第 15 卷第 4 期
1998 年 8 月

控制理论与应用
CONTROL THEORY AND APPLICATIONS

Vol. 15, No. 4
Aug., 1998

Delay-Independent Stability for Linear Systems with Multiple Time Delays

TAN Wen

(Department of Power Engineering, North China Electric Power University · Baoding, 071003, PRC)

XU Jianmin and MAO Zhongyuan

(Department of Automatic Control Engineering, South China University of Technology · Guangzhou, 510641, PRC)

Abstract: The delay-independent asymptotic stability for linear systems with multiple delays is discussed in this paper using the structured singular value concept. Our result shows that the delays can be viewed as repeated scalar uncertainty blocks for a "nominal" plant. When the systems have other structured perturbations, the stability issue can be analyzed in the frame of μ- analysis approach.

Key words: time-delay systems; stability independent of delay (i. o. d.); structured singular value; structured perturbations; repeated scalar uncertainty block

1 Introduction

The study of stability for time-delay systems can be divided into two groups: one is stability independent of delay[1~7]; the other, stability dependent of delay[8~10]. Since usually the delays are unknown, it is more satisfactory if a time-delay system is stable independent of delay.

[1～4] studied asymptotic stability independent of delay for single time delay systems; while[5～7]for multiple time delay systems. Their criteria are only sufficient conditions. Though the conditions in [5] may be necessary under some conditions, they are limited to a very special class of systems. To the authors' knowledge, necessary and sufficient conditions have not been completely characterized for general time-delay systems.

In this paper, we will present a characterization for linear systems with multiple delays using the structured singular value concept. Our result shows that the delays can be treated as repeated scalar uncertainty blocks for a "nominal" plant. Thus when there are other uncertainties in the plant model, we just need to lump all the uncertainties into diagonal block form and analyze in the frame of μ- analysis approach.

2 Preliminaries

Consider the following linear system with multiple delays:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + \sum_{i=1}^{r} D_i x(t-\tau_i), \\ x(t) &= \Phi(t), \quad t \leqslant 0, \end{aligned} \tag{2.1}$$

where $A, D_i \in \mathbb{R}^{n\times n}$ and $\tau_i (i = 1, \cdots, r)$ are uncertain time delays. System (2. 1) is called

Manuscript received May 14, 1996, revised Augt. 12, 1997.

asymptotically stable independent of delay (i. o. d.), if for all delays $0 \leqslant \tau_i < \infty (i = 1, \cdots, r)$, the system is asymptotically stable, i. e. , $\det (sI - A - \sum_{i=1}^{r} D_i e^{-\tau_i s}) \neq 0, \forall \operatorname{Re} s \geqslant 0$.

Our analysis is based on the structured singular value concept[11]. For matrix $M \in \mathbb{C}^{n\times n}$ and the following uncertainty block:

$$\boldsymbol{\Delta} := \{\operatorname{diag}[\delta_1 I_{r_1}, \cdots, \delta_S I_{r_S}, \Delta_{S+1}, \cdots, \Delta_{S+F}] : \delta_i \in \mathbb{C}, \Delta_{S+j} \in \mathbb{C}^{m_j \times m_j}, 1 \leqslant i \leqslant S, 1 \leqslant j \leqslant F\}, \tag{2.2}$$

where $\sum_{i=1}^{S} r_i + \sum_{j=1}^{F} m_j = n$, the structured singular value of M with respect to the structure $\boldsymbol{\Delta}$ is defined as:

$$\mu_{\Delta}(M) := \frac{1}{\min\{\bar{\sigma}(\Delta) : \Delta \in \boldsymbol{\Delta}, \det(I - M\Delta) = 0\}}, \tag{2.3}$$

unless no $\Delta \in \boldsymbol{\Delta}$ makes $I - M\Delta$ singular, in which case $\mu_\Delta(M) := 0$.

For transfer function $P(s)$, the structured singular value is defined as:

$$\mu_\Delta(P) := \sup_{\operatorname{Re} s \geqslant 0} \mu_\Delta(P(s)) = \sup_{\omega} \mu_\Delta(P(j\omega)). \tag{2.4}$$

Structured singular value plays an important role in the analysis of structured uncertainties[11,12], its computation was studied extensively in the past years. A commercial MATLAB-based toolbox, μ- analysis and synthesis toolbox[13], is now available to compute it efficiently.

3 Main Results

Consider system (2. 1), let

$$P(s) := \begin{bmatrix} I_n \\ \vdots \\ I_n \end{bmatrix} (sI - A)^{-1} [D_1 \quad \cdots \quad D_r], \tag{3.1}$$

$$\boldsymbol{\Delta} := \operatorname{diag}\{\delta_1 I_n, \cdots, \delta_r I_n : \delta_i \in \mathbb{C}, i = 1, \cdots, r\} \tag{3.2}$$

then our main results can be stated as follows:

Theorem 1 System (2. 1) is asymptotically stable i. o. d. if and only if A is asymptotically stable and $\mu_\Delta(P) < 1$.

Proof Sufficiency. Note that $\mu_\Delta(P) < 1$ means that $\mu_\Delta(P(j\omega)) < 1, \forall \omega$, so for all $\Delta \subset \boldsymbol{\Delta}$, $\| \Delta \| = 1$, we have $\det (I - P(j\omega)\Delta) \neq 0, \forall \omega$. By the definition of the uncertainty structure in (3. 2), we must have $\det (I - P(j\omega)\Delta) \neq 0, \forall \omega$ for all $\Delta \subset \boldsymbol{\Delta}$ where $|\delta_i| = 1, i = 1, \cdots, r$. Since for any $0 \leqslant \tau_i < \infty$, we have $|e^{-j\omega\tau_i}| = 1$, therefore,

$$\det(I - P(j\omega)\operatorname{diag}\{e^{-j\omega\tau_1} I_n, \cdots, e^{-j\omega\tau_r} I_n\}) \neq 0, \quad \forall \omega, \forall 0 \leqslant \tau_i < \infty, i = 1, \cdots, r. \tag{3.3}$$

Note that

$$\det(sI - A - \sum_{i=1}^{r} D_i e^{-\tau_i s})$$

$$= \det(sI - A)\det(I - (sI - A)^{-1}[D_1, \cdots, D_r]\operatorname{diag}\{e^{-\tau_1 s} I_n, \cdots, e^{-\tau_r s} I_n\} \begin{bmatrix} I_n \\ \cdots \\ I_n \end{bmatrix})$$

$$= \det(sI - A)\det(I - P(s)\mathrm{diag}\{e^{-\tau_1 s}I_n, \cdots, e^{-\tau_r s}I_n\}) \tag{3.4}$$

and that A is asymptotically stable, by (3.3) and (3.4), system (2.1) is asymptotically stable i.o.d..

Necessity. Suppose $\det(sI - A - \sum_{i=1}^{r} D_i e^{-\tau_i s}) \neq 0, \forall\ \mathrm{Re}s \geq 0$ and $0 \leq \tau_i < \infty (i = 1, \cdots, r)$, then we have:

$$\forall\ \mathrm{Re}s > 0, \quad \det(sI - A - \sum_{i=1}^{r} D_i e^{-\tau_i s}) \neq 0, \quad \text{as} \quad \tau_i \to \infty (i = 1, \cdots, r) \tag{3.5}$$

thus $\forall\ \mathrm{Re}s > 0, \det(sI - A) \neq 0$, i.e., A is stable.

When $s = j\omega$, $\det(j\omega I - A - \sum_{i=1}^{r} D_i e^{-j\omega\tau_i}) \neq 0$ for $0 \leq \tau_i < \infty (i = 1, \cdots, r)$. By (3.4), we must have $\det(j\omega I - A) \neq 0$ and $\det(I - P(j\omega)\mathrm{diag}\{e^{-j\omega\tau_1}I_n, \cdots, e^{-j\omega\tau_r}I_n\}) \neq 0, \forall\ \omega$. The former means that A has no eigenvalues on the $j\omega$ axis, so in fact A is asymptotically stable. The latter means that for any structured perturbation Δ in (3.2) such that $|\delta_i| = 1$, we can always find τ_i's such that $\delta_i = e^{-j\omega\tau_i}$ for any frequency ω, so we have $\det(I - P(j\omega)\Delta) \neq 0$, thus $\mu_\Delta(P) < 1$. Q.E.D.

Remark 1 Theorem 1 shows that for stability i.o.d., the delays can be viewed as repeated scalar uncertainty blocks for the nominal plant defined in (3.1).

Remark 2 Since $\mu_\Delta(P) \leq \| P \|_\infty$, a sufficient condition for system (2.1) to be asymptotically stable i.o.d. can be expressed as: There exists a symmetric positive-definite matrix X such that

$$A^T X + XA + \sum_{i=1}^{r} X D_i D_i^T X + I < 0, \tag{3.6}$$

which is nothing but the condition for $\| P \|_\infty < 1$[14]. A less conservative approach is using the upper bound for $\mu_\Delta(P)$, i.e., $\mu_\Delta(P) \leq \inf \| FPF^{-1} \|_\infty$[12], where F is over the following sets:

$$\mathscr{F} := \{\mathrm{diag}\{F_1, \cdots, F_r\} : F_i \in \mathbb{C}^{n\times n}, F_i = F_i^* > 0\}. \tag{3.7}$$

Thus another sufficient condition can be expressed as: There exists symmetric positive-definite matrices $F_i (i = 1, \cdots, r)$ and a symmetric positive-definite matrix X such that

$$A^T X + XA + \sum_{i=1}^{r} X D_i F_i^{-1} F_i^{-T} D_i^T X + \sum_{i=1}^{r} F_i F_i^T < 0. \tag{3.8}$$

This condition is the same as given in [6], and if all F_i's are equal, then it is the same as given in [7]. Note that it is only sufficient, so in practice the result it predicts may be conservative.

Remark 3 In [5] sufficient (sometimes also necessary) conditions for stability i.o.d. were presented. However, the criterion is limited to a special class of systems, i.e., A is a Metzler matrix or can be transformed to such form, so its application is very limited.

Example 1 Consider system (2.1) with $A = \begin{bmatrix} -4 & 0 \\ 0 & -3 \end{bmatrix}$, $D = a\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$, where $a \neq 0$

is a scalar parameter. By theorem 1, the system is asymptotically stable i. o. d. if and only if $|a| < 1.50$, whereas [2] gave the bound as $|a| < 1.3110$, and [3], $|a| < 1.3592$. In fact, since in this example

$$\det(sI - A - De^{-\tau s}) = (s + 4 - ae^{-\tau s})(s + 3 - 2ae^{-\tau s}),$$

it is not difficult to verify that it has no $j\omega$ roots if and only if $|a| < 1.5$.

Example 2 Consider system (2. 1) with $A = \begin{bmatrix} -3 & 1 & 0 \\ 1 & -2 & 0 \\ -2 & 1 & -1 \end{bmatrix}$, $D_1 = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, $D_2 = a\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. It was given by Luo, et. al. [5] to illustrate their conditions. With a careful choice of transformation, they predicted the system is asymptotically stable i. o. d. if and only if $|a| < 2.5$. The result can be directly obtained from Theorem 1.

4 Robust Stability for Uncertain Time-Delay Systems

In this section, we will consider system (2. 1) subjected to structured perturbations:

$$\begin{aligned} \dot{x}(t) &= (A + \sum_{i=1}^{m} \alpha_i A_i)x(t) + \sum_{k=1}^{r}(D_k + \sum_{l=1}^{L_k} \beta_{kl} B_{kl})x(t - \tau_k), \\ x(t) &= \Phi(t), \quad t \leqslant 0, \end{aligned} \tag{4.1}$$

where $A, D_k, A_i, B_{kl} \in \mathbb{R}^{n\times n}$ are fixed, and α_i, β_{kl} are parameters which are scaled such that $|\alpha_i| < 1$, $|\beta_{kl}| < 1$ for $i = 1,\cdots,m; k = 1,\cdots,r; l = 1,\cdots,L_k$. Let uncertainty block defined as:

$$\Delta = \mathrm{diag}\{[\Delta_1, \Delta_2, \Delta_3]\} \tag{4.2}$$

where

$$\begin{cases} \Delta_1 := \mathrm{diag}\{\delta_1 I_n, \cdots, \delta_r I_n : \delta_k \in \mathbb{C}, k = 1,\cdots,r\}, \\ \Delta_2 := \mathrm{diag}\{\alpha_1 I_n, \cdots, \alpha_m I_n : \alpha_i \in \mathbb{R}, i = 1,\cdots,m\}, \\ \Delta_3 := \mathrm{diag}\{\beta_{11} I_n, \cdots, \beta_{1L_1} I_n, \cdots, \beta_{rL_1} I_n, \cdots, \beta_{rL_r} I_n : \beta_{kl} \in \mathbb{R}, k = 1,\cdots,r; l = 1,\cdots,L_k\}. \end{cases} \tag{4.3}$$

Then we have:

Theorem 2 System (4. 1) is robustly asymptotically stable i. o. d. if and only if A is asymptotically stable and $\mu_\Delta(\overline{P}) < 1$, where the state-space realization of $\overline{P}(s) := \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix}$ is:

$$\left[\begin{array}{c|ccc} A & [D_1 \cdots D_r] & [A_1 \cdots A_m] & [B_{11} \cdots B_{1L_1} \cdots B_{r1} \cdots B_{rL_r}] \\ \hline C & 0 & 0 & 0 \\ C_1 & 0 & 0 & 0 \\ 0 & C_2 & 0 & 0 \end{array}\right], \tag{4.4}$$

where

$$C := \left.\begin{bmatrix} I_n \\ \vdots \\ I_n \end{bmatrix}\right\} r, \quad C_1 := \left.\begin{bmatrix} I_n \\ \vdots \\ I_n \end{bmatrix}\right\} m, \quad C_2 := \begin{bmatrix} L_1\left\{\begin{bmatrix} I_n \\ \vdots \\ I_n \end{bmatrix}\right. & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & L_r\left\{\begin{bmatrix} I_n \\ \vdots \\ I_n \end{bmatrix}\right. \end{bmatrix}. \quad (4.5)$$

Proof By the results in the previous section, system (4.1) is asymptotically stable i.o.d. if and only if $P(s)$ is asymptotically stable and $\mu_{\Delta 1}(P) < 1$ for all admissible parameters, where

$$P(s) := C(sI - A - \sum_{i=1}^{m} \alpha_i A_i)^{-1}\left[D_1 + \sum_{k=1}^{L_1} \beta_{1k}\beta_{1k} \quad \cdots \quad D_r + \sum_{k=1}^{L_r} \beta_{rk} B_{rk}\right]. \quad (4.6)$$

It is easy to see

$$P(s) = P_{11} + [P_{12} \quad P_{13}]\begin{bmatrix} \Delta_2 & 0 \\ 0 & \Delta_3 \end{bmatrix}\left(I - \begin{bmatrix} P_{22} & P_{23} \\ P_{32} & P_{33} \end{bmatrix}\begin{bmatrix} \Delta_2 & 0 \\ 0 & \Delta_3 \end{bmatrix}\right)^{-1}\begin{bmatrix} P_{21} \\ P_{31} \end{bmatrix}, \quad (4.7)$$

for all $\Delta_2 \subset \boldsymbol{\Delta}_2, \Delta_3 \subset \boldsymbol{\Delta}_3$. So by the main loop theorem[12], we arrive at the conclusions.

Q.E.D.

When the uncertainties have other structure information, similar analysis can be performed, if only we take the delays as repeated scalar uncertainty blocks.

5 Conclusions

In this paper, we derive a criterion for the asymptotic stability i.o.d. for linear systems with multiple delays using the structured singular value concept. We can, in fact, treat the delay in the state as repeated scalar uncertainty blocks, thus we can use μ- analysis approach to analyze the robust stability issue.

References

1 Brierley, S. D., Chiasson, J. N., Lee, E. B. and Zak, S. H.. On stability independent of delay for linear systems. IEEE Trans. Automat. Contr., 1982, AC-27(2): 252—254

2 Mori, T., Fukuma, N. and Kuwahara, M.. Simple stability criteria for single and composite linear systems with time delays. Int. J. Control, 1981, 34(6): 1175—1184

3 Hmamed, A.. On the stability of time delay systems: new results. Int. J. Control, 1986, 43(2): 321—324

4 Hmamed, A.. Further results on the robust stability of uncertain time-delay systems. Int. J. Systems Sci., 1991, 22(3): 605—614

5 Luo, J. S., Johnson, A. and van den Bosch, P. J.. Delay-independent robust stability of uncertain linear systems. Systems and Control Letters, 1995, 24(1): 33—39

6 Verriest, E. I. and Ivanov, A. F.. Robust stability of systems with delayed feedback. Circuits, Systems & Signal Process, 1994, 13(2): 213—222

7 Ge, J. H., Frank, P. M. and Lin, C. F.. H_∞ control via output feedback for state delayed systems. Int. J. Control, 1996, 64(1): 1—7

8 Mori, T.. Criteria for asymptotic stability of linear time-delay systems. IEEE Trans. Automat. Contr., 1985, AC-30(1): 158—161

9 Su, J. H.. Further results on the robust stability of linear systems with a single time delay. Systems and Control Letters, 1994, 23(3): 375－379

10 Su, J. H. and Huang, C. G.. Robust stability of delay dependence for linear uncertain systems. IEEE Trans. Automat. Contr., 1992, AC-37(8): 1656－1659

11 Doyle, J. C.. Analysis of feedback systems with structured uncertainties. IEE Proc., Part D, 1982, 129(2): 242－250

12 Packard, P. and Doyle, J.. The complex structured singular value. Automatica, 1993, 29(1): 71－109

13 Balas, G. J., et al.. μ- analysis and synthesis toolbox. MUSYN and The Math. Works, Inc., Minneapolis, MN, 1991

14 Zhou, K. M. and Khargonekar, P.. An algebraic Riccati equation approach to H_∞ optimization. Systems and Control Letters, 1988, 11(1): 85－91

时滞线性系统的与时滞无关稳定性

谭　文　　　　徐建闽　毛宗源

（华北电力大学动力系·保定，071003）（华南理工大学自动控制工程系·广州，510641）

摘要：本文利用结构奇异值概念讨论时滞线性系统的与时滞无关之渐近稳定性.本文结果表明时滞可看成某一“标称”系统的重复标量不确定摄动块.当系统具其它结构摄动时，鲁棒稳定分析可在结构奇异值方法框架上进行.

关键词：时滞系统；与时滞无关之稳定；结构奇异值；结构摄动；重复标量不确定块

本文作者简介

谭　文　1969 年生.1990 年毕业于厦门大学数学系应用数学专业，1993 年于厦门大学系统科学系获硕士学位.1996 年于华南理工大学自动化系获博士学位.1994 年 10 月至 1996 年 2 月分别在香港理工大学机械工程系及电子工程系作访问学者.现在华北电力大学动力工程系任教.主要研究方向为鲁棒控制，H_∞控制及其在挠性体控制及工业过程控制中的应用.

徐建闽　见本刊 1998 年第 1 期第 151 页.

毛宗源　见本刊 1998 年第 1 期第 151 页.

（上接第 592 页）

长，黄心汉、张彭、朱淼良、杨静宇、梁天培为副理事长，罗安为秘书长.大会一致同意 2000 年在长沙召开中国人工智能学会第四届智能机器人研讨会.

与会代表对各级领导和科学家的热情鼓励、对国家科委高技术司、国家自然科学基金委员会、863 机器人专家组的大力支持，对大会程序委员会和组织委员会的辛勤劳动，对承办单位浙江大学计算机系和协办单位中南工业大学信息工程学院的大力支持与热情接待表示衷心感谢.大家对本次会议的各项工作表示满意，并对会议取得圆满成功表示由衷高兴和热烈祝贺.