On the Positive Periodic Solutions of Semilinear Periodic-Parabolic System

—— In Memory of My Teacher Professor C. C. Kwan

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Abstract: An abstract maximum principle, which can be applied to elliptic systems and periodic-parabolic systems, is given. Accordingly, a generalization of the Hess-Kato theorem on principal eigenvalue is obtained, and is applied to study semilinear problems.

Key words: principal eigenvalue; irreducible; elliptic system; bifurcation

关于半线性周期抛物组的正周期解

—— 纪念我的老师关肇直教授

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摘要：本文给出一个抽象的极大值原理，使之可应用于椭园组与周期抛物组．由此，推出了关于主特征值的 Hess-Kato 定理，并将其应用到半线性问题．

关键词：主特征值；不可约；椭园组；分歧

1 Introduction

We study the existence of positive periodic solution $U \in C(\overline{Q}_T)$ of given period $T > 0$ of the following system:

$$\begin{cases} \left(\dfrac{\partial}{\partial t} + A(x,t;D)\right)U = f(x,t,U), & \text{in } Q_T, \\ U = 0, & \text{on } \partial\Omega \times [0,T], \\ U(\cdot,0) = U(\cdot,T), & \text{on } \overline{\Omega} \end{cases} \tag{1.1}$$

where $Q_T = \Omega \times (0,T)$, $f \in C(Q_T \times \mathrm{R}^p, \mathrm{R}^p)$, $A(x,t,D) = \mathrm{diag}(D_1, D_2, \cdots, D_p)$, and

$$\begin{aligned} D_k u = & -\sum a_{ij}^{(k)}(x,t)\partial_{ij}u \\ & + \sum b_j^k(x,t)\partial_j u + c^{(k)}(x,t)u, \end{aligned} \tag{1.2}$$

$k = 1, 2, \cdots, p$ are second order uniformly elliptic operators with coefficients $a_{ij}^{(k)} = a_{ji}^{(k)}$, $b_j^{(k)}$, $c^{(k)} \geq 0$ belong to the real Banach space $E = \{\omega \in C(\overline{Q}_T) | \omega(\cdot,0) = \omega(\cdot,T)\}$.

We assume that Ω is a bounded domain with smooth boundary in R^N.

The solution U is said positive, if all components of $U = U(x,t) = (u_1(x,t), \cdots, u_p(x,t))$ are positive, $\forall (x,t) \in Q_T$.

It is well known that the related problem for equations has been widely studied by many authors e.g., Kolesov[1], Amann[2], Beltramo, Hess[3], Lazer[4], Castro, Lazer[5] and Hess[6].

In this paper, we follow [3], and turn to study the linear eigenvalue problem:

$$\begin{cases} \left(\dfrac{\partial}{\partial t} + A(x,t;D)\right)U = \lambda M(x,t)U, & \text{in } Q_T \\ U = 0, & \text{on } \partial\Omega \times [0,T], \\ U(\cdot,0) = U(\cdot,T), & \text{on } \overline{\Omega} \end{cases} \tag{1.3}$$

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where $M \in C(\overline{Q}_T, M(p,R))$ is a real $p \times p$ matrix -valued function, and λ is the eigenvalue.

For a vector function U, we use the notations: $U \geq 0$ means all components of U are nonnegative functions; $U > 0$ means $U \geq 0$, but not $U = 0$, the null element; and $U >> 0$ means all components of U are positive.

The following assumptions on M are given:

i) $M = (m_{kl}(x,t))$ is cooperative, i.e.,$m_{kl}(x,t) \geq 0$, $\forall k \neq l$, $\forall (x,t) \in Q_T$.

ii) M is fully coupled, i.e., the index set $\{1,2,\cdots,p\}$ can not be split up into two disjoint nonempty subsets I and J such that $m_{kl}(x,t) \equiv 0$, in Q_t for $k \in I, \ell \in J$.

iii) $\max_{1\leq k\leq p} \int_0^T \max_{x\in\bar\Omega} m_{kk}(x,t)dt > 0$.

Let L be the operator $\frac{\partial}{\partial t} + A(x,t,D)$ on $X = E^p$ with domain

$$D(L) = \{U \in X | LU \in X, U = 0 \ \text{ on } \ \partial\Omega \times [0,T]\}.$$

Let $Y = D(L)$ be the Banach space with the graph norm.

Our main results read as follows.

Theorem 1.1 Under the assumptions i), ii) an iii), there exists a unique positive eigenvalue $\lambda_1 > 0$ and eigenvalue $\phi >> 0$ such that

1) $L\phi = \lambda_1 M\phi$.

Moreover, we have

2) dim ker $(L - \lambda_1 M) = 1$.

3) The algebraic multiplicity of λ_1^{-1} of the compact operator $L^{-1}M$ is odd.

4) $\forall\lambda > 0$ if it is an eigenvalue of $LU = \lambda MU$, then $\lambda \geq \lambda_1$.

Theorem 1.2 Under the assumptions i) and ii), if $\lambda_1 > 0$ is the first eigenvalue for the problem $LU = \lambda MU$, then $\forall 0 < \lambda < \lambda_1$, $\forall h \in L^q(Q_T, \mathrm{R}^p)$, with $q > N$, and $h > 0$ there exists unique $U >> 0$, satisfying the system

$$LU = \lambda MU + h. \qquad (1.4)$$

Now, we turn to the following nonlinear problem:

$$LU = \lambda f(x,t;U) \quad \text{in} \quad Q_T \qquad (1.5)$$

where $f \in C(Q_T \times \mathrm{R}^p, \mathrm{R}^p)$ is T-periodic in t, and satisfies $f(x,t,\theta) = 0$. Set

$$M(x,t) = \frac{\partial}{\partial \xi} f(x,t;0). \qquad (1.6)$$

Theorem 1.3 Assume that the matrix M, defined in (1.6), satisfies i) ii) and iii), then there is a bifurcation of positive solutions of (1.5). The closure (in $R \times D(L)$) of the set of positive solutions S contains a component S_0 unbounded in $R \times D(L)$ with $(\lambda_1, \theta) \in S_0$, where λ_1 is the first positive eigenvalue of (1.3). Moreover, (λ_1, θ) is the only bifurcation point for positive solutions.

For $p = 1$, all these three theorems have been obtained in [2] . However, in this case, assumptions i) and ii) do not make sense, they are dropped out.

As a special case, where $A(x,t;D)$ and $M(x,t)$ do not depend on t, the problem (1.3) is reduced to a nonlinear eigenvalue problem for elliptic systems, all the related results were obtained recently in [4], where the Hess-Kato theorem for elliptic equations was extended to systems.

For $p = 1$, the studies of the nonlinear elliptic eigenvalue problem and of the periodic solution problem for parabolic equations (1.1) are parallel, cf [3] and [6]. They will be the same for systems. In this sense, all our proofs will be parallel to those appeared in [7]. However, the proof of the Hess-Kato theorem for elliptic systems is based on the Strong Maximum Principle for elliptic systems due to Sweers[8], in which the irreducibility of positive operators and the Krein-Rutman theorem are applied in combining with a result due to de Pagter on Banach lattices[9].

In the following, we shall present a proof of the Strong Maximum Principle which applies to both elliptic systems and periodic parabolic systems. Without concerning with Banach lattices, the following version of the Krein Rutman theorem is applied, cf [10](see also [11]).

Let X be a Banach space with a totally positive cone P, let P denote $P - \{\theta\}$, and let T be a positive compact operator in X satisfying the equation:

$\forall x \in P$, there exists $n \in N$ such that $\langle x^*, T^n x\rangle > 0, \quad \forall x^* \in P$.

Then

a) $r(T) > 0$ is a simple eigenvalue with a positive eigenvector ϕ such that $\langle x^*, \phi\rangle > 0, \quad \forall x^* \in \dot{P}^*$.

b) $T^*\phi^* = r(T)\phi^*$, for $\forall \phi^* \in \dot{P}^*$.

c) $|\lambda| < r(T) \quad \forall\lambda \in \sigma(T)$ with $\lambda \neq r(T)$.

The paper is organized as follow: the three theorems in §1 are announced without proofs, because similar proofs can be found in [7], if Theorems 2.3 and 2.4 are known. In §2 , we use the above version of the Krein Rutman theorem to give a proof of the abstract form of the Strong Maximum Principle. This is Theorem 2.3. In §3, we present examples showing how Theorem 2.3 includes the Strong Maximum Principle for systems.

2 Principal Eigenvalue

Let X be a Banach space with a totally and normally positive cone P. Suppose that X is a direct sum of Banach space: $X \oplus \sum_{j=1}^{p} X_j$ and Let P_j be the projection of X onto X_j, and $P_j = P_j P, \quad j = 1, 2, \cdots, p$.

Given $e \in P - \{0\}$, we write $X_e = \cup_{\lambda>0}\lambda[-e, e]$, and

$$\|x\|_e = \inf\{\lambda > 0 | x \in \lambda[-e, e]\}$$

then $(X_e, \|\cdot\|_e)$ is a Banach space continuously imbedded in X and possessing a positive cone $P_e = P \cap X_e$ with nonempty interiour $\mathrm{int}(P_e)$. Assume

A) $\forall x^* \in \dot{P}^*$, the dual positive cone in X^*, $\langle x^*, e\rangle > 0$.

Let $e_j = P_j e$, similarly, we have the Banach space X_{e_j} and $X_e = \oplus \sum_{j=1}^{p} X_{e_j}$.

Assume that $L = \oplus \sum L_j$, where $L_j : D(L_j) \to X_j$ is a linear closed operator with domain $D(L_j) \subset X_{e_j}$ satisfying the following:

I) $\forall c \geq 0$, $(cI + L_j)^{-1}$ exists, and is a positive compact operator on X_j. Moreover, $\forall x_j \in \dot{P}_j$, $\exists \alpha = \alpha(x_j, c) > 0$, such that

$$(cI + L_j)^{-1} x_j \geq \alpha e_j, \quad j = 1, 2, \cdots, p.$$

Let $B \in L(X, X) \cap L(X_e, X_e)$ satisfy

II) $\exists c_0 > 0$, such that $c_0 I + B$ is strictly positive.

III) None of the direct sums $X_{j_1} \oplus \cdots \oplus X_{j_k}$, $1 \leq j_1 < \cdots < j_k \leq p$, are invariant subspaces of B.

Lemma 2.1 Assume I), II), III) and A), then for $c > c_0$, the operator $A = L_c^{-1} B_c \in L(X, X)$ is strictly positive, compact and satisfies

$\forall x \in \dot{P}$, $\exists n$, an interger $\leq p$, such that

$$\langle x^*, A^n\rangle > 0 \quad \forall x^* \in \dot{P}^* \tag{2.1}$$

where $L_c = cI + L$ and $B_c = cI + B$.

Proof The strict positivity and the compactness of A follow directly from I) and II). It remains to prove (2.1). Indeed, $\forall x \in \dot{P}$, if, say, $P_i x \neq 0$ for all i, then

i) $$P_i B_c x = P_i B_{c_0} x + (c - c_0) P_i x \geq (c - c_0) P_i x > 0.$$

ii) $\forall j \neq i$ such that $P_j B_c x > 0$. For otherwise, $B_c x \in X_i$, i.e., X_i is invariant of B_c, which implies that X_i is an invariant subspace of B. This contradicts with III).

Combining i), ii) and I), we have $\alpha_1 = \alpha_1(x, c) > 0$ such that

$$Ax \geq \alpha_1(e_i + e_j).$$

Repeating the above argument, we have $k \neq i, j$ and $\alpha_2 > 0$ such that

$$A^2 x \geq \alpha_2(e_i + e_j + e_k).$$

After at most p steps, we arrive at $A^p x \geq \alpha_p e$. Then $\forall x^* \in P^*$,

$$\langle x^*, A^p x\rangle \geq \alpha_p \langle x^*, e\rangle > 0,$$

provided by A).

Lemma 2.2 Assume I), II), III) and A) . Suppose that there exists $\overline{x} \in D(L) \cap P$ such that $(L - B)\overline{x} \in P$, then $T = (L - B)^{-1} \in L(X, X)$ exists, and is strictly positive and compact. Moreover, $\forall x \in \dot{P}, \forall x^* \in \dot{P}^*$, $\langle x^*, Tx\rangle > 0$.

Proof According to the above version of Krein-Rutman theorem cf [10] or [11], $\exists r(A) > 0$ and $\forall x^* \in \dot{P}^*$ such that $A^* x^* = r(A) x^*$.

1^0 $r(A) < 1$.

Indeed, by the assumption on $\overline{x}$, we have $L_c\overline{x} > B_c\overline{x}$. It follows $0 < A\overline{x} < \overline{x}$, so is $0 < A^p\overline{x} < A^{p-1}\overline{x} < \cdots < \overline{x}$. Thus

$$r(A)^p \langle x^*, \overline{x}\rangle = \langle (A^*)^p x^*, \overline{x}\rangle = \langle x^*, A^p\overline{x}\rangle < \langle x^*, \overline{x}\rangle.$$

However, according to lemma 2.1, $\langle x^*, A^p\overline{x}\rangle > 0$, it follows $\langle x^*, \overline{x}\rangle > 0$, and then $r(A) < 1$.

2^0 $(L-B)$ is invertible. This is due to

$$L-B=L_c-B_c=L_c(I-A).$$

From 1^0, $(I-A)^{-1}\in L(X,X)$, therefore $(L-B)^{-1}=(I-A)^{-1}L_c^{-1}$.

3^0 Obviously, T is compact. From

$$T=\sum_{j=0}^{\infty}A^jL_c^{-1}$$

and lemma 2.1, it follows that T is strictly positive.

4^0 Now, $\forall x^*\in\dot{P}$, $\forall x^*\in\dot{P}^*$, we have

$$\langle x^*,Tx\rangle\geq\langle x^*,A^pL_c^{-1}x\rangle>0,$$

provided by (2.1).

Theorem 2.3 Assume I), II), III) and A). For any $\lambda>0$, there exists $\mu(\lambda)\in R^1$, $x_\lambda\in D(L)\cap\dot{P}$ and $x^*_\lambda\in D(L^*)\cap\dot{P}^*$ such that

$$(L-\lambda B)x_\lambda=\mu(\lambda)x_\lambda\quad\text{and}\quad(L^*-\lambda B^*)x^*_\lambda=\mu(\lambda)x^*_\lambda. \tag{2.2}$$

Proof First, we apply Krein Rutman theorem to L_j^{-1} on X_j, there exists $\mu_j^{-1}>0$ and $x_j\in P_j$ satisfying $L_j^{-1}x_j=\mu_j^{-1}x_j$, $j=1,2,\cdots,p$. This means $x_j\in D(L_j)\cap\dot{P}_j$, and $L_jx_j=\mu_jx_j$, and then $\exists\alpha_j>0$ such that $x_j\geq\alpha_je_j$, $j=1,2,\cdots,p$, according to I).

Second, setting $\overline{x}=\sum x_j$, we have $\overline{x}\in D(L)\cap\dot{P}$, satisfying $\overline{x}\geq\alpha e$, where $\alpha=\min\{\alpha_j|j=1,2,\cdots,p\}>0$.

Since $B\in L(X_e,X_e)$ and $D(L)\subset X_e$, we obtain $\beta>0$ such that $B\overline{x}\leq\beta e\leq\alpha^{-1}\beta\overline{x}$.

Let $\gamma=\alpha^{-1}\beta$, we have

$$(L-\lambda(B-\gamma I))\overline{x}\geq L\overline{x}=\sum L_jx_j=\sum\mu_jx_j>\theta.$$

However, $\lambda(B-\gamma I)$ satisfies II) and III), lemma 2.2 is applied. In virture of Krein Rutman theorem, $\exists\overline{\mu}(\lambda)>0$ $x_\lambda\in\dot{P}$, $x^*_\lambda\in\dot{P}^*$ satisfying

$$(L-\lambda(B-\gamma I))^{-1}x_\lambda=\overline{\mu}(\lambda)^{-1}x_\lambda$$

and

$$(L^*-\lambda(B^*-\gamma I))^{-1}x^*_\lambda=\overline{\mu}(\lambda)^{-1}x^*_\lambda.$$

These imply that $x_\lambda\in D(L)\cap\dot{P}$ and $x^*_\lambda\in D(L^*)\cap\dot{P}^*$ satisfy

$$(L-\lambda(B-\gamma I))x_\lambda=\overline{\mu}(\lambda)x_\lambda$$

and

$$(L^*-\lambda(B^*-\gamma I))x^*_\lambda=\overline{\mu}(\lambda)x^*_\lambda.$$

Setting $\mu(\lambda)=\overline{\mu}(\lambda)-\lambda\gamma$, (2.2) follows.

Our problem is to find the principal eigenvalue for the weight operator B, i.e., $L\phi=\lambda B\phi$, $\phi\in D(L)\cap\dot{P}$. This, in turn, is to find the first positive root of $\mu(\lambda)$.

Theorem 2.4 Assume I), II), III) and A). If $\exists\lambda_0>0$, $\exists\underline{x}\in D(L)\cap\text{int}(P_e)$ such that $(L-\lambda_0B)\underline{x}\in-\text{int}(P_e)$, then there exist a unique $\lambda_1>0$, and $\phi\in D(L)\cap\dot{P}$, $\phi^*\in\dot{P}^*$ satisfying

i) $L\phi=\lambda_1B\phi$ and $L^*\phi^*=\lambda_1B^*\phi^*$.

Moreover, we have

ii) dim $\ker(L-\lambda_1B)$ =dim coker$(L-\lambda_1B)=1$.

iii) The algebraic multiplicity of λ_1^{-1} for the compact operator $L^{-1}B$ is odd.

iv) $\forall\lambda>0$, if it is an eigenvalue of $Lx=\lambda Bx$, the $\lambda\geq\lambda_1$.

Proof First, following lemma 3.1 in [2] , we show that the function $\lambda\to\mu(\lambda)$ in Theorem 2.3 is analytic. Second, following lemma 1.2 in [7], we prove that $\mu(\lambda_0)\leq0$. Since $\mu(0)>0$, the first root $\lambda_1\in(0,\lambda_0]$ exists.

The remaining part of the proof follows from [12].

Returning to the proof of Theorem 1.1, we have to find $\lambda_0>0$ and $\underline{x}\in D(L)\cap\text{int}(P_e)$, such that $(L-\lambda_0B)\underline{x}\in-\text{int}(P_e)$.

The assumption iii) plays an important role to give the existence of such λ_0 and $\underline{x}$.

3 Examples

Theorems 2.3 and 2.4 can be applied to both elliptic and parabolic problems. The point is to define the space X_j, the element $e_j>0$, the operator L_j and to varify the assumptions A) and I), $j=1,2,\cdots,p$. We shall investigate the above two problems individually.

Example 1(elliptic)

Let $X_j=C(\overline{\Omega})$, L_j be the operator D_j in (0, 2) with domain $D(L_j)=\{\mu\in C_0(\overline{\Omega})|L_ju\in C(\overline{\Omega})\}$, and let e_j be the solution of the equation:

$$\begin{cases}D_ju=1, & \text{in }\Omega,\\ u=0, & \text{on }\partial\Omega.\end{cases}$$

It is well known that the assumption (A) trivially holds, because $e_j(x)>0$, $\forall x\in\Omega$, and that L_j is closed.

$\forall c \geq 0$, as a bounded operator $(cI + L_j)^{-1}$ in $C(\overline{\Omega})$, it maps $C(\overline{\Omega})$ into $C_0^1(\overline{\Omega})$, and is strongly positive. Consequently , it is compact. However, $\forall u \in \mathrm{int}(P_{C_0^1(\overline{\Omega})})$, there exists $\alpha = \alpha(u) > 0$ such that $u \geq \alpha(u)e$, by the Strong Maximum Principle. The assumption (I) is verified.

Let $M \in C(\overline{\Omega}, M(p, R))$ be a matrix satisfying i) and ii), independent to the variable t. Then II) and III) follow from i) and ii) respectively.

Example 2(parabolic case)

Let $X_j = \{\omega \in C(\overline{Q}_T) | \omega(\cdot, 0) = \omega(\cdot, T)\}$,

$$L_j = \frac{\partial}{\partial t} + D_j,$$

with

$$D(L_j) = \{\omega \in X_j | L_j\omega \in X_j, \omega|_{\partial\Omega\times[0,T]} = 0\}$$

and let e_j be the function of x defined in Example 1.

Again, (A) is trivially true.

First, we claim that L_j is a closed operator in X_j.

1^0 Let us consider $\widehat{L}_j$ being the operator L_j with domain

$$D(\widehat{L}_j) = \{\omega \in W_q^{2,1}(Q_T) \big| \omega|_{\partial\Omega\times[0,T]} = 0, \omega(\cdot, 0) = \omega(\cdot, T)\}$$

with $0 < 2 - \dfrac{N+2}{q}, \quad q \neq \dfrac{3}{2}$.

$\widehat{L}_j$ is a closed operator in $L^q(Q_T)$ and has a bounded inverse.

In fact, $\forall u_0 \in W_q^{2-\frac{2}{q}} \cap \overset{\circ}{W}{}_q^1(\Omega)$, $\forall f \in L^q(Q_T)$, the equation:

$$\begin{cases} \widehat{L}_j\omega = f, & \text{in } Q_T, \\ \omega(\cdot, 0) = u_0, & \text{on } Q, \\ \omega = 0, & \text{on } \partial\Omega \times [0, T]. \end{cases} \tag{3.1}$$

has a solution

$$\begin{aligned} \omega =& \omega_1(\cdot, t) + \omega_2(\cdot, t) \\ =& u(t, 0)u_0 + \int_0^t u(t, s)f(\cdot, s)\mathrm{d}s, \end{aligned} \tag{3.2}$$

where $u(t, s)$ is the fundamental solution of the above parabolic equation (3.1). According to the L^q-theory, we have

1)

$$\begin{aligned} \|\omega_2\|_{C_0(\overline{Q}_T)} \leq& C_1\|\omega_2\|_{C^{2r,r}(\overline{Q}_T)} \leq C_2\|\omega_2\|_{W_q^{2,1}(Q_T)} \\ \leq& C_3\|f\|_{L^q(Q_T)} \leq C_4\|f\|_{C(\overline{Q}_T)} \end{aligned}$$

where

$$0 < r < 1 - \frac{N+2}{2q}. \tag{3.3}$$

2) Let K be the operator $u(T, 0)$, then

$$K : C_0(\overline{\Omega}) \to L^q(\Omega) \to W_q^{2-\frac{2}{q}} \cap \overset{\circ}{W}{}_q^1(\Omega) \to C_0^1(\overline{\Omega}),$$

because $u(t, 0)u_0$ is a mild solution of the homogenuous equation (3.1) where $f = 0$.

According to the Maximum Principle, as an operator in $C_0(\overline{\Omega})$, K is a positive compact operator with $0 < spr(K) < 1$.

In order to study $D(\widehat{L}_j)$, we slove the periodic equation:

$$u_0 = \omega(0) = \omega(T) = Ku_0 + \omega_2(\cdot, T),$$

i.e.,

$$(I - K)u_0 = \omega_2(\cdot, T). \tag{3.4}$$

Since $\omega_2(\cdot, T) \in C_0(\overline{\Omega})$, there exists unique $u_0 \in C_0(\overline{\Omega})$ satisfying (3.4). The regularity of K and $\omega_2(\cdot, T)$ imply $u_0 \in W_q^{2-\frac{2}{q}} \cap \overset{\circ}{W}{}_q^1(\Omega)$ and then $\omega \in W_q^{2,1}(Q_T)$. We proved that $\widehat{L}_j^{-1} : f \to \omega$ is the inverse of $\widehat{L}_j$. Therefore $\widehat{L}_j$ is closed.

2^0 $\widehat{L}_j^{-1}$ is a bounded operator in $C(\overline{Q}_T)$:

$$\|\omega\|_{C(\overline{Q}_T)} \leq C\|\widehat{L}_j\omega\|_{C(\overline{Q}_T)}.$$

In fact, in virtue of (3.3) and (3.4), $\|u_0\|_C$ is bounded by $\|f\|_{C(\overline{Q}_T)}$, and by the Maximum Principle, we obtain

$$\begin{aligned} \|\omega\|_{C(\overline{Q}_T)} \leq& \|u_0\|_{C(\overline{Q}_T)} + C_1\|f\|_{C(\overline{Q}_T)} \\ \leq& C_2\|f\|_{C(\overline{Q}_T)} = C_2\|\widehat{L}_j\omega\|_{C(\overline{Q}_T)}. \end{aligned}$$

3^0 Let $R = \widehat{L}_j^{-1}|_{C(\overline{Q}_T)}$ and $L_j = R^{-1}$. Then L_j is closed.

Second, from $R : C(\overline{Q}_T) \to C_0^{2r,r}(\overline{Q}_T)$. $0 < r < 1 - \dfrac{N+2}{2q}$. One proves that L_j^{-1} is compact.

Assume that $f \in \dot{P}_j$, the positive cone in X_j , according to the strong maximum principle of the parabolic equation, $\omega_2(x, t) > 0$, $\forall (x, t) \in Q_T$ and $\dfrac{\partial}{\partial t}\omega_2(\cdot, t)\big|_{\partial\Omega} > 0$, $\forall t > 0$. Then, by the strong positivity of K and the equation (3.4), $u_0 \in \mathrm{int}(P_{C_0^1(\overline{\Omega})})$. Again, by the Strong Maximum Principle, $\omega_1(\cdot, t) \in$

$\mathrm{int}(P_{C_0^1(\overline{\Omega})})$, $\forall t > 0$. Therefore, $\exists \alpha = \alpha(f) > 0$ such that $L_j^{-1}(f) = w \geq \alpha(f)e$.

Finally, $\forall c \geq 0$, the same conclusion holds for $(cI + L_j)^{-1}$.

Let $M \in C(\overline{Q}_T, M(p, R))$ be a matrix satisfying (i) and (ii), then (II) and (III) hold.

参考文献

1 Kolesov J S. A test for the existence of periodic solutions to parabolic equations. Soviet Math. Dokl., 1966(7): 1318 -1320

2 Amann H. Periodic solution of semilinear parabolic equations . Nonlinear Analysis.Cesari-Kannan-Weinberger ed. Academic Press, 1978, 1–29

3 Beltramo A, Hess P. On the principal eigenvalue of a periodic parabolic operator.Comm in PDE, 1984(9), 919–941

4 Lazer A C. Some remarks on periodic solutions of parabolic differential equations. Dynamical Systems II, Bednarek-Cesari ed., Academic Press, 1982, 227–246

5 Castro A, Lazer A. Results on periodic solutions of parabolic equations suggested by elliptic theory. Bull. U. M. I., 1982 (6), 1089 -1104

6 Hess P. On positive solutions of semilinear periodic-parabolic problems. Infinite Dynamical Systems, LN in Math. 1076, Springer, 1984 101–114

7 Chang K C. An extension of the Hess-Kato theorm to elliptic system and applications to multiple solution problem. Acta Math Sinica, 1999, 15(4):

8 Sweers G. Strong positivity in $C(\overline{\Omega})$ for eliptic systems Math Z., 1992, 209: 251–271

9 Pagter B de. Irreducible compact operators. Math Z., 1986,192: 149 -153

10 Diemling K. Nonlinear Functional Analysis. Springer-Verlag, 1985

11 Guan Zhao-zhi. Lectures on Functional Analysis. Beijing: High Eduction Press, 1958 (in chinese)

12 Chang K C. Principle eigenvalue for weight matrix in elliptic systems. To appear