

分散自适应模糊滑模控制器的设计与分析*

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摘要: 研究了一类具有函数控制增益的耦合大系统的分散自适应模糊控制问题, 提出了能够利用专家的语言信息和数字信息的分散自适应模糊滑模控制器的设计方案. 通过理论分析, 证明了分散自适应模糊控制系统是全局稳定的, 跟踪误差可收敛到零的一个邻域内.

关键词: 耦合系统; 模糊控制; 滑模控制; 自适应控制; 全局稳定性

Design and Analysis of Decentralized Adaptive Fuzzy Sliding Mode Controllers

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Abstract: The problem of decentralized adaptive fuzzy control for a class of interconnected systems with unknown function control gains is studied in this paper. A design scheme of a decentralized adaptive fuzzy sliding mode controller is proposed. The design is capable of incorporating linguistic and numerical information into controllers. By theoretical analysis, the decentralized adaptive fuzzy control system is proven to be globally stable, with tracking errors converging to a neighborhood of zero.

Key words: interconnected systems; fuzzy control; sliding mode control; adaptive control; global stability

1 引言(Introduction)

文献[1~3]利用一型模糊系统的逼近能力, 提出了一些稳定自适应模糊控制器的设计方案. 其缺点只对模糊系统中的结论模糊集的峰值进行了调节, 而前提模糊集在控制过程中一直不变, 这样当过程的状态变量维数较大时, 逼近未知函数的模糊系统所需的模糊规则数目较多.

本文在[2,3]基础上利用二型模糊逻辑系统去逼近过程未知函数和控制增益, 并对分散模糊系统中前提模糊集和结论模糊集的隶属函数的形状参数及峰值进行了自适应调节, 从而大大减少了用于建模的模糊逻辑系统中的规则数目. 另外, 根据李亚普诺夫方法, 确定了逼近误差以及各子系统间耦合作用项的上界多项式函数中未知系数的自适应律. 由于控制律中增加了逼近误差的自适应补偿项, 因此不管逼近未知函数的分散模糊系统是否准确, 都能

保证闭环模糊控制系统的全局稳定性.

2 问题的描述及基本假设(Problem statement and basic assumptions)

考虑由下面 N 个相互关联的子系统 P_i 所构成的非线性系统 P :

$$P_i: \begin{cases} \dot{x}_{i1} = x_{i2}, \\ \vdots \\ \dot{x}_{im_i} = f_i(x_i) + b_i(x_{iT})u_i(t) + d_i(x, t), \end{cases} \quad (1)$$

$i = 1, \dots, N.$

其中 $x_i = (x_{i1}, \dots, x_{im_i})^T \in \mathbb{R}^{m_i}$ 是子系统 P_i 的状态向量, u_i 是子系统 P_i 的控制输入, f_i 是未知连续函数, $b_i(x_{iT})$ 是未知控制增益, 而 $x_{iT} = (x_{i1}, \dots, x_{im_i-1})^T \in \mathbb{R}^{m_i-1}$, $d_i(x, t)$ 代表外来干扰及子系统间交互作用的和, $x = (x_1^T, \dots, x_N^T)^T \in \mathbb{R}^m$ 是系统 P 的状态向量, 而 $m = \sum_{i=1}^N m_i$.

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控制目标是对每一个子系统 P_i 要求子系统的状态向量 $x_i = (x_{i1}, \dots, x_{im_i})^T$ 尽可能好地去跟踪一个指定的期望轨迹 $x_{id} = (y_{id}, \dot{y}_{id}, \dots, y_{id}^{(m_i-1)})^T$. 定义子系统 P_i 的跟踪误差向量 $e_i = x_i - x_{id} = (e_{i1}, \dots, e_{im_i})^T$. 因此, 问题是设计一个分散控制律 $u_i(t)$, 使得 e_i 收敛到零的一个小邻域内.

令

$$h_i(x_i) = b_i^{-1}(x_{iT})f_i(x_i),$$

$$g_i(x_{iT}) = b_i^{-1}(x_{iT}), i = 1, \dots, N.$$

为了设计稳定的自适应模糊滑模控制, 参照文 [2~4] 中的讨论, 对未知连续函数 $h_i(x_i), g_i(x_{iT})$ 作出如下假设:

- 1) $|h_i(x_i)| \leq K_{i0}(x_i), x_i \in A_{id}^c$;
- 2) $0 < g_i(x_{iT}) \leq K_{i1}(x_{iT}), \forall t \geq 0$;
- 3) $|\dot{g}_i(x_{iT})| = |\nabla g_i(x_{iT})\dot{x}_{iT}| \leq K_{i2}(x_i) \|x_i\|, \forall t \geq 0$;
- 4) 存在未知非负实数 a_{ijk} , 使得

$$|d_i(x, t)| \leq \sum_{k=0}^p \sum_{j=1}^N a_{ijk} \|x_j\|^k. \quad (2)$$

其中 $K_{i0}(x_i), K_{i1}(x_{iT}), K_{i2}(x_i)$ 是已知正的连续函数, p 为正整数, 集合 A_{id} 由下式给出:

$$A_{id} = \{x_i \mid \|x_i - x_{i0}\|_{p_i, w_i} \leq 1\}, \quad (3)$$

而 $w_i = \{w_{ij}\}_{j=1}^{m_i}$ 是一组严格正的权, x_{i0} 是 \mathbb{R}^{m_i} 中一定点, $\|x_i\|_{p_i, w_i}$ 是一种加权 p_i -范数, 其定义如下:

$$\|x_i\|_{p_i, w_i} = \left[\sum_{j=1}^{m_i} \left(\frac{|x_{ij}|}{w_{ij}} \right)^{p_i} \right]^{\frac{1}{p_i}},$$

$$x_i = (x_{i1}, x_{i2}, \dots, x_{im_i})^T.$$

设 $h_i(x_i, \theta_{hi}), g_i(x_i, \theta_{gi})$ 是两个二型模糊逻辑系统在区域 A_i 上分别对 $h_i(x_i), g_i(x_{iT})$ 的一个逼近, 即

$$A_i = \{x_i \mid \|x_i - x_{i0}\|_{p_i, w_i} \leq 1 + \Psi_i\}, \quad (4)$$

$$h_i(x_i, \theta_{hi}) = \frac{\sum_{l=1}^{M_i} y_{hi}^l \left[\prod_{j=1}^{m_i} \exp\left(-\frac{(x_{ij} - a_{jhi}^l)^2}{(b_{jhi}^l)^2 + b_{0hi}}\right) \right]}{\sum_{l=1}^{M_i} \prod_{j=1}^{m_i} \exp\left(-\frac{(x_{ij} - a_{jhi}^l)^2}{(b_{jhi}^l)^2 + b_{0hi}}\right)}, \quad (5)$$

$$g_i(x_i, \theta_{gi}) = \frac{\sum_{l=1}^{M_i} y_{gi}^l \left[\prod_{j=1}^{m_i} \exp\left(-\frac{(x_{ij} - a_{jgi}^l)^2}{(b_{jgi}^l)^2 + b_{0gi}}\right) \right]}{\sum_{l=1}^{M_i} \prod_{j=1}^{m_i} \exp\left(-\frac{(x_{ij} - a_{jgi}^l)^2}{(b_{jgi}^l)^2 + b_{0gi}}\right)}, \quad (6)$$

而 M_i 是第 i 个模糊系统中的规则数目, $\Psi_i > 0$ 表示

过渡区域的宽度,

$$\theta_{hi} = (y_{hi}^1, \dots, y_{hi}^{M_i}, b_{1hi}^1, \dots, b_{m_ihi}^1, \dots, b_{1hi}^{M_i}, \dots, b_{m_ihi}^{M_i}, a_{1hi}^1, \dots, a_{m_ihi}^1, \dots, a_{1hi}^{M_i}, \dots, a_{m_ihi}^{M_i})^T,$$

$$\theta_{gi} = (y_{gi}^1, \dots, y_{gi}^{M_i}, b_{1gi}^1, \dots, b_{m_ihi}^1, \dots, b_{1gi}^{M_i}, \dots, b_{m_ihi}^{M_i}, a_{1gi}^1, \dots, a_{m_ihi}^1, \dots, a_{1gi}^{M_i}, \dots, a_{m_ihi}^{M_i})^T$$

是可调参数, 正数 $b_{0hi}; b_{0gi}$ 是设计参数. 令

$$\Omega_{hi} = \{\theta_{hi} \mid \|\theta_{hi}\| \leq M_{hi}\},$$

$$\Omega_{gi} = \{\theta_{gi} \mid \|\theta_{gi}\| \leq M_{gi}\},$$

$$y_{gi}^l \geq \epsilon, l = 1, \dots, M_i,$$

$$i = 1, \dots, N,$$

$$\theta_{hi}^* = \arg \min_{\theta_{hi} \in \Omega_{hi}} \left[\sup_{x_i \in \Lambda_i} |h_i(x_i, \theta_{hi}) - h_i(x_i)| \right],$$

$$\theta_{gi}^* = \arg \min_{\theta_{gi} \in \Omega_{gi}} \left[\sup_{x_i \in \Lambda_i} |g_i(x_i, \theta_{gi}) - g_i(x_{iT})| \right],$$

$$i = 1, \dots, N,$$

其中正常数 M_{hi}, M_{gi}, ϵ 是设计参数. 设 $\hat{\theta}_{hi}(t) \in \Omega_{hi}, \hat{\theta}_{gi}(t) \in \Omega_{gi}$, 分别是 $\theta_{hi}^*, \theta_{gi}^*$ 在 t 时刻的估计值, 将 $h_i(x_i, \theta_{hi}^*), g_i(x_i, \theta_{gi}^*)$ 在 $\hat{\theta}_{hi}(t), \hat{\theta}_{gi}(t)$ 的邻域内展开成泰勒展式得

$$h_i(x_i, \theta_{hi}^*) - h_i(x_i, \hat{\theta}_{hi}(t)) = \Phi_{hi}^T(t) \frac{\partial h_i(x_i, \hat{\theta}_{hi})}{\partial \hat{\theta}_{hi}} + O(\|\Phi_{hi}(t)\|^2), \quad (7)$$

$$g_i(x_i, \theta_{gi}^*) - g_i(x_i, \hat{\theta}_{gi}(t)) = \Phi_{gi}^T(t) \frac{\partial g_i(x_i, \hat{\theta}_{gi})}{\partial \hat{\theta}_{gi}} + O(\|\Phi_{gi}(t)\|^2), \quad (8)$$

$$\text{其中 } \Phi_{hi}(t) = \theta_{hi}^* - \hat{\theta}_{hi}(t),$$

$$\Phi_{gi}(t) = \theta_{gi}^* - \hat{\theta}_{gi}(t).$$

令

$$\epsilon_{hi} = \max_{x_i \in \Lambda_i, \hat{\theta}_{hi}(t) \in \Omega_{hi}} [O(\|\Phi_{hi}(t)\|^2) + |h_i(x_i) - h_i(x_i, \theta_{hi}^*)|], i = 1, \dots, N,$$

$$\epsilon_{gi} = \max_{x_i \in \Lambda_i, \hat{\theta}_{gi}(t) \in \Omega_{gi}} [O(\|\Phi_{gi}(t)\|^2) + |g_i(x_{iT}) - g_i(x_i, \theta_{gi}^*)|], i = 1, \dots, N,$$

则 $\epsilon_{hi}, \epsilon_{gi}$ 是未知有界常数.

3 分散自适应模糊控制器的设计及主要结果 (Decentralized adaptive fuzzy Controller design and main result)

对子系统 P_i 定义切换函数

$$s_i(t) = c_{i1}e_{i1} + c_{i2}e_{i2} + \dots + c_{i(m_i-1)}e_{i(m_i-1)} + e_{im_i}. \quad (9)$$

其中 $e_{i1} = x_{i1} - y_{id}, e_{i2} = \dot{x}_{i1} - \dot{y}_{id}, \dots, e_{im_i} = x_{im_i}^{(m_i-1)} - y_{id}^{(m_i-1)}$, 而常数 $c_{i1}, c_{i2}, \dots, c_{i(m_i-1)}$ 确定的多项式

$\lambda^{m_i-1} + c_{i(m_i-1)}\lambda^{m_i-2} + \dots + c_{i1}$ 是霍尔维茨多项式.

将 $s_i(t)$ 对时间 t 求导得

$$s_i(t) = \sum_{j=1}^{m_i-1} c_{ij}e_{i(j+1)} + b_i(x_{iT})u_i(t) + d_i(x, t) + f_i(x_i) - y_{id}^{(m_i)}(t). \quad (10)$$

采用如下控制律

$$u_i(t) = -k_{id}s_{i\Delta}(t) - \frac{1}{2}K_{i2}(x_i) \|x_i\| s_{i\Delta}(t) + [m_i(t)(K_{i0}(x_i) + K_{i1}(x_{iT}) |u_i^*(t)|) + K_{i1}(x_{iT}) \sum_{k=0}^p \sum_{j=1}^N \hat{a}_{ijk}(t) \|x_j\|^k] u_{if}(t) + (1 - m_i(t))u_{ia}(t). \quad (11)$$

其中 $k_{id} > 0$, $\hat{a}_{ijk}(t)$ 是 a_{ijk} 在 t 时刻的估计值, $u_{if}(t)$ 由文献[3]中式(21)确定($z_i = s_i/\varphi_i$),

$$u_i^*(t) = \sum_{j=1}^{m_i-1} c_{ij}e_{i(j+1)} - y_{id}^{(m_i)}(t), \quad (12)$$

$$\dot{\hat{\theta}}_{hi} = \begin{cases} \eta_{i1}(1 - m_i(t))s_{i\Delta}(t) \frac{\partial h_i(x_i, \hat{\theta}_{hi})}{\partial \hat{\theta}_{hi}}, & \text{当 } \|\hat{\theta}_{hi}\| < M_{hi} \\ \eta_{i1}(1 - m_i(t))s_{i\Delta}(t) \frac{\partial h_i(x_i, \hat{\theta}_{hi})}{\partial \hat{\theta}_{hi}} - \eta_{i1}(1 - m_i(t))s_{i\Delta}(t) \frac{\hat{\theta}_{hi} \hat{\theta}_{hi}^T}{\|\hat{\theta}_{hi}\|^2} \frac{\partial h_i(x_i, \hat{\theta}_{hi})}{\partial \hat{\theta}_{hi}}, & \text{当 } \|\hat{\theta}_{hi}\| = M_{hi} \text{ 且 } s_{i\Delta}(t) \hat{\theta}_{hi}^T \frac{\partial h_i(x_i, \hat{\theta}_{hi})}{\partial \hat{\theta}_{hi}} \leq 0, \\ \eta_{i1}(1 - m_i(t))s_{i\Delta}(t) \frac{\partial h_i(x_i, \hat{\theta}_{hi})}{\partial \hat{\theta}_{hi}}, & \text{当 } \|\hat{\theta}_{hi}\| = M_{hi} \text{ 且 } s_{i\Delta}(t) \hat{\theta}_{hi}^T \frac{\partial h_i(x_i, \hat{\theta}_{hi})}{\partial \hat{\theta}_{hi}} > 0, \end{cases} \quad (16)$$

$$\dot{\hat{\epsilon}}_{hi} = \eta_{i2}(1 - m_i(t)) |s_{i\Delta}(t)|. \quad (17)$$

当 $\dot{y}_{gi}^l(t) = \epsilon$ 时

$$\dot{y}_{gi}^l = \begin{cases} \eta_{i3}(1 - m_i(t))s_{i\Delta}(t)u_i^*(t) \frac{\partial g_i(x_i, \hat{\theta}_{gi})}{\partial \dot{y}_{gi}^l}, & \text{当 } s_{i\Delta}(t)u_i^*(t) \frac{\partial g_i(x_i, \hat{\theta}_{gi})}{\partial \dot{y}_{gi}^l} > 0, \\ 0, & \text{当 } s_{i\Delta}(t)u_i^*(t) \frac{\partial g_i(x_i, \hat{\theta}_{gi})}{\partial \dot{y}_{gi}^l} \leq 0, \end{cases} \quad (18)$$

否则

$$\dot{\hat{\theta}}_{gi+} = \begin{cases} \eta_{i3}(1 - m_i(t))s_{i\Delta}(t)u_i^*(t) \frac{\partial g_i(x_i, \hat{\theta}_{gi})}{\partial \hat{\theta}_{gi+}}, & \text{当 } \|\hat{\theta}_{gi}\| < M_{gi} \text{ 或 } \|\hat{\theta}_{gi}\| = M_{gi}, \\ & \text{且 } s_{i\Delta}(t)u_i^*(t) [\hat{\theta}_{gi+}^T \frac{\partial g_i(x_i, \hat{\theta}_{gi})}{\partial \hat{\theta}_{gi+}} + \hat{\theta}_{gie1}^T \frac{\partial g_i(x_i, \hat{\theta}_{gi})}{\partial \hat{\theta}_{gie1}}] \leq 0, \\ \eta_{i3}(1 - m_i(t))s_{i\Delta}(t)u_i^*(t) \frac{\partial g_i(x_i, \hat{\theta}_{gi})}{\partial \hat{\theta}_{gi+}} - \eta_{i3}(1 - m_i(t))s_{i\Delta}(t)u_i^*(t) \hat{\theta}_{gi+} [\hat{\theta}_{gi+}^T \frac{\partial g_i(x_i, \hat{\theta}_{gi})}{\partial \hat{\theta}_{gi+}} + \hat{\theta}_{gie1}^T \frac{\partial g_i(x_i, \hat{\theta}_{gi})}{\partial \hat{\theta}_{gie1}}] > 0, & \text{当 } \|\hat{\theta}_{gi}\| = M_{gi} \text{ 且 } s_{i\Delta}(t)u_i^*(t) [\hat{\theta}_{gi+}^T \frac{\partial g_i(x_i, \hat{\theta}_{gi})}{\partial \hat{\theta}_{gi+}} + \hat{\theta}_{gie1}^T \frac{\partial g_i(x_i, \hat{\theta}_{gi})}{\partial \hat{\theta}_{gie1}}] > 0, \\ \frac{\partial g_i(x_i, \hat{\theta}_{gi})}{\partial \hat{\theta}_{gi+}} + \hat{\theta}_{gie1}^T \frac{\partial g_i(x_i, \hat{\theta}_{gi})}{\partial \hat{\theta}_{gie1}} \Big/ \|\hat{\theta}_{gi+}\|^2, & \end{cases} \quad (19)$$

$$u_{ia}(t) = -h_i(x_i, \hat{\theta}_{hi}) + \hat{\epsilon}_{hi}(t)u_{if}(t) - u_i^*(t)g_i(x_i, \hat{\theta}_{gi}) + \hat{\epsilon}_{gi}(t)u_{if}(t) |u_i^*(t)|, \quad (13)$$

$$m_i(t) = \max\{0, \text{sat}(\frac{\|x_i - x_{i0}\|_{p_i, w_i} - 1}{\Psi_i})\}, \quad (14)$$

$$s_{i\Delta}(t) = s_i(t) - \varphi_i \text{sat}(s_i(t)/\varphi_i). \quad (15)$$

这里 $m_i(t)$ 是一种调制函数, $0 \leq m_i(t) \leq 1, \forall t \geq 0$. 而 $\hat{\theta}_{hi}(t), \hat{\epsilon}_{hi}(t), \hat{\theta}_{gi}(t), \hat{\epsilon}_{gi}(t)$ 分别是 $\theta_{hi}^*, \epsilon_{hi}, \theta_{gi}^*, \epsilon_{gi}$ 在 t 时刻的估计值. 饱和函数 $\text{sat}(y) = y, \text{当 } |y| \leq 1; \text{sat}(y) = \text{sgn}(y), \text{当 } |y| > 1. \varphi_i > 0$ 为边界层宽度.

采用如下自适应律

$$\dot{\hat{\epsilon}}_{gi} = \eta_{i4}(1 - m_i(t)) | s_{i\Delta}(t) u_i^*(t) |, \quad (20)$$

$$\begin{cases} \dot{\hat{a}}_{ijk} = \eta_i K_{i1}(x_{iT}) | s_{i\Delta}(t) | \| x_j \|^k, \\ \hat{a}_{ijk}(0) \geq 0, i, j = 1, \dots, N, k = 0, 1, \dots, p. \end{cases} \quad (21)$$

其中 $\epsilon > 0$ 是一常数, $\eta_{i1} > 0, \eta_{i2} > 0, \eta_{i3} > 0, \eta_{i4} > 0, \eta_i > 0$ 均为自适应率, $\hat{\theta}_{gi+}(t)$ 是将 $\hat{\theta}_{gi}(t)$ 中删除满足式(18)的所有分量后所得的参数估计向量, $\hat{\theta}_{gie1}(t)$ 是 $\hat{\theta}_{gi}(t)$ 的前 M_i 个分量中满足式(18)第一行条件的所有分量所构成的列向量, $\hat{\theta}_{gie2}(t)$ 是 $\hat{\theta}_{gi}(t)$ 的前 M_i 个分量中满足式(18)第二行条件的所有分量所构成的列向量.

于是提出如下稳定性定理:

定理 考虑过程(1),其控制律由式(11)~(15)确定,自适应律由式(16)~(21)确定,并满足假设1)~4),则闭环模糊控制系统中所有信号有界,跟踪误差收敛到零的一个邻域内.

证 取

$$\begin{aligned} V(t) = & \sum_{i=1}^N \left[\frac{1}{2} g(x_{iT}) s_{i\Delta}^2(t) + \right. \\ & + \frac{1}{2} (\Phi_{hi}^T \Phi_{hi} / \eta_{i1} + \Phi_{gi}^T \Phi_{gi} / \eta_{i3} + \\ & (\hat{\epsilon}_{hi}(t) - \epsilon_{hi})^2 / \eta_{i2} + (\hat{\epsilon}_{gi}(t) - \epsilon_{gi})^2 / \eta_{i4} + \\ & \left. \sum_{k=0}^p \sum_{j=1}^N (\hat{a}_{ijk}(t) - a_{ijk})^2 / \eta_i \right], \quad (22) \end{aligned}$$

采用文[2,3,5]中类似的方法,不难得出结论成立.

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