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Sliding Window Delta Operator-Based Adaptive Lattice Filters*

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Abstract: The detailed procedure to derive a sliding window delta operator-based adaptive lattice filter algorithm is illustrated based on the least squares geometric projection approach in this paper. Applications of this filter to system identification[1] and parameter change detection[2] have been addressed.

Key words: adaptive lattice filters; system identification; parameter change detection

1 Introduction

Making use of the advantages of lattice filters and delta operator, Jabbari proposed a delta operator-based lattice filter to identify the continuous-time model of a system. With a forgetting factor, by discarding the signals far away from the current sampling instant, this filter can also be employed to either track the parameter changes or detect the parameter jumps[3]. In the presence of smooth model parameter changes, the filter can get a good trade-off between small noise sensitivity and fast rate of convergence. However, in the presence of abrupt parameter changes, the performance of this filter is greatly affected.

In this paper, we present a new solution for filtering those signals which are produced from fast time-varying models. We use a finite length sliding window to capture the most recent signals close to the sampling instant to improve significantly the tracking capability of the lattice filter.

2 Notations and Definitions

Let $y(k)$ be the filter input signal at sampling instant k, the data vector at instant $k-n$ within the window of length w is defined as

$$Y_w(k-n) = [y(k-n), y(k-n-1), \cdots, y(k-n-w+1)]^T \in \mathbb{R}^{n\times 1}. \tag{1}$$

And we form the following subspace

$$H_{n,w}(k) = \text{span}\{Y_w(k-n), \delta Y_w(k-n), \cdots, \delta^{n-1} Y_w(k-n)\}. \tag{2}$$

Note that $H_{0,w}(k) = \{0\}$ and δ is the delta operator. We introduce two vectors

$$\phi = [1, 0, \cdots, 0]^T \in \mathbb{R}^{w\times 1} \quad \text{and} \quad \Psi = [0, 0, \cdots, 1]^T \in \mathbb{R}^{w\times 1} \tag{3}$$

and define

$$P_{n,w}(k) = \text{orthogonal projection operator onto } H_{n,w}(k), \tag{4}$$

then for any vector $x \in \mathbb{R}^{w\times 1}$, its orthogonal projection onto $H_{n,w}(k)$ is $P_{n,w}(k)x$.

In order to derive the lattice filter algorithm, the following variables are also required, which are listed in Table 1.

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Table 1 Definitions of filter variables

Name	Definition
Forward error vector	$f_{n,w}(k) = [I - P_{n,w}(k)]\delta^n Y_w(k-n)$
Backward error vector	$b_{n,w}(k) = [I - P_{n,w}(k+1)]Y_w(k-n)$
First element of $f_{n,w}(k)$	$e_{n,w}(k) = f^{\mathrm{T}}_{n,w}(k)\phi$
Last element of $f_{n,w}(k)$	$d_{n,w}(k) = f^{\mathrm{T}}_{n,w}(k)\Psi$
First element of $b_{n,w}(k)$	$r_{n,w}(k) = b^{\mathrm{T}}_{n,w}(k)\phi$
Last element of $b_{n,w}(k)$	$q_{n,w}(k) = b^{\mathrm{T}}_{n,w}(k)\Psi$
Residual of ϕ with respect to $H_{n,w}(k+1)$	$\alpha_{n,w}(k) = [I - P_{n,w}(k+1)]\phi$
Residual of Ψ with respect to $H_{n,w}(k+1)$	$\beta_{n,w}(k) = [I - P_{n,w}(k+1)]\Psi$
The $(n+1)$ th partial correlation coefficient	$K_{n+1,w}(k) = \langle f_{n,w}(k), b_{n,w}(k-1)\rangle$
Norm square of $f_{n,w}(k)$	$R^e_{n,w}(k) = \langle f_{n,w}(k), f_{n,w}(k)\rangle$
Norm square of $b_{n,w}(k)$	$R^r_{n,w}(k) = \langle b_{n,w}(k), b_{n,w}(k)\rangle$
Norm square of $\alpha_{n,w}(k)$	$v_{n,w}(k) = \langle \alpha_{n,w}(k), \alpha_{n,w}(k)\rangle$
Norm square of $\beta_{n,w}(k)$	$\eta_{n,w}(k) = \langle \beta_{n,w}(k), \beta_{n,w}(k)\rangle$

3 Derivation of Filter Order Update Equations

From the definition of the delta operator, we have

$$\delta Y_w(k) = \frac{Y_w(k+1) - Y_w(k)}{T} \tag{5}$$

where T is the sampling period. Substituting this equation into (2) yields

$$H_{n,w}(k) = \mathrm{span}\{Y_w(k-n), Y_w(k-n+1), \delta Y_w(k-n+1), \cdots, \delta^{n-2} Y_w(k-n+1)\}, \tag{6}$$

therefore
$$H_{n+1,w}(k) = \mathrm{span}\{Y_w(k-n-1)\} \oplus H_{n,w}(k) \tag{7}$$

where the symbol $\oplus$ indicates the sum of two subspaces.

From the definition of the backward error vector in Table 1, it can be seen that

$$\mathrm{span}\{Y_w(k-n-1)\} = \mathrm{span}\{b_{n,w}(k-1)\} \overset{\perp}{\oplus} H_{n,w}(k), \tag{8}$$

where $\overset{\perp}{\oplus}$ stands for the direct sum of two subspaces. Substituting (8) into (7) leads to

$$H_{n+1,w}(k) = \mathrm{span}\{b_{n,w}(k-1)\} \overset{\perp}{\oplus} H_{n,w}(k), \tag{9}$$

and from the orthogonality in (9), it follows that

$$P_{n+1,w}(k) = P_{n,w}(k) + P^b_{n,w}(k-1) \tag{10}$$

where $P_{n+1,w}(k)$, $P^b_{n,w}(k-1)$ are the orthogonal projection operators onto the subspaces $H_{n+1,w}(k)$ and span $\{b_{n,w}(k-1)\}$ respectively. From (10), we have

$$[I - P_{n+1,w}(k)] = [I - P^b_{n,w}(k-1)][I - P_{n,w}(k)]. \tag{11}$$

Similarly, from (2) and the definition of the forward error vector in Table 1, we have

$$H_{n+1,w}(k+1) = \mathrm{span}\{\delta^n Y_w(k-n)\} \oplus H_{n,w}(k) = \mathrm{span}\{f_{n,w}(k)\} \overset{\perp}{\oplus} H_{n,w}(k). \tag{12}$$

Furthermore

$$[I - P_{n+1,w}(k+1)] = [I - P^f_{n,w}(k)][I - P_{n,w}(k)] \tag{13}$$

where $P^f_{n,w}(k)$ is the orthogonal projection operator onto span $\{f_{n,w}(k)\}$.

Using (11), the definitions in Table 1 and the orthogonal projection formula produces the order update equations for forward error vector $f_{n,w}(k)$

$$f_{n+1,w}(k)=\frac{1}{T}[I-P^b_{n,w}(k-1)][I-P_{n,w}(k)][\delta^n Y_w(k-n)-\delta^n Y_w(k-n-1)]$$
$$=\frac{1}{T}f_{n,w}(k)-\frac{1}{T}b_{n,w}(k-1)(R^r_{n,w}(k-1))^{-1}K_{n+1,w}(k). \tag{14}$$

Similarly, the order update equations for backward error vector $b_{n,w}(k)$ can also be obtained as follows

$$b_{n+1,w}(k)=b_{n,w}(k-1)-f_{n,w}(k)(R^e_{n,w}(k))^{-1}K_{n+1,w}(k). \tag{15}$$

Based on (14)(15) and the definitions in Table 1, we have the following order update equations

$$e_{n+1,w}(k)=\frac{1}{T}e_{n,w}(k)-\frac{1}{T}r_{n,w}(k-1)(R^r_{n,w}(k-1))^{-1}K_{n+1,w}(k), \tag{16}$$

$$d_{n+1,w}(k)=\frac{1}{T}d_{n,w}(k)-\frac{1}{T}q_{n,w}(k-1)(R^r_{n,w}(k-1))^{-1}K_{n+1,w}(k), \tag{17}$$

$$r_{n+1,w}(k)=r_{n,w}(k-1)-e_{n,w}(k)(R^e_{n,w}(k))^{-1}K_{n+1,w}(k), \tag{18}$$

$$q_{n+1,w}(k)=q_{n,w}(k-1)-d_{n,w}(k)(R^e_{n,w}(k))^{-1}K_{n+1,w}(k). \tag{19}$$

It should be noted that for $R^e_{n,w}$ and $R^r_{n,w}(k-1)$ in the above equations, their update equations can be obtained directly from (14)(15) and the definitions in Table 1

$$R^e_{n+1,w}(k)=\frac{1}{T^2}R^e_{n,w}(k)-\frac{1}{T^2}K_{n+1,w}(k)(R^r_{n,w}(k-1))^{-1}K_{n+1,w}(k), \tag{20}$$

$$R^r_{n+1,w}(k)=R^r_{n,w}(k-1)-K_{n+1,w}(k)(R^e_{n,w}(k))^{-1}K_{n+1,w}(k). \tag{21}$$

It is interesting to note that actually (18)~(21) are both order and time updated.

4 Derivation of Filter Time Update Equations[4]

Let g and s denote subspaces spanned by any vectors located in $\mathbb{R}^{w\times 1}$, ξ and η be any vectors belonging to $\mathbb{R}^{w\times 1}$, $g\oplus s$ be the sum of two subspaces, $\xi|_s$ be the orthogonal projection of ξ onto s, and $\xi|_{g\oplus s}$ be the orthogonal projection of ξ onto $g\oplus s$. If the error between vectors ξ and $\xi|_s$ is expressed by ξ_s, i. e.

$$\xi_s=\xi-\xi|_s \tag{22}$$

and similarly

$$\xi_{g\oplus s}=\xi-\xi|_{g\oplus s} \quad \text{and} \quad (\xi_g)_{s_g}=\xi_g-(\xi_g)|_s, \tag{23}$$

then

$$\xi_{s\oplus g}=(\xi_s)_{g_s}, \tag{24}$$

$$\langle\xi_{s\oplus g},\eta_{s\oplus g}\rangle=\langle(\xi_s)_{g_s},(\eta_s)_{g_s}\rangle=\langle(\xi_g)_{s_g},(\eta_g)_{s_g}\rangle=\langle\xi_g,\eta_g\rangle-\langle s_g,\eta_g\rangle\frac{\langle\xi_g,s_g\rangle}{\langle s_g,s_g\rangle}. \tag{25}$$

If we choose $g=H_{n,w}(k)$, $\xi=\delta^n Y_w(k-n)$, $\eta=Y_w(k-n-1)$, and with different choices of subspaces for s, we can compute $\langle\xi_g,\eta_g\rangle=K_{n+1,w}(k)$. Now, it directly follows that

$$K_{n+1,w}(k)=K_{n+1,w-1}(k-1)+r_{n,w}(k-1)(\nu_{n,w}(k-1))^{-1}e_{n,w}(k) \tag{26}$$

and

$$K_{n+1,w-1}(k)=K_{n+1,w}(k)-q_{n,w}(k-1)(\mu_{n,w}(k-1))^{-1}d_{n,w}(k). \tag{27}$$

Finally, in order to complete the filter algorithm, we need to derive the update equations for $\nu_{n,w}(k)$ and $\mu_{n,w}(k)$. These equations can be obtained by using the orthogonal property between each backward error vector.

Using (10), we may write

$$P_{n,w}(k+1) = \sum_{i=0}^{n-1} P^b_{i,w}(k). \tag{28}$$

Therefore, using (28) and the orthogonal projection formula, $\alpha_{n,w}(k)$ and $\beta_{n,w}(k)$ can be rewritten respectively as

$$\alpha_{n,w}(k) = \phi - \sum_{i=0}^{n-1} P^b_{i,w}(k)\phi = \phi - \sum_{i=0}^{n-1} \frac{r_{i,w}(k)}{R^r_{i,w}(k)} b_{i,w}(k) \tag{29}$$

and

$$\beta_{n,w}(k) = \Psi - \sum_{i=0}^{n-1} P^b_{i,w}(k)\Psi = \Psi - \sum_{i=0}^{n-1} \frac{q_{i,w}(k)}{R^r_{i,w}(k)} b_{i,w}(k) \tag{30}$$

where $r_{i,w}(k)$, $q_{i,w}(k)$, and $R^r_{i,w}(k)$ have been already defined Table 1.

Substituting (29) into $\nu_{n,w}(k)$ and exploiting the orthogonality of the backward error vectors lead to

$$\nu_{n,w}(k) = 1 - \sum_{i=0}^{n-1} r_{i,w}(k)(R^r_{i,w}(k))^{-1} r_{i,w}(k), \tag{31}$$

thus

$$\nu_{n+1,w}(k) = \nu_{n,w}(k) - r_{n,w}(k)(R^r_{n,w}(k))^{-1} r_{n,w}(k). \tag{32}$$

Similarly, Substituting (30) into $\mu_{n,w}(k)$, we can get

$$\mu_{n+1,w}(k) = \mu_{n,w}(k) - q_{n,w}(k)(R^r_{n,w}(k))^{-1} q_{n,w}(k). \tag{33}$$

After the complete algorithm of the proposed lattice filter has been derived, it is shown in Table 2 for easy reference.

Table 2 The complete filter algorithm

$K_{n+1,w}(k) = K_{n+1,w-1}(k-1) + r_{n,w}(k-1)(\nu_{n,w}(k-1))^{-1} e_{n,w}(k)$
$K_{n+1,w-1}(k) = K_{n+1,w}(k) - q_{n,w}(k-1)(\mu_{n,w}(k-1))^{-1} d_{n,w}(k)$
$e_{n+1,w}(k) = \frac{1}{T} e_{n,w}(k) - \frac{1}{T} r_{n,w}(k-1)(R^r_{n,w}(k-1))^{-1} K_{n+1,w}(k)$
$d_{n+1,w}(k) = \frac{1}{T} d_{n,w}(k) - \frac{1}{T} q_{n,w}(k-1)(R^r_{n,w}(k-1))^{-1} K_{n+1,w}(k)$
$r_{n+1,w}(k) = r_{n,w}(k-1) - e_{n,w}(k)(R^e_{n,w}(k))^{-1} K_{n+1,w}(k)$
$q_{n+1,w}(k) = q_{n,w}(k-1) - d_{n,w}(k)(R^e_{n,w}(k))^{-1} K_{n+1,w}(k)$
$R^e_{n+1,w}(k) = \frac{1}{T^2} R^e_{n,w}(k) - \frac{1}{T^2} K_{n+1,w}(k)(R^r_{n,w}(k-1))^{-1} K_{n+1,w}(k)$
$R^r_{n+1,w}(k) = R^r_{n,w}(k-1) - K_{n+1,w}(k)(R^e_{n,w}(k))^{-1} K_{n+1,w}(k)$
$\nu_{n+1,w}(k) = \nu_{n,w}(k) - r_{n,w}(k)(R^r_{n,w}(k))^{-1} r_{n,w}(k)$
$\mu_{n+1,w}(k) = \mu_{n,w}(k) - q_{n,w}(k)(R^r_{n,w}(k))^{-1} q_{n,w}(k)$
with the initial conditions: for each $k \geqslant 0, n = 0$,
$e_{0,w}(k) = r_{0,w}(k) = y(k)$
$d_{0,w}(k) = q_{0,w}(k) = \begin{cases} y(k-w+1), & k \geqslant w-1 \\ 0, & k < w-1 \end{cases}$
$R^e_{0,w}(k) = R^r_{0,w}(k) = \langle Y_w(k), Y_w(k) \rangle$
$\nu_{0,w}(k) = \mu_{0,w}(k) = 1$
for $n+1 > k$, $K_{n+1,w}(k) = 0$

5 Discussion and Conclusion

In this paper, the complete algorithm of a sliding window delta operator-based adaptive lattice filter has been derived based on the geometric projection operation under least squares criteria. The unique property of the proposed filter is that it uses the finite number of data points to extract the underlying information of the signals to be processed. When the window length of the filter is reasonably short, such type of filters will be more sensitive to the change of the signals than infinite length filters. Such a property is extremely important in analyzing the time-varying nature of the signals. In fact, we have done a lot of simulation examples to support our theoretical results, though those examples are not included because of the limited space. We have successfully employed this filter to identify the transfer functions of continuous-time systems and detect the parameter changes in continuous-time system models. Additionally, it has been found out that as compared to the infinity memory delta operator-based lattice filter this newly proposed filter is more sensitive to the changes of system parameters.

Some open issues, i. e. the choice of window length, need to be further investigated. In addition, it should be pointed out that this filter algorithm is more complicated than that of the infinity memory delta operator-based lattice filter developed by Jabbari[3].

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基于δ算子的滑动窗自适应格形滤波器

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摘要: 本文利用最小二乘几何投影方法详细推导了一种基于滤波器δ算子的滑动窗自适应格形滤波器算法,并深入讨论了这种滤波器在系统辨识和参数变化检测中的应用.

关键词: 自适应格形滤波器; 系统辨识; 参数变化检测

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