第 15 卷第 6 期
1998 年 12 月

控制理论与应用
CONTROL THEORY AND APPLICATIONS

Vol. 15, No. 6
Dec., 1998

The Model-Free Learning Adaptive Control of a Class of Nonlinear Discrete-Time Systems *

HOU Zhongsheng

(Institute of Applied Systems Analysis, Northern Jiaotong University · Beijing, 100044, PRC)

HUANG Wenhu

(Department of Astronautics & Mechanics, Harbin Institute of Technology · Harbin, 150001, PRC)

Abstract: The model-free learning adaptive control (MFLAC) of a class of nonlinear discrete-time systems is presented in this paper. No structural information, mathematical model, external testing signals and training process are needed, it is designed only by using I/O data of the controlled systems. The unmodelled dynamics do not exist. The simulation results for several typical nonlinear systems are given to demonstrate the correctness and effectiveness of the approach proposed.

Key words: nonlinear systems; model-free learning adaptive control; adaptive control; pseudo-partial-derivative

1 Instruction

For the adaptive control of linear systems, the theory and design principles have already been understood quite well and the theoretical analysis methods are well established based on the "key technical lemma "[1] and "linear time-varying technique"[2]. However, a topic on nonlinear discrete-time system only a few papers have been devoted[3,4], and only some special nonlinearities have been considered. One of the reason[3,4] is that the Lyapunov design technique, an extremely useful tool in continuous-time, is of little use in nonlinear discrete-time systems because the increments of the parameter estimates do not appear linearly in the increments of Lyapunov function. Moreover the methods for directly adjusting the controller parameters based on the output error for general nonlinear discrete-time systems are not available[5].

As we have known, the dependence on mathematical model structure of the controlled system and the unmodelled dynamics are the two main inevitable problems for the traditional adaptive control theory, therefore the design of the adaptive control system only using the I/O data of controlled plant will be of great significance both in the development and applications of control theory. Two kinds of model-free control techniques applied successfully in practice are the PID typed control technique and the adaptive control by using the neural networks method, but they both suffer some limitations: The PID typed can only cope with linear time-invariant system, and the neural networks technique also has some problems which are very difficult to be overcome, such as, the need of known orders of system and high speed computer, the determination of numbers of nodes and hidden layers and how to carry out the theoretical analysis, etc.

* Supported by Natural Science Foundation of Heilongjiang (F9613).
Manuscript received Jun. 30, 1995, revised Mar. 20, 1998.

2 Plant Description and Model Transformation

Following discrete-time SISO nonlinear systems are considered:

$$y(k+1) = f(y(k), y(k-1), \cdots, y(k-n_y), u(k), u(k-1), \cdots, u(k-n_u)).$$

where n_y, n_u are orders of output $y(k)$ and input $u(k)$ respectively, $f(\cdots)$ is a nonline tion.

Rewrite (1) as

$$y(k+1) = f(Y(k), u(k), U(k-1)).$$

It is also called NARX model. Hammerstein model, bilinear model and some otl linear system models can be shown to be special cases of (1) or (2). Since the gene (1) or (2), the direct learning adaptive control of (1) or (2) will be equivalen MFLAC.

The MFLAC is based on the following assumptions made about the systems:

A1) System (1) or (2) is observable, and controllable in following meaning, th some expected system output bounded signal $y^*(k+1)$, there exists a bounded co put signal in time instant k, the output $y(k+1)$ controlled by it will be equal to the s $y^*(k+1)$.

A2) The partial derivative of $f(\cdot, u(k), \cdot)$ with respect to control input $u(k)$ is uous.

A3) The system (1) or (2) is generalized Lipschitz, that is, satisfying

$$|\Delta y(k+1)| \leqslant L|\Delta u(k)| \quad \text{for any } k \text{ and } \Delta u(k)$$

where L is a constant.

Above three assumptions made about the system are not too severe, the A1) is assumption about the controlled system, to control such a system is impossible if A satisfied. The A2) is a condition that a class of nonlinear systems can satisfy althou do exist some nonlinear systems which contradict it. The A3) is a limitation on th change of the system output, obviously which includes a class of nonlinear systems.

Theorem For the nonlinear system (2), satisfying Assumptions A1), A2) a then there must exist $\phi(k)$, called pseudo-partial-derivative, when $\Delta u(k) \neq 0$, we ha

$$\Delta y(k+1) = \phi(k)\Delta u(k),$$

and $|\phi(k)| \leqslant L$.

Proof (2) gives

$$\begin{aligned}\Delta y(k+1) &= f(Y(k), u(k), U(k-1)) - f(Y(k-1), u(k-1), U(k-2)) \\ &= f(Y(k), u(k), U(k-1)) - f(Y(k), u(k-1), U(k-1)) \\ &\quad + f(Y(k), u(k-1), U(k-1)) - f(Y(k-1), u(k-1), U(k-\end{aligned}$$

using Assumption A2) and the mean value theorem, (4) gives

$$\Delta y(k+1) = \frac{\partial \bar{f}}{\partial u(k)}\Delta u(k) + \xi(k).$$

where $\frac{\partial \bar{f}}{\partial u(k)}$ denotes the partial derivative value of $f(\cdots)$ with respect to u at some tween $u(k-1)$ and $u(k)$, and $\xi(k) = f(Y(k), u(k-1), U(k-1)) - f(Y(k-1), u$ $U(k-2))$.

Considering the equation with a variable $\eta(k)$

$$\phi(k) = \eta(k)\Delta u(k), \tag{6}$$

since condition $\Delta u(k) \neq 0$, equation (6) must have solution $\eta(k)$.

Let

$$\phi(k) = \frac{\partial \bar{f}}{\partial u(k)} + \eta(k), \tag{7}$$

using (6) and (7), the (3) is the direct result of (5).

By using (3) and Assumption A3, we know that $|\phi(k)| \leqslant L$. Q. E. D.

Remark 1 Pseudo-partial-derivative $\phi(k)$ is obviously a time-varying parameter even though the (1) or (2) is a time-invariant system. It is clearly that the $\phi(k)$ has some relations with inputs and outputs of the system till time instant k. The Theorem gives that $\phi(k)$ is a "differential" signal in some meaning and bounded for any k, so we have some reasons to say that $\phi(k)$ is a slowly time-varying parameter and the relation with the $u(k)$ can be ignored when the magnitude of $\Delta u(k)$ and the sampling period are not too large.

Remark 2 From Theorem and Remark 1, we know that (3) is a dynamic linear system with slowly time-varying parameter when $\Delta u(k) \neq 0$ and $\Delta u(k)$ is not too large. Therefore, besides the condition $\Delta u(k) \neq 0$ which will be considered in the control system design, some free adjustable parameter should be added in the control input criterion function, which is used to keep the rate of change of control input signal not vary too big.

3 The Model-Free Learning Control Algorithm

For the one-step-ahead controller[P122,1], excessive control effort may be called for to bring $y^*(k+1)$ to $y(k+1)$ in one step, particularly in the early stages of parameter tuning. The weighted one-step-ahead controller, in general, leads to steady-state tracking error. So we used the following control input criterion function

$$J(u(k)) = [(y(k+1) - y^*(k+1))^2 + \lambda(u(k) - u(k-1))^2], \tag{8}$$

where λ is a weighting constant.

Since the term $\lambda(\Delta u(k))^2$ is introduced in criterion function (8), the controller from minimizing it will overcome the defects of the controllers mentioned above.

Rewrite (3) as

$$y(k+1) = y(k) + \phi(k)\Delta u(k). \tag{9}$$

Substituting (9) into (8) and differentiating (8) with respect to $u(k)$ and setting it be zero give the control law as follows:

$$u(k) = u(k-1) + \frac{\rho_k \phi(k)}{\lambda + \phi^2(k)}[y^*(k+1) - y(k)]. \tag{10}$$

Remark 3 The ρ_k in control law algorithm (10) is a step-size constant series, which is added in (10) in order to get its generality.

Remark 4 From (9) and (10), we can see that λ is not only a penalty factor on $\Delta u(k)^2$, so the substitution scope of that system (2) is substituted by system (3) can be limited in some extent, which, as a result, makes pseudo-partial-derivative $\phi(k)$ not change too much, but also is a part of denominator in (10). This is an important parameter for this control system. Computer simulation results show that suitable choice of λ can improve the performance of the control system.

Remark 5 From control law algorithm (10), we can see that this kind of cont has an iterative learning form, it is different from the control law in [1], and has no re with any structural information (mathematical model, structure, orders) of controlled it is designed only by I/O data of controlled system.

4 The Estimation Algorithm of Pseudo-Partial-Derivative

The criterion function for parameter estimation often used in this situation is lows[1]

$$J(\phi(k)) = [(y(k+1) - y(k) - \phi(k)\Delta u(k))^2].$$

Parameter estimates by the algorithm derived from minimizing (11) often cha fast or too sensitive to some individual abrupt incorrect sampling data which may be by some instruments out of work or noise disturbance. In order to get an estimatio rithm which has robustness, we modify the criterion function (11) as follows

$$J(\phi(k)) = [(y(k+1) - y(k) - \phi(k)\Delta u(k))^2 + \mu(\phi(k) - \hat{\phi}(k-1))^2].$$

This is a new criterion function in parameter estimation. The item $\mu(\phi(k) - \hat{\phi}(k$ in criterion function (12) is to punish the rate of change of parameter estimation. Sin the situation in time instant k is considered, the estimation algorithm derived by min (12) should have the ability to track the time-varying parameters. Using the procedu lar to Section 3, we can obtain the parameter estimation algorithm as follows

$$\hat{\phi}(k) = \hat{\phi}(k-1) + \frac{\eta_k \Delta u(k-1)}{\mu + \Delta u(k-1)^2}[\Delta y(k) - \hat{\phi}(k-1)\Delta u(k-1)].$$

Remark 6 Remarks similar to Remark 3~5 can also be listed for (13).

Remark 7 The differences between the algorithm (13) and the projection al (also known as NLMS algorithm) are as follows: the addition of the small constant denominator of the NLMS algorithm is only for avoiding division by zero, no practica ing, but μ here in algorithm (13) is a weighting constant which punishes the rate of ch parameter estimate, and the methods which the NLMS and the (13) are derived are ferent, the algorithm (13) is obtained by minimizing the new criterion function (12).

5 The MFLAC Scheme

Using the parameter estimation algorithm, the learning adaptive control law al developed in Section 3 and 4 and the discussion above, we give the MFLAC scheme as

$$\hat{\phi}(k) = \hat{\phi}(k-1) + \frac{\eta_k \Delta u(k-1)}{\mu + \Delta u(k-1)^2}[\Delta y(k) - \hat{\phi}(k-1)\Delta u(k-1)],$$

$$\hat{\phi}(k) = \hat{\phi}(k-1) \quad \text{if} \quad |\hat{\phi}(k)| \leqslant \varepsilon,$$

$$u(k) = u(k-1) + \frac{\rho_k \hat{\phi}(k)}{\lambda + \hat{\phi}^2(k)}[y^*(k+1) - y(k)].$$

where the step-size series $\rho_k, \eta_k \in (0,2)$, and λ, μ are two weighting constants, ε is positive constant, $\hat{\phi}(1)$ is the initial estimation value of $\phi(k)$.

Remark 8 In MFLAC scheme above, the number of the controller parameters needed to be adjusted on-line for the SISO nonlinear system is only one, that is, pseudo-partial-derivative. It is quite different from and much fewer than that of the traditional adaptive control system[1].

Remark 9 The MFLAC system (14~16) has nothing with the controlled system except the I/O data, this is the reason why we call it MFLAC.

As we have known, the designing of the controller and estimator of traditional adaptive control system depends on the structure and the orders of mathematical model of controlled system, but the structure and the orders of controlled system are very difficult to identify, and sometimes have relations with time and environment, so the applications of various adaptive control systems reported may be failure due to the unmodelled dynamics. The MFLAC system presented in this paper only use the I/O data of controlled system, the unmodelled dynamics disappear, therefore it should have strong robustness.

6 Some Selected Simulation Results

In this section, simulation results for two typical discrete SISO nonlinear systems are given to demonstrate the effectiveness of the model-free learning adaptive control scheme proposed. All the models below are only used for collection of I/O data. The simulations were performed using the MATLAB software.

The system's initial values of following two simulation examples are set to be $u(1) = u(2) = 0, y(1) = -1, y(2) = 1, y(3) = 0.5, \hat{\phi}(1)$ is set to be 2, ε is set to 10^{-5}. The step size are set to be that $\eta_k = 1, \rho_k = 0.6$.

Example 1

$$y(k+1) = \begin{cases} \dfrac{y(k)}{1+y(k)^2} + u(k)^3, & k \leqslant 500, \\ \dfrac{y(k)y(k-1)y(k-2)u(k-1)(y(k-2)-1) + a(k)u(k)}{1+y(k-1)^2+y(k-2)^2}, & k > 500. \end{cases}$$

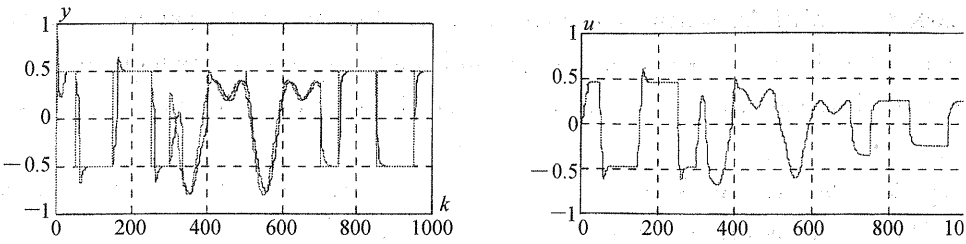
This system consists of two nonlinear subsystems. The structure, orders, and parameters are all time-varying. Both two subsystems were used for simulation separately in [5] by using neural networks, but in their simulation, the second part had no time-varying parameter $a(k)$. $a(k) = 1 + \mathrm{round}(i/500)$.

When

$$y^*(k+1) = \begin{cases} 0.5(-1).\hat{}\ \mathrm{round}(k/100), & k \leqslant 300, \\ 0.5\sin(k\pi/100) + 0.3\cos(k\pi/50), & 300 < k \leqslant 700, \\ 0.5(-1).\hat{}\ \mathrm{round}(k/100), & 700 < k \leqslant 1000 \end{cases}$$

and set the weighting constants $\lambda = 2, \mu = 1$, the simulation results are shown by Fig. 1(a) and (b).

If we set the weighting constants $\lambda = 1, \mu = 1$, the closed-loop response becomes more active than the Fig. 1, but the overshoots are bigger. If we set $\lambda = 0, \mu = 1$, the system oscillates. The simulation results are omitted due to the volume limited.



(a) Tracking performance (b) Control input

Fig. 1 Simulation results for the MFLAC of Example 1

Example 2

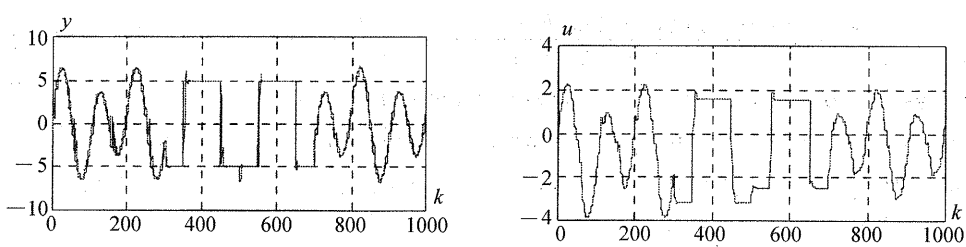
$$y(k+1)=\begin{cases}\dfrac{5y(k)y(k-1)}{1+y(k)^2+y(k-1)^2+y(k-2)^2}+u(k)+1.1u(k-1), & k\leqslant \\ \dfrac{2.5y(k)y(k-1)}{1+y(k)^2+y(k-1)^2}+0.7\sin(0.5(y(k)+y(k-1))) & \\ \times\cos(0.5(y(k)+y(k-1)))+1.4u(k-1)+1.2u(k), & k>\end{cases}$$

This system consists of two nonlinear nonminimum phase subsystems connectec cade, the orders of the system are time-varying. Both subsystems were used for sir separately in [5,6] by using neural networks. However, the first subsystem can not trolled effectively by the conventional neural network controller[5]. The second one v for simulation without term $1.4\,u(k-1)$ in right side is a minimum phase system[6

When

$$y^*(k+1)=\begin{cases}5\sin(k\pi/50)+2\cos(k\pi/100), & k\leqslant 300,\\ 5(-1).\hat{}\ \mathrm{round}(k/100), & 300<k\leqslant 700,\\ 5\sin(k\pi/50)+2\cos(k\pi/100), & 700<k\leqslant 1000\end{cases}$$

set the weighting constants $\lambda=3$, $\mu=1$, the simulation results are shown in Fig. 2 (b).



(a) Tracking performance (b) Control input

Fig. 2 Simulation results for the MFLAC of Example 2

If we set the weighting constants $\lambda=1$, $\mu=1$, the overshoots are bigger than th Fig. 2, and the transient responses are a little faster. If we set the weighting constant 1, $\mu=1$, the system diverges. The simulation results are omitted.

7 Conclusions

In contrast to other adaptive control schemes, the features of this new typed control technique are as follows: First, the proposed MFLAC scheme only use the I/C controlled system. No mathematical model and structural information of controlled are needed, which implies that no unmodelled dynamics exists. Second, the MFLAC r nism does not require any external testing signals and any training process, which are necessary to the nonlinear system adaptive control by using neural networks. Third, the scheme proposed is simple and can be easily used, and has minimum computational burden and strong robustness. Finally, all the results of this paper can be extended easily to the MISO and MIMO nonlinear cases. There are some reasons to believe that the same methods can be used successfully in practice. Hence they should find widely applications in many areas of industrial process control.

Further works should focus on the stability analysis.

References

1 Goodwin, G. C. and Sin, K. S.. Adaptive Filtering, Prediction and Control. Englewood Cliffs, NJ: Prentice Hall, 1984

2 Guo, L.. Time-Varying Stochastic Systems: Stability, Estimation and Control. Changchun: Jilin Science and Technology Press, 1993

3 Kanellakopoulos, I.. A discrete-time adaptive nonlinear systems. IEEE Trans. Automat. Contr., 1994, 39(11): 2362—2365

4 Yeh, Po-Chiang and Kokotovic, P. V.. Adaptive control of a class of nonlinear discrete-time systems. Int. J. Control, 1995, 62(2): 303—324

5 Narendra, K. S. and Parthasarathy, K.. Identification and control of dynamical systems using neural networks. IEEE Trans. Neural Networks, 1990, 1(1): 4—27

6 Chen, F. C. and Khalil, H. K.. Adaptive control of nonlinear system using neural networks. Proc. of the 29th Conf. on Decision and Control, Hawaii, 1990, 1707—1712

一类离散时间非线性系统不依赖受控系统数学模型的学习自适应控制

侯忠生

(北方交通大学应用系统分析研究所·北京,100044)

黄文虎

(哈尔滨工业大学航天工程与力学系·哈尔滨,150001)

摘要：给出了一类离散时间非线性系统的不依赖受控系统数学模型的学习自适应控制方案,它不需要受控系统的结构信息、数学模型、外部试验信号和训练过程,仅用受控系统的I/O数据来设计,传统的未建模动态不存在,所给出的计算机仿真结果说明了所给出的方案的正确性和有效性.

关键词：非线性系统；不依赖受控系统数学模型的学习；自适应控制；伪偏导数

本文作者简介

侯忠生 1962年生.副教授.1994年获东北大学控制理论及应用专业博士学位,1997年哈尔滨工业大学航天学院博士后出站到北方交通大学交通运输学院工作.目前主要从事非线性系统自适应控制及无模型学习自适应控制,智能交通系统的研究工作.

黄文虎 1936年生.教授.中国工程院院士.长期从事振动工程,动力学控制及故障诊断的研究工作.