Blind Identifiability of Truncated Quadratic Nonlinear Models Using Spectra Analyses

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Abstract: The blind identification methodology for one sort of second-order Volterra models-truncated quadratic nonlinear models based on spectra analyses is developed in this paper. The discussions are shown that it is efficient and accurate as per demonstrated by the simulations.

Key words: blind model identification; truncated quadratic nonlinear models; autocorrelations; steepest descent adaptive strategy

1 Introduction

Truncated Volterra nonlinear models identification, which involves determination of time domain model kernels or frequency domain model kernels, is a very important issue in many practical engineering problems[1,2]. Due to the difficulty in the spectra estimations of the non-linear models, available model input signals were usually assumed to be Gaussian or i. i. d. random processes[3,4]. In this paper, however, we will show that the truncated second-order Volterra models can be identified in the time domain based on some properties of the model output statistics only, that is, the model kernels can be determined blindly. The result is shown accurate as per verified by the simulations. Besides, the methodology is very general and can be readily applied to the engineering practice.

2 Blind Identifiability of Quadratic Models

Consider a stochastic quadratic nonlinear model

$$y(t) = \sum_{i=0}^{q}\sum_{j=0}^{q} h(i,j)x(t-i)x(t-j) \tag{1}$$

where the excited sequences of the model $\{x(t)\}$ are unobservable zero-mean, i. i. d. signals with $\gamma_{4x} = E[x^4(t)] \neq 0$, and $\gamma_{2x} = E[x^2(t)] \neq 0$. Unknown quadratic kernels include $\{h(i,j); \forall i = 0,1,\cdots,q, j = 0,1,\cdots,q\}$. Without any loss of generality, the kernels satisfy the following properties i) symmetrical: $h(i,j) = h(j,i), \forall i \neq j$; ii) bounded: $h(i,j) = 0, \forall i,j > q$; iii) causal: $h(i,j) = 0, \forall i,j < 0$; and iv) stable: $\sum_{i,j} |h(i,j)| < \infty$. The second-order moments (autocorrelations) of the output sequences $\{y(t)\}$ for model (1) can be defined as

$$M_2^y(m) = E[y(t)y(t+m)] \tag{2}$$

where E denotes an expectation operation and the discrete time quantity m is usually termed as the lag of the autocorrelation and $m \leqslant q$. The following lemma then hold in this study.

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Lemma If $\{x(t)\}$ is i. i. d. and zero-mean, then the autocorrelation of quadratic model (1) excited by $\{x(t)\}$ is given by

$$M_2^y(m) = \gamma_{4x}\sum_{i=0}^{q-m} h(i,i)h(i+m,i+m) + \gamma_{2x}^2\sum_{i=0}^{q}\sum_{\substack{j=0\\ j\neq i+m}}^{q} h(i,i)h(j,j) + 4\gamma_{2x}^2\sum_{i=0}^{q-m}\sum_{j=i+1}^{q-m} h(i,j)h(i+m,j+m). \tag{3}$$

Proof For i. i. d. random signals $\{x(t)\}$, one can prove the following properties

$$E[x(t_1)x(t_2)x(t_3)x(t_4)] = \begin{cases}\gamma_{4x}, & t_1=t_2=t_3=t_4,\\ \gamma_{2x}^2, & t_1=t_2\neq t_3=t_4, \text{or } t_1=t_4\neq t_2=t_3, \text{or } t_1=t_3\neq t_2=t_4,\\ 0, & \text{otherwise}.\end{cases} \tag{4}$$

a) For $i=j$ terms of Equ. (1), the following polynomial can be extended as

$$E\{[\sum_{i=0}^{q} h(i,i)x^2(t-i)][\sum_{i=0}^{q} h(i,i)x^2(t+m-i)]\} = \gamma_{4x}\sum_{i=0}^{q-m} h(i,i)h(i+m,i+m) + \gamma_{2x}^2\sum_{i=0}^{q}\sum_{\substack{j=0\\ j\neq i+m}}^{q} h(i,i)h(j,j). \tag{5}$$

b) For $i\neq j$ terms of Equ. (1), with polynomial multiplication formula and Equ. (4) in mind we obtain

$$E[\sum_{i=0}^{q}\sum_{j=0}^{q} h(i,j)x(t-i)x(t-j)][\sum_{i=0}^{q}\sum_{j=0}^{q} h(i,j)x(t+m-i)x(t+m-j)] = 4\gamma_{2x}^2\sum_{i=0}^{q-m}\sum_{j=i+1}^{q-m} h(i,j)h(i+m,j+m), \quad i\neq j. \tag{6}$$

By combining Equ. (5) and Equ. (6) we get the desired autocorrelation expression for nonlinear model (1).

In virtue of the features of the model kernels, $M_2^y(m)\neq 0$ only for $\{h(i,j): i+m\leqslant q, j+m\leqslant q\}$. Additionally, some of the nonzero $M_2^y(m)$ are identical according to the symmetries of autocorrelations. Therefore, (3) gives $(q+1)$ equations which include $\frac{(q+1)(q+2)}{2}$ unknown parameters. Obviously the important question herein is the unique solution of the identified model kernels. As we know, a leading idea of our framework for model identification should be that the effort spent in developing a model of a system must be related to the application it is going to be used for. The quadratic approximation to the nonlinear model under investigation aims at obtaining an accurate identification target in preference to the linear approximation[4]. Therefore certain truncated quadratic models may be sufficiently suitable for the engineering practice and the proposed underdetermined equation (3) can be transformed into a determined or overdetermined equation which has the unique solution. In this study the following least-squares (LS) cost function and the steepest descent strategy is addressed to solve the problem.

$$J = \frac{1}{2}[M_2^y(m) - \gamma_{4x}\sum_{i=0}^{q-m} h(i,i)h(i+m,i+m) - \gamma_{2x}^2\sum_{i=0}^{q}\sum_{\substack{j=0\\ j\neq i+m}}^{q} h(i,i)h(j,j)$$

$$- 4\gamma_{2x}^2 \sum_{i=0}^{q-m} \sum_{j=i+1}^{q-m} h(i,j)h(i+m,j+m)]^2, \quad (7)$$

$$h(i,j)_{n+1} = h(i,j)_n - \eta \frac{\partial J}{\partial h(i,j)_n}, \quad i,j = 0,1,\cdots,q \quad (8)$$

where η denotes the step-size parameter which satisfies $0 \leqslant \eta \leqslant 1$. It is worth noting that the above mentioned autocorrelations involve expectation operations. They cannot be computed in an exact manner from available real output signals, but can be approximated consistently by replacing expectations by sample averages, e.g.,

$$\hat{M}_2^y(m) = \frac{1}{N_\Omega} \sum_{t \in \Omega} y(t)y(t+m) \quad (9)$$

where N_Ω is the number of samples in region Ω. We substitute $M_2^y(m)$ of Eqn. (7) by the estimated autocorrelations $\hat{M}_2^y(m)$ in (9) to identify the quadratic kernels.

3 Simulations

In the experiment, the following two truncated quadratic models are simulated by computers.

$$y(t) = x^2(t) - 1.62x^2(t-1) + 1.41x^2(t-2),$$

$$y(t) = 7.44x(t)x(t-1) + 1.52x(t-1)x(t-2) + 3.72x(t)x(t-2).$$

As general, an independent exponentially distributed random sequence $\{x(t)\}$ with zero-mean is generated as the input signal. To estimate the autocorrelations in (9) we use 1024 output samples which are computed by convolving the random input signals with the given models. In case of noisy output observations, available output signals are $y_{noise}(t) = y(t) + n(t)$, where $n(t)$ is a sequence of Gaussian noises which is independent of the model output $y(t)$. The signal-to-noise ratio (SNR) is defined as $\mathrm{SNR} = 10\log_{10}(\frac{r_{2y}^2}{r_{2n}^2}) = 50$ dB. To verify the theoretical results, 50 Monte-Carlo runs are used and the value used for the step-size parameter η is supposed to 0.1 herein. Table 1 and Table 2, which respectively summarize the estimated model kernel means and variances by using the methodology developed in Section 2 compared with their corresponding true values, show that the mean values are quite accurate estimates of the model kernels with small estimating variances.

Table 1 True and estimated (means ± variances) kernels of the quadratic model under SNR = 50 dB and 50 Monte-Carlo runs for model 1

	$h(0,0)$	$h(1,1)$	$h(2,2)$
true values	1	−1.62	1.41
estimated h (0,0)	0.9914 ±0.0628	—	—
estimated h (1,1)	—	−1.6213 ±0.0891	—
estimated h (2,2)	—	—	1.4071 ±0.0448

Table 2 True and estimated (means ± variances) kernels of the quadratic model under SNR = 50 dB and 50 Monte-Carlo runs for model 2

	$h(0,1)$	$h(1,2)$	$h(0,2)$
true values	3.72	0.76	1.86
estimated h (0,1)	3.7102 ±0.00913	—	—
estimated h (1,2)	—	0.7611 ±0.1014	—
estimated h (0,2)	—	—	1.8588 ±0.0987

4 Conclusion

The algorithm for blindly identifying truncated quadratic nonlinear models based on model output statistics is derived. It is indicated that the proposed technique is accurate and very general which can be applied to the practical identification engineering.

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基于谱分析的一类截尾二阶非线性模型盲辨识的方法

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摘要：提出一种基于自相关谱分析的对一类截尾 Volterra 非线性模型-二阶非线性模型进行盲辨识的算法．理论分析和计算机仿真均显示，该算法准确通用，适合于实际工程应用．

关键词：盲辨识；二阶非线性模型；自相关谱；最速降寻优算法

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