具有多个独立执行机构的 Lurie 控制系统的鲁棒绝对稳定性

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摘要：本文讨论了具有多个相互独立执行机构的 Lurie 型区间直接控制系统和区间间接控制系统的绝对稳定性，给出了系统稳定的若干充分条件，对文[5]所讨论的问题作了进一步探讨.

关键词：区间矩阵；区间直接控制系统；区间间接控制系统；绝对稳定性；Lyapunov 函数

Robust Absolute Stability for Lurie Control systems with Several Independent Stationary Components

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Abstract: The problem of robust absolute stability for Lurie type interval direct control systems and interval indirect control systems with several independent nonlinear stationary elements are consideried. Some sufficient conditions of robust absolute stability for Lurie interval direct control systems and interval in direct control systems are respectively given. The problem studied in paper [5] have been profoundly discussed.

Key words: interval matrices; interval direct control system; interval indirect control system; absolute stability; Lyapunov function

具有多个执行机构的 Lurie 型控制系统是一类非常重要的非线性控制系统，该系统的稳定性研究在非线性控制系统的设计中具有十分重要的意义.近来年，国内外许多学者对具有多个执行机构的 Lurie 型控制系统的稳定性进行了广泛的研究，得到了一些很好的结果[1~4].八十年代以来，区间动力系统的鲁棒稳定性研究引起了国内外学者的广泛兴趣，但到目前为止对非线性区间动力系统稳定性的讨论还不多见.作者在文[5,6]中讨论了高为炳院士等提出的 Lurie 型区间控制系统的鲁棒稳定性，给出了保证系统绝对稳定的若干充分条件.本文中我们将讨论具有多个相互独立执行机构的 Lurie 型区间直接控制系统和 Lurie 型区间间接控制系统的绝对稳定性.

1 直接控制系统的绝对稳定性(Absolute stability of direct control systems)

考虑直接控制系统

$$\begin{cases} \dot{x}_i = G[B_{ii}, C_{ii}]x_i + \sum\limits_{j=1, i\neq j}^{n} G[B_{ij}, C_{ij}]x_j + G[R_i, S_i]f_i(\sigma_i), \\ \sigma_i = c_i^{\mathrm{T}}x_i, f_i(\sigma_i) \in K_i[0, \infty]; \ i = 1,2,\cdots,r. \end{cases} \tag{1.1}$$

这里，$K_i[0,\infty] = \{f_i(\cdot) \mid f_i(0) = 0, 0 \leqslant \sigma_i f_i(\sigma_i) \leqslant +\infty\}$；$i = 1,2,\cdots,r$. $G[B_{ij}, C_{ij}] = \{A_{ij} \mid B_{ij} \leqslant A_{ij} \leqslant C_{ij}\}$ 为 $n_i \times n_j$ 阶区间矩阵，$G[R_i, S_i]$ 为 $n_i \times 1$ 阶区间矩阵，$A_{ij} = (a_{kl}^{(ij)})_{n_i \times n_j}$，$B_{ij} = (b_{kl}^{(ij)})_{n_i \times n_j}$，$C_{ij} = (c_{kl}^{(ij)})_{n_i \times n_j}$ 为 $n_i \times n_j$ 阶区间矩阵；$i,j = 1,2,\cdots,r$. $R_i = (r_1^{(i)}, r_2^{(i)}, \cdots, r_{n_i}^{(i)})^{\mathrm{T}}$，$S_i = (s_1^{(i)}, s_2^{(i)}, \cdots, s_{n_i}^{(i)})^{\mathrm{T}}$，$b_i = (b_1^{(i)}, b_2^{(i)}, \cdots, b_{n_i}^{(i)})^{\mathrm{T}}$，$c_i = (c_1^{(i)}, c_2^{(i)}, \cdots, c_{n_i}^{(i)})^{\mathrm{T}}$ 为 n 维向量，$i = 1,2,\cdots,r$，$x_i = (x_1^{(i)}, x_2^{(i)}, \cdots, x_{n_i}^{(i)})^{\mathrm{T}}$，$\sum\limits_{i=1}^{r} n_i = n$.

在本文中我们约定：对任意 $l \times m$ 阶矩阵 $M = (m_{ij})_{l\times m}$，矩阵$(|M|) = (|m_{ij}|)_{l\times m}$.

对于任意的 $A_{ij} \in G[B_{ij}, C_{ij}]$，$b_i \in G[R_i, S_i]$，考虑系统：

$$\begin{aligned} \dot{x}_i &= A_{ii}x_i + \sum_{j=1, i\neq j} A_{ij}x_j + b_i f_i(\sigma_i); \\ \sigma_i &= c_i^{\mathrm{T}}x_i, \ i = 1,2,\cdots,r. \end{aligned} \tag{1.2}$$

设：$A_{ij}^{(0)} = \dfrac{1}{2}(B_{ij} + C_{ij})$，$K_{ij} = \dfrac{1}{2}(C_{ij} - B_{ij})$，$b_i^{(0)} = \dfrac{1}{2}(R_i + S_i)$，$K_i = \dfrac{1}{2}(R_i - S_i)$，则该系统可化为

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$$\dot{x}_i = A_{ii}^{(0)}x_i + \sum_{j=1,i\neq j}^{r} A_{ij}^{(0)}x_j + b_i^{(0)}f_i(\sigma_i) + (A_{ii} - A_{ii}^{(0)})x_i + \sum_{j=1,i\neq j}^{r}(A_{ij} - A_{ij}^{(0)})x_j + (b_i - b_i^{(0)})f_i(\sigma_i), \quad (1.3)$$

$$\sigma_i = c_i^{\mathrm T}x_i;\quad i = 1,2,\cdots,r.$$

假定 $A_{ii}^{(0)}$ 稳定,则存在正定矩阵 P_i 满足 Lyapunov 方程 $A_{ii}^{(0)^{\mathrm T}}P_i + P_iA_{ii}^{(0)} = -2I_{n_i}$. 设

$$z_i = ((|x_i|)^{\mathrm T}, |f_i(\sigma_i)|)^{\mathrm T},\quad V_i = \begin{pmatrix} 2I_{n_i} & -U_{ii}^{\mathrm T} \\ -U_{ii} & -\alpha_{ii}\end{pmatrix};\quad V_{ii} = \begin{pmatrix} \overline{W}_{ii} & \overline{U}_{ii}^{\mathrm T} \\ \overline{U}_{ii} & \bar{\alpha}_{ii}\end{pmatrix};\quad V_{ij} = \begin{pmatrix} \overline{W}_{ij} & \overline{U}_{ji}^{\mathrm T} \\ \overline{U}_{ij} & 0\end{pmatrix};$$

$$U = \begin{bmatrix} \lambda_{\min}(V_1) - \|V_{11}\| & -\frac{1}{2}(\|V_{12}\| + \|V_{21}\|) & \cdots & -\frac{1}{2}(\|V_{1r}\| + \|V_{r1}\|) \\ -\frac{1}{2}(\|V_{12}\| + \|V_{21}\|) & \lambda_{\min}(V_2) - \|V_{22}\| & \cdots & -\frac{1}{2}(\|V_{2r}\| + \|V_{r2}\|) \\ \cdots & \cdots & \cdots & \cdots \\ -\frac{1}{2}(\|V_{1r}\| + \|V_{r1}\|) & -\frac{1}{2}(\|V_{2r}\| + \|V_{r2}\|) & \cdots & \lambda_{\min}(V_r) - \|V_{rr}\| \end{bmatrix}.$$

这里:

$$U_{ii} = b_i^{\mathrm T^{(0)}}P_i + \frac{1}{2}\theta_i c_i^{\mathrm T}A_{ii}^{(0)} + \frac{1}{2}c_i^{\mathrm T};$$

$$\alpha_{ii} = \theta_i c_i^{\mathrm T}b_i^{(0)},$$

$$\overline{W}_{ii} = K_{ii}^{\mathrm T}(|P_i|) + (|P_i|)K_{ii};$$

$$\overline{U}_{ii} = K_i^{\mathrm T}(|P_i|) + \frac{1}{2}\theta_i(|c_i|)^{\mathrm T}K_{ii};$$

$$\bar{\alpha}_{ii} = \theta_i(|c_i|)^{\mathrm T}K_i;$$

$$\overline{W}_{ij} = 2[(|P_i|)(|A_{ij}^{(0)}|) + K_{ii}];$$

$$\overline{U}_{ij} = \frac{1}{2}\theta_i(|c_i|)^{\mathrm T}[(|A_{ij}^{(0)}|) + K_{ij}].$$

则可得下面定理:

定理 1.1 设存在正定矩阵 $P_i(A_{ii}^{(0)^{\mathrm T}}P_i + P_iA_{ii}^{(0)} = -2I_{n_i})$ 和正数 $\theta_i(i = 1,2,\cdots,r)$;使得矩阵 U 正定,则系统(2.1)稳定.

证 取 Lyapunov 函数: $V(x) = \sum_{i=1}^{r}[x_i^{\mathrm T}P_ix_i + \theta_i\int_0^{\sigma_i}f_i(\sigma_i)\mathrm d\sigma_i]$, 则

$$\frac{\mathrm dV(x)}{\mathrm dt}\Big|_{(1.3)} =$$

$$\sum_{i=1}^{r}[(\dot{x}_i^{\mathrm T}P_ix_i + x_i^{\mathrm T}P_i\dot{x}_i + \theta_i\dot{\sigma}_i f_i(\sigma_i))]_{(1.3)} =$$

$$\sum_{i=1}^{r}[x_i^{\mathrm T}(A_{ii}^{\mathrm T^{(0)}}P_i + P_iA_{ii}^{(0)})x_i + \sum_{j=1,i\neq j}^{m}(x_j^{\mathrm T}A_{ij}^{\mathrm T^{(0)}}P_ix_i + x_i^{\mathrm T}P_iA_{ij}^{(0)}x_j) + b_i^{\mathrm T^{(0)}}P_ix_if_i(\sigma_i) + x_i^{\mathrm T}P_ib_i^{(0)}f_i(\sigma_i) +$$

$$\theta_ic_i^{\mathrm T}(A_{ii}^{(0)}x_i + \sum_{j=1,i\neq j}^{r}A_{ij}^{(0)}x_j + b_i^{(0)}f_i(\sigma_i))f_i(\sigma_i) + x_i^{\mathrm T}((A_{ii} - A_{ii}^{(0)})^{\mathrm T}P_i + P_i(A_{ii} - A_{ii}^{(0)}))x_i +$$

$$\sum_{j=1,i\neq j}^{m}(x_j^{\mathrm T}(A_{ij} - A_{ij}^{(0)})^{\mathrm T}P_ix_i + x_i^{\mathrm T}P_i(A_{ij} - A_{ij}^{(0)})x_j) + (b_i - b_i^{(0)})^{\mathrm T}P_ix_if_i(\sigma_i) + x_i^{\mathrm T}P_i(b_i - b_i^{(0)})f_i(\sigma_i) + \theta_ic_i^{\mathrm T}((A_{ii} - A_{ii}^{(0)})x_i +$$

$$\sum_{j=1,i\neq j}^{r}(A_{ij} - A_{ij}^{(0)})x_j + (b_i - b_i^{(0)})f_i(\sigma_i))f_i(\sigma_i)] \leqslant$$

$$\sum_{i=1}^{r}[-2x_i^{\mathrm T}x_i + 2(b_i^{\mathrm T^{(0)}}P_i + \frac{1}{2}\theta_ic_i^{\mathrm T}A_{ii}^{(0)} + \frac{1}{2}c_i^{\mathrm T})x_if_i(\sigma_i) + \theta_ic_i^{\mathrm T}b_i^{(0)}f_i^2(\sigma_i)] +$$

$$\sum_{i=1}^{r}\sum_{j=1,j\neq i}^{r}[(|x_j|)^{\mathrm T}(|A_{ij}^{\mathrm T^{(0)}}|)^{\mathrm T}(|P_i|)(|x_i|) + (|x_i|)^{\mathrm T}(|P_i|)(|A_{ij}^{(0)}|)(|x_j|) + \theta_i(|c_i|)^{\mathrm T}(|A_{ij}^{(0)}|)(|x_j|)|f_i(\sigma_i)|] +$$

$$\sum_{i=1}^{r}[(|x_i|)^{\mathrm T}(K_{ii}^{\mathrm T}(|P_i|) + (|P_i|)K_{ii})(|x_i|) + 2(K_i^{\mathrm T}(|P_i|) + \frac{1}{2}\theta_i(|c_i|)^{\mathrm T}K_{ii})(|x_i|)|f_i(\sigma_i)| + \theta_i(|c_i|)^{\mathrm T}K_i|f_i(\sigma_i)|^2] +$$

$$\sum_{i=1}^{r}\sum_{j=1,j\neq i}^{r}[(|x_j|)^{\mathrm T}K_{ij}^{\mathrm T}(|P_i|)(|x_i|) + (|x_i|)^{\mathrm T}(|P_i|)K_{ij}(|x_j|) + \frac{1}{2}\theta_i(|c_i|)^{\mathrm T}K_{ij}(|x_j|)|f_i(\sigma_i)|] - \sum_{i=1}^{r}\sigma f(\sigma_i) =$$

$$\sum_{i=1}^{r}[-2x_i^{\mathrm T}x_i + 2(b_i^{\mathrm T^{(0)}}P_i + \frac{1}{2}\theta_ic_i^{\mathrm T}A_{ii}^{(0)} + \frac{1}{2}c_i^{\mathrm T})x_if_i(\sigma_i) + \theta_ic_i^{\mathrm T}b_i^{(0)}f_i^2(\sigma_i)] + \sum_{i=1}^{r}[(|x_i|)^{\mathrm T}(K_{ii}^{\mathrm T}(|P_i|) + (|P_i|)K_{ii})(|x_i|) + 2(K_i^{\mathrm T}(|P_i|) + \frac{1}{2}\theta_i(|c_i|)^{\mathrm T}K_{ii})(|x_i|)|f_i(\sigma_i)| +$$

$$\theta_i(|c_i|)^{\mathrm{T}}K_i|f_i(\sigma_i)|^2 + \sum_{i=1}^{r}\sum_{j=1,j\neq i}^{r}[(|x_j|)^{\mathrm{T}}((|A_{ij}^{\mathrm{T}(0)}|)^{\mathrm{T}} + K_{ij}^{\mathrm{T}})(|P_i|)(|x_i|) + (|x_i|)^{\mathrm{T}}(|P_i|)((|A_{ij}^{(0)}|) + K_{ij})(|x_j|) + \theta_i(|c_i|)^{\mathrm{T}}((|A_{ij}^{(0)}|) + K_{ij})(|x_j|)|f_i(\sigma_i)|] - \sum_{i=1}^{r}\sigma_i f(\sigma_i) =$$

$$-\sum_{i=1}^{r}\begin{bmatrix} x_i \\ f_i(\sigma_i)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix} 2I_{n_i} & -U_{ii}^{\mathrm{T}} \\ -U_{ii} & \alpha_{ii}\end{bmatrix}\begin{bmatrix} x_i \\ f_i(\sigma_i)\end{bmatrix} + \sum_{i=1}^{r}\begin{bmatrix} (|x_i|) \\ |f_i(\sigma_i)|\end{bmatrix}^{\mathrm{T}}\begin{bmatrix} \overline{W}_{ii} & \overline{U}_{ii}^{\mathrm{T}} \\ \overline{U}_{ii} & \bar{\alpha}_{ii}\end{bmatrix}\begin{bmatrix} (|x_i|) \\ |f_i(\sigma_i)|\end{bmatrix} +$$

$$\sum_{i=1}^{r}\sum_{j=1,i\neq j}^{r}\begin{bmatrix} (|x_i|) \\ |f_i(\sigma_i)|\end{bmatrix}^{\mathrm{T}}\begin{bmatrix} \overline{W}_{ij} & \overline{U}_{ji}^{\mathrm{T}} \\ \overline{U}_{ij} & 0\end{bmatrix}\begin{bmatrix} (|x_j|) \\ |f_j(\sigma_j)|\end{bmatrix} - \sum_{i=1}^{r}\sigma_i f(\sigma_i) \leqslant$$

$$-\sum_{i=1}^{r}\lambda_{\min}(V_i)\begin{bmatrix} (|x_i|) \\ |f_i(\sigma_i)|\end{bmatrix}^{\mathrm{T}}\begin{bmatrix} (|x_i|) \\ |f_i(\sigma_i)|\end{bmatrix} + \sum_{i=1}^{r}\begin{bmatrix} (|x_i|) \\ |f_i(\sigma_i)|\end{bmatrix}^{\mathrm{T}}\begin{bmatrix} \overline{W}_{ii} & \overline{U}_{ii}^{\mathrm{T}} \\ \overline{U}_{ii} & \bar{\alpha}_{ii}\end{bmatrix}\begin{bmatrix} (|x_i|) \\ |f_i(\sigma_i)|\end{bmatrix} +$$

$$\sum_{i=1}^{r}\sum_{j=1,i\neq j}^{r}\begin{bmatrix} (|x_i|) \\ |f_i(\sigma_i)|\end{bmatrix}^{\mathrm{T}}\begin{bmatrix} \overline{W}_{ij} & \overline{U}_{ji}^{\mathrm{T}} \\ \overline{U}_{ij} & 0\end{bmatrix}\begin{bmatrix} (|x_j|) \\ f_j(\sigma_j)|\end{bmatrix} - \sum_{i=1}^{r}\sigma_i f(\sigma_i) \leqslant$$

$$-\sum_{i=1}^{r}(\lambda_{\min}(V_i) - \|V_{ii}\|)\|z_i\|^2 + \sum_{i=1}^{r}\sum_{j=1,i\neq j}^{r}\|V_{ij}\|\|z_i\|\|z_j\| = -(\|z_1\|,\|z_2\|,\cdots,\|z_r\|)U(\|z_1\|,\|z_2\|,\cdots,\|z_r\|)^{\mathrm{T}} - \sum_{i=1}^{r}\sigma_i f(\sigma_i).$$

故当矩阵 U 正定时，$\left.\dfrac{\mathrm{d}V(x)}{\mathrm{d}t}\right|_{(1.3)}$ 负定. 证毕.

$$U = \begin{bmatrix} \lambda_{\min}(V_1) - \|V_{11}\| & -\frac{1}{2}(\|V_{12}\| + \|V_{21}\|) & \cdots & -\frac{1}{2}(\|V_{1r}\| + \|V_{r1}\|) \\ -\frac{1}{2}(\|V_{12}\| + \|V_{21}\|) & \lambda_{\min}(V_2) - \|V_{22}\| & \cdots & -\frac{1}{2}(\|V_{2r}\| + \|V_{r2}\|) \\ \cdots & \cdots & \cdots & \cdots \\ -\frac{1}{2}(\|V_{1r}\| + \|V_{r1}\|) & -\frac{1}{2}(\|V_{2r}\| + \|V_{r2}\|) & \cdots & \lambda_{\min}(V_r) - \|V_{rr}\| \end{bmatrix}.$$

这里

$$U_{ii} = b_i^{\mathrm{T}(0)}P_i + \frac{1}{2}\theta_i c_i^{\mathrm{T}};\ \alpha_{ii} = \theta_i\rho_i,$$

$$\overline{W}_{ii} = K_{ii}^{\mathrm{T}}(|P_i|) + (|P_i|)K_{ii};$$

$$\overline{U}_{ii} = K_i^{\mathrm{T}}(|P_i|);$$

$$\overline{W}_{ij} = 2(|P_i|)(|A_{ij}^{(0)}|) + K_{ij}).$$

则可得下面定理：

定理 2.1 设存在正定矩阵 P_i（P_i 满足 $A_{ii}^{(0)\mathrm{T}}P_i + P_iA_{ii}^{(0)} = -2I_{n_i}$）和正数 $\theta_i(i = 1,2,\cdots,r)$；使得矩

2 间接控制系统的绝对稳定性(Absolute stability of indirect control systems)

考虑系统

$$\begin{cases} \dot{x}_i = G[B_{ii},C_{ii}]x_i + \sum\limits_{j=1,i\neq j}^{n} G[B_{ij},C_{ij}]x_j + G[R_i,S_i]f_i(\sigma_i), \\ \dot{\sigma}_i = c_i^{\mathrm{T}}x_i - \rho_i f_i(\sigma_i), f_i(\sigma_i) \in K_i[0,\infty]; \\ i = 1,2,\cdots,r. \end{cases} \tag{2.1}$$

对于任意的 $A_{ij} \in G[B_{ij},C_{ij}]$, $b_i \in G[R_i,S_i]$, 考虑系统：

$$\begin{aligned} \dot{x} &= A_{ii}x_i + \sum_{j=1,i\neq j}^{r} A_{ij}x_j + b_i f_i(\sigma_i); \\ \dot{\sigma}_i &= c_i^{\mathrm{T}}x_i - \rho_i f_i(\sigma_i), i = 1,2,\cdots,r. \end{aligned} \tag{2.2}$$

设：$A_{ij}^{(0)} = \frac{1}{2}(B_{ij} + C_{ij})$, $K_{ij} = \frac{1}{2}(C_{ij} - B_{ij})$, $b_i^{(0)} = \frac{1}{2}(R_i + S_i)$, $K_i = \frac{1}{2}(R_i - S_i)$, 则该系统可化为：

$$\begin{aligned} \dot{x}_i &= A_{ii}^{(0)}x_i + \sum_{j=1,i\neq j}^{r} A_{ij}^{(0)}x_j + b_i^{(0)}f_i(\sigma_i) + (A_{ii} - A_{ii}^{(0)})x_i + \sum_{j=1,i\neq j}^{r}(A_{ij} - A_{ij}^{(0)})x_j + (b_i - b_i^{(0)})f_i(\sigma_i), \\ \dot{\sigma}_i &= c_i^{\mathrm{T}}x_i - \rho_i f_i(\sigma_i);\quad i = 1,2,\cdots,r. \end{aligned} \tag{2.3}$$

假定 $A_{ii}^{(0)}$ 稳定，则存在正定矩阵 P_i 满足 Lyapunov 方程 $A_{ii}^{(0)\mathrm{T}}P_i + P_iA_{ii}^{(0)} = -2I_{n_i}$ 设

$$z_i = ((|x_i|)^{\mathrm{T}}, |f_i(\sigma_i)|)^{\mathrm{T}},$$

$$V_i = \begin{pmatrix} 2I_{n_i} & -U_{ii}^{\mathrm{T}} \\ -U_{ii} & \alpha_{ii}\end{pmatrix};\quad V_{ii} = \begin{pmatrix} \overline{W}_{ii} & \overline{U}_{ii}^{\mathrm{T}} \\ \overline{U}_{ii} & 0\end{pmatrix};$$

$$V_{ij} = \begin{pmatrix} \overline{W}_{ij} & 0 \\ 0 & 0\end{pmatrix};$$

阵 U 正定,则系统(2.1)稳定.

证 取 Lyapunov 函数:$V(x,f) = \sum_{i=1}^{r}[x_i^{\mathrm{T}}P_ix_i + \theta_i\int_0^{\sigma_i}f_i(\sigma_i)\mathrm{d}\sigma_i]$, 则

$$\left.\frac{\mathrm{d}V(x,f)}{\mathrm{d}t}\right|_{(2.3)} =$$

$$\sum_{i=1}^{r}[(\dot{x}_i^{\mathrm{T}}P_ix_i + x_i^{\mathrm{T}}P_i\dot{x}_i + \theta_i\dot{\sigma}f_i(\sigma_i))]_{(2.3)} =$$

$$\sum_{i=1}^{r}[x_i^{\mathrm{T}}(A_{ii}^{\mathrm{T}^{(0)}}P_i + P_iA_{ii}^{(0)})x_i + \sum_{j=1,i\neq j}(x_j^{\mathrm{T}}A_{ij}^{\mathrm{T}^{(0)}}P_ix_i +$$
$$x_i^{\mathrm{T}}P_iA_{ij}^{(0)}x_j) + b_i^{\mathrm{T}^{(0)}}P_ix_if_i(\sigma_i) + x_i^{\mathrm{T}}P_ib_i^{(0)}f_i(\sigma_i) +$$
$$\theta_i(c_i^{\mathrm{T}}x_i - \rho_if_i(\sigma_i))f_i(\sigma_i) + x_i^{\mathrm{T}}((A_{ii} - A_{ii}^{(0)})^{\mathrm{T}}P_i +$$
$$P_i(A_{ii} - A_{ii}^{(0)}))x_i + \sum_{j=1,i\neq j}^{m}(x_j^{\mathrm{T}}(A_{ij} - A_{ij}^{(0)})^{\mathrm{T}}P_ix_i +$$
$$x_i^{\mathrm{T}}P_i(A_{ij} - A_{ij}^{(0)})x_j) + (b_i - b_i^{(0)})^{\mathrm{T}}P_ix_if_i(\sigma_i) +$$
$$x_i^{\mathrm{T}}P_i(b_i - b_i^{(0)})f_i(\sigma_i)] \leqslant$$

$$\sum_{i=1}^{r}[-2x_i^{\mathrm{T}}x_i + 2(b_i^{\mathrm{T}^{(0)}}P_i + \frac{1}{2}\theta_ic_i^{\mathrm{T}})x_if_i(\sigma_i) -$$
$$\theta_i\rho_if_i^2(\sigma_i)] + \sum_{i=1}^{r}\sum_{j=1,j\neq i}^{r}[(|x_j|)^{\mathrm{T}}(|A_{ij}^{\mathrm{T}^{(0)}}|)^{\mathrm{T}}\cdot$$
$$(|P_i|)(|x_i|) + (|x_i|)^{\mathrm{T}}(|P_i|)(|A_{ij}^{(0)}|))\cdot$$
$$(|x_j|)] + \sum_{i=1}^{r}[(|x_i|)^{\mathrm{T}}(K_{ii}^{\mathrm{T}}(|P_i|) +$$
$$(|P_i|)K_{ii})(|x_i|) + 2K_i^{\mathrm{T}}(|P_i|)$$
$$(|x_i|)|f_i(\sigma_i)|] + \sum_{i=1}^{r}\sum_{j=1,j\neq i}^{r}[(|x_j|)^{\mathrm{T}}K_{ij}^{\mathrm{T}}\cdot$$
$$(|P_i|)(|x_i|) + (|x_i|)^{\mathrm{T}}(|P_i|)K_{ij}(|x_j|) =$$

$$\sum_{i=1}^{r}[-2x_i^{\mathrm{T}}x_i + 2(b_i^{\mathrm{T}^{(0)}}P_i + \frac{1}{2}\theta_ic_i^{\mathrm{T}})x_if_i(\sigma_i) +$$
$$\theta_i\rho_i^{(0)}f_i^2(\sigma_i)] + \sum_{i=1}^{r}[(|x_i|)^{\mathrm{T}}(K_{ii}^{\mathrm{T}}(|P_i|) +$$
$$(|P_i|)K_{ii})(|x_i|) + 2K_i^{\mathrm{T}}(|P_i|)(|x_i|)\cdot$$
$$|f_i(\sigma_i)|] + \sum_{i=1}^{r}\sum_{j=1,j\neq i}^{r}[(|x_j|)^{\mathrm{T}}((|A_{ij}^{\mathrm{T}^{(0)}}|)^{\mathrm{T}} +$$
$$K_{ij}^{\mathrm{T}})(|P_i|)(|x_i|) + (|x_i|)^{\mathrm{T}}(|P_i|)\cdot$$
$$((|A_{ij}^{(0)}|) + K_{ij})(|x_j|) =$$

$$-\sum_{i=1}^{r}\begin{bmatrix}x_i\\f_i(\sigma_i)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}2I_{n_i} & -U_{ii}^{\mathrm{T}}\\-U_{ii} & \alpha_{ii}\end{bmatrix}\begin{bmatrix}x_i\\f_i(\sigma_i)\end{bmatrix}+$$
$$\sum_{i=1}^{r}\begin{bmatrix}(|x_i|)\\|f_i(\sigma_i)|\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\bar{W}_{ii} & \bar{U}_{ii}^{\mathrm{T}}\\\bar{U}_{ii} & 0\end{bmatrix}\begin{bmatrix}(|x_i|)\\|f_i(\sigma_i)|\end{bmatrix}+$$
$$\sum_{i=1}^{r}\sum_{j=1,i\neq j}^{r}\begin{bmatrix}(|x_i|)\\|f_i(\sigma_i)|\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\bar{W}_{ij} & 0\\0 & 0\end{bmatrix}\begin{bmatrix}(|x_j|)\\|f_i(\sigma_i)|\end{bmatrix}\leqslant$$

$$-\sum_{i=1}^{r}\lambda_{\min}(V_i)\begin{bmatrix}(|x_i|)\\|f_i(\sigma_i)|\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}(|x_i|)\\|f_i(\sigma_i)|\end{bmatrix}+$$
$$\sum_{i=1}^{r}\begin{bmatrix}(|x_i|)\\|f_i(\sigma_i)|\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\bar{W}_{ii} & \bar{U}_{ii}^{\mathrm{T}}\\\bar{U}_{ii} & 0\end{bmatrix}\begin{bmatrix}(|x_i|)\\|f_i(\sigma_i)|\end{bmatrix}+$$
$$\sum_{i=1}^{r}\sum_{j=1,i\neq j}^{r}\begin{bmatrix}(|x_i|)\\|f_i(\sigma_i)|\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\bar{W}_{ij} & 0\\0 & 0\end{bmatrix}\begin{bmatrix}(|x_j|)\\|f_j(\sigma_j)|\end{bmatrix}\leqslant$$

$$-\sum_{i=1}^{r}(\lambda_{\min}(V_i) - \|V_{ii}\|)\|z_i\|^2 +$$
$$\sum_{i=1}^{r}\sum_{j=1,i\neq j}^{r}\|V_{ij}\|\|z_i\|\|z_j\| =$$
$$-(\|z_1\|,\|z_2\|,\cdots,\|z_r\|)U(\|z_1\|,\|z_2\|,\cdots,\|z_r\|)^{\mathrm{T}}.$$

故当矩阵 U 正定时,$\left.\frac{\mathrm{d}V(x,f)}{\mathrm{d}t}\right|_{(2.3)}$ 负定. 证毕.

3 例子(Example)

考虑系统

$$\begin{cases}\dot{x}_1(t) = [-1.2,-1.0]x_1(t) + [0.2,0.4]x_2(t) + [0.05,0.1]x_3(t) + [0.8,1.2]f_1(\sigma_1),\\\dot{x}_2(t) = [-0.4,-0.2]x_1(t) + [-1.1,-0.9]x_2(t) + [0.02,0.06]x_3(t) + [-1.1,-0.9]f_1(\sigma_1),\\\dot{x}_3(t) = [0.01,0.03]x_1(t) + [0.02,0.04]x_2(t) + [-2.2,-1.8]x_3(t) + [-0.6,-0.4]f_2(\sigma_2),\end{cases}$$

$\sigma_1 = -x_1 + x_2$; $\sigma_2 = x_3$; $c_1 = (-1,1)^{\mathrm{T}}$, $c_2 = 1$.

这里:

$$G[B_{11},C_{11}] = \begin{pmatrix}[-1.2,-1.0] & [0.2,0.4]\\[-0.4,-0.2] & [-1.1,-0.9]\end{pmatrix},$$

$$G[B_{12},C_{12}] = \begin{pmatrix}[0.05,0.1]\\[0.02,0.06]\end{pmatrix};$$

$$G[B_{21},C_{21}] = ([0.01,0.03]\quad[0.02,0.04]),$$

$$G[B_{22},C_{22}] = [-2.2,-1.8],$$

计算可知:

$$P_1 = \begin{pmatrix}0.9124 & -0.0120\\-0.0120 & 0.9964\end{pmatrix};$$

$$P_2 = (0.5);U = \begin{pmatrix}0.1518 & -0.115\\-0.115 & 0.1892\end{pmatrix}.$$

由于该矩阵正定,由定理 2.1 可知该系统绝对稳定.

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