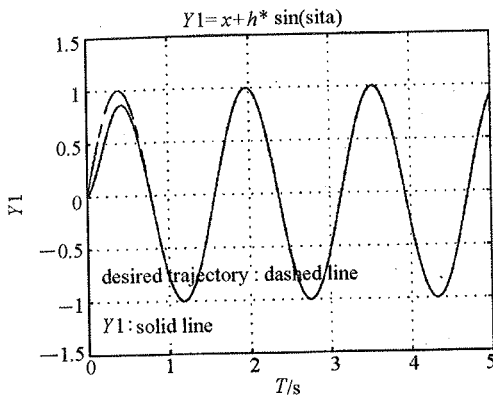


Fig. 2(d)  $\Delta \epsilon \in (0,1)$

## 5 Conclusions

In this paper, the robust output tracking problem of a kind of uncertain nonholonomic systems is discussed. The reduced dimensional state realization and main properties are given under some assumptions. Then, the simplified state space model of 3-wheel mobile robot is given with the conditions of pure rolling and non slipping. Finally, the detailed robust output tracking control law based on the simplified model is presented, the simulate result shows that the control law is effective to the disturbance, the system output track the reference trajectory well as the system has uncertain parameters.

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## 移动机器人的鲁棒输出跟踪

李青湘 胡跃明 裴海龙 周其节

(华南理工大学自动控制工程系·广州, 510641)

**摘要:** 本文讨论了一类不确定非完整系统的鲁棒输出跟踪问题. 首先给出了在适当条件下受限系统的降阶状态实现及有关性质; 进而给出了三轮移动机器人在纯滚动与非打滑条件下的简化模型, 并结合变结构控制方法对该模型给出了具体的鲁棒输出跟踪控制规律.

**关键词:** 移动机器人; 非完整系统; 输出跟踪; 变结构控制; 鲁棒性

## 本文作者简介

**李青湘** 1970年生. 1992年、1995年于西北工业大学分别获学士、硕士学位. 1995年3月至今在华南理工大学自动控制工程系攻读博士学位. 目前主要研究方向为非完整系统的受限控制.

**胡跃明** 1960年生. 分别于1985年和1991年在安徽大学与华南理工大学获理学硕士与工学博士学位. 曾任安徽经济管理学院讲师、香港理工大学电子工程系副研究员及访问研究员等职. 现为华南理工大学自动控制工程系教授. 著有“分布参数变结构控制系统”一书, 并在国内外学术刊物上发表论文四十多篇. 目前研究兴趣为非线性控制系统理论及应用, 机器人控制等.

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