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Smooth Time-Varying Exponential Stabilization of Nonholonomic Systems in the Vector Power Form*

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Abstract: We consider the stabilization problem of nonholonomic systems in the vector power form. We show this class of systems can be transformed into linear time-varying control systems by introducing an assistant state variable. Thus, asymptotic stability with exponential convergence is achieved by using a smooth time-varying feedback control law. Besides the smoothness of the control law, the design procedure is very simple and the convergence rate of each state can be specified a priori. Simulation results show the efficiency of the method.

Key words: nonholonomic systems; power form; smooth time-varying feedback; exponential stabilization
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非完整向量幂式系统的光滑时变指数镇定

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摘要: 研究非完整向量幂式系统的镇定问题.通过引入一个辅助状态变量,此类系统可被转换成一个线性时变系统,由此可得到具有指数收敛率的光滑反馈控制律.该方法优点是控制律光滑并且设计过程简单,系统每个状态的收敛率可被事先确定.仿真结果表明了该方法的有效性.

关键词: 非完整系统; 幂式系统; 光滑时变反馈; 指数镇定

1 Introduction

The problem of controlling nonholonomic systems has attracted much attention over the past decade. Brockett's necessary condition[1] for feedback stabilization precludes the existence of continuously differentiable, time-invariant, state feedback control laws for such systems, though they are controllable.

One way to circumvent this obstruction is using smooth time-varying feedback[2,3] and nonsmooth time-varying feedback[4,5]. It is worthy of noting that most of the existing (continuous or discountious) time-varying control results, though extremely sophisticated and elegant, suffer from the drawback that the control laws designed by these methods are very complex and the design procedure is far from intuitive. An alternative approach to the stabilization of nonholonomic systms is using discontinuous time-invariant strategies. In [6], a nonsmooth transformation was used to develop time-invariant, exponentially convergent controllers for a special class of nonholonomic systems including chained systems. In [7] authors developed exponentially convergent feedback control laws for nonholonomic systems in power form using the method of invariant manifolds. The resulting controllers in [6,7] are discontinuous on a superplane of the state space.

In [8], a smooth aperiodic time-varying exponentially convergent control law was developed for a class of nonholonomic systems including (multiple) chained form system, power form system, Brockett system, etc.. By introducing a proper assistant state variable and taking a coordinate transformation, a class of nonholonomic systems were transformed into linear time-varying systems which can be designed using linear control theory. This paper is a subsequent research of [8].

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Here we deal with nonholonomic systems in the vector power form

$$\begin{cases} x_1^{(r_1)} = u_1, \\ x_2^{(r_2)} = u_2, \\ x_i^{(r_i)} = A_i(Y_1, u_1)\begin{bmatrix} Y_2 \\ u_2 \end{bmatrix}, i = 3, \cdots, n, \end{cases} \tag{1}$$

where r_i denotes the order of time differentiation on the variable x_i, $Y_i = [x_i \, x_i^{(1)} \cdots x_i^{(r_i-1)}]^{\mathrm{T}}$, $A_i \in \mathbb{R}^{1\times(r_2+1)}$, $i = 3, \cdots, n$.

This model is a dynamic model for nonholonomic systems by the terminology of classical mechanics, since for the case when $r_1 = r_2 = 2$, the controls u_1 and u_2 are typically generalized force variables. Eq. (1) with $r_1 > 2$, $r_2 > 2$ can be used to model nonholonomic control systems with augmented actuator dynamics[9]. We refer to (1) as a vetor power form, for it can be viewed as a vector extension of the well known power form model[7].

In this paper we show that nonholonomic systems in the vector power form can be transformed into linear time-varying control systems by introducing a proper assistant state variable. Thus, asymptotic stability with exponential convergence is achieved by using a smooth time-varying feedback control law. The design procedure is very simple. Moreover, the convergence rate of each state can be specified a priori. Simulations of a knife edge system, corresponding to a five-dimensional extended power form system, are presented.

2 Stabilization of the vector power form

2.1 Control law for u_1

Let us introduce an assistant state $x_0(t) \in \mathbb{R}$, to system (1) such that

$$\dot{x}_0 = x_1. \tag{2}$$

Attaching (2) to the x_1 subsystem of (1) we have

$$x_0^{(r_1+1)} = u_1. \tag{3}$$

Since linear time-invariant system(3) is completely controllable, there exists a state feedback control law

$$u_1 = -K_1\begin{bmatrix} x_0 \\ Y_1 \end{bmatrix} \tag{4}$$

to stabilize the system with exponential convergence. By using eigenvalue assignment technique, the spectrum of the closed loop system can be assigned arbitrarily according to the designer's need.

Proposition 1 If the eigenvalues of the closed-loop system (3) under the feedback (4) are assigned to be $r_1 + 1$ different negative real numbers $-l_0, -l_1, \cdots, -l_{r_1}$, where $0 < l_0 < l_1 < \cdots < l_{r_1}$, then there always exists an initial value of $x_0(t)$, denoted as $x_0(0)$, which makes the following limit values

$$\lim_{t\to+\infty}\frac{u_1(t)}{z(t)}, \lim_{t\to+\infty}\frac{x_0(t)}{z(t)}, \lim_{t\to+\infty}\frac{x_1^{(j)}(t)}{z(t)}, j = 0, \cdots, r_1 - 1$$

be nonzero real numbers, where $z(t) = \mathrm{e}^{-l_0 t}$.

Proof Under the assumption of the proposition, the solution of Eq. (3,4) can be described by the following equation

$$\begin{bmatrix} x_0(t) \\ x_1^{(0)}(t) \\ \vdots \\ x_1^{(r_1-1)}(t) \end{bmatrix} = \begin{bmatrix} m_0 & \cdots & m_{r_1} \\ -m_0 l_0 & \cdots & -m_{r_1} l_{r_1} \\ \vdots & & \vdots \\ (-1)^{r_1} m_0 l_0^{r_1} & \cdots & (-1)^{r_1} m_{r_1} l_{r_1}^{r_1} \end{bmatrix}\begin{bmatrix} \mathrm{e}^{l_0 t} \\ \mathrm{e}^{-l_1 t} \\ \vdots \\ \mathrm{e}^{-l_{r_1} t} \end{bmatrix}, \tag{5}$$

where $m_i \{ i = 0, 1, \cdots, r_1 \}$ are some real constants. Eq. (5) can also be rewritten as follows

$$\begin{bmatrix} x_0(t) \\ x_1^{(0)}(t) \\ \vdots \\ x_1^{(r_1-1)}(t) \end{bmatrix} = L\begin{bmatrix} m_0\mathrm{e}^{-l_0 t} \\ m_1\mathrm{e}^{-l_1 t} \\ \vdots \\ m_{r_1}\mathrm{e}^{-l_{r_1} t} \end{bmatrix}, \tag{6}$$

where

$$L = \begin{bmatrix} 1 & \cdots & 1 \\ -l_0 & \cdots & -l_{r_1} \\ \vdots & \vdots & \vdots \\ (-1)^{r_1} l_0^{r_1} & \cdots & (-1)^{r_1} l_{r_1}^{r_1} \end{bmatrix}. \tag{7}$$

It is easy to check that the matrix L is invertible. Denote its inverse by $N = \{n_{ij}\}_{(r_1+1)\times(r_1+1)}$. Taking $t = 0$ in Eq. (6), one concludes easily that $m_i \{ i = 0, 1, \cdots, r_1 \}$ are determined by initial values of system states. Moreover, m_0 is determined by

$$m_0 = n_{11} x_0(0) + \sum_{j=2}^{r_1+1} n_{1j} x_1^{(j-2)}(0), \tag{8}$$

where

$$n_{11} = \frac{1}{\det(L)}\begin{vmatrix} -l_1 & \cdots & -l_{r_1} \\ \vdots & \vdots & \vdots \\ (-1)^{r_1} l_1^{r_1} & \cdots & (-1)^{r_1} l_{r_1}^{r_1} \end{vmatrix} \neq 0. \tag{9}$$

Since x_0 is an assistant state variable, its initial value $x_0(0)$ can be selected freely. Given any fixed initial value $Y_1(0)$ of state vector Y_1, since n_{11} is nonzero, m_0 can always be set to be nonzero if we choose $x_0(0)$ such that

$$x_0(0) \neq -\frac{1}{n_{11}}\sum_{j=2}^{r_1+1} n_{1j} x_1^{(j-2)}(0). \tag{10}$$

From (1) we know $u_1(t) = x_1^{(r_1)}(t)$. Observing (5) we can conclude that

$$\begin{cases} \lim\limits_{t\to+\infty} \dfrac{u_1(t)}{\mathrm{e}^{-l_0 t}} = (-1)^{r_1+1} m_0 l_0^{r_1+1}, \\ \lim\limits_{t\to+\infty} \dfrac{x_0(t)}{\mathrm{e}^{-l_0 t}} = m_0, \\ \lim\limits_{t\to+\infty} \dfrac{x_1^{(j)}(t)}{\mathrm{e}^{-l_0 t}} = (-1)^{j+1} m_0 l_0^{j+1}, j = 0,\cdots, r_1 - 1. \end{cases} \tag{11}$$

All the above limit values are nonzero real numbers because $m_0 \neq 0$. The proposition is thus proved.

Remark 1 Given the numeric area of $x_1^{(j)}(0), j = 0,\cdots, r_1 - 1$, we can choose the $x_0(0)$ independent of $x_1^{(j)}(0)$ and make $m_0 \neq 0$. For example, we assume that $|x_1^{(j)}(0)| \leqslant q_j, j = 0,\cdots, r_1 - 1$ where q_j are positive constants. It can be easily verified that Eq. (8) always holds if we choose $x_0(0)$ such that

$$|x_0(0)| > \frac{1}{|n_{11}|}\sum_{j=2}^{r_1+1} |n_{1j}| q_{j-2}.$$

2.2 Control law for u_2

Lemma 1[8] Consider a linear time-varying control system

$$\dot{x} = (A_0 + A_1(t))x + (B_0 + B_1(t))u \tag{12}$$

with $x \in \mathbb{R}^n, u \in \mathbb{R}^m$. Suppose system (12) satisfies the following properties

1) $\lim\limits_{t\to\infty} A_1(t) = 0, \int_0^\infty \| A_1(t) \| \mathrm{d}t < \infty$,

$\lim\limits_{t\to\infty} B_1(t) = 0, \int_0^\infty \| B_1(t) \| \mathrm{d}t < \infty$;

2) (A_0, B_0) is a stabilizable pair.

Then there exists a state feedback $u = -Kx$ which makes the closed system (12) be uniformly exponentially stable, where K is selected to make $A_0 - B_0K$ a Hurwitz matrix.

Now, the variable Y_1, u_1 can be considered as some functions of time. Therefore, the nonlinear part of the original nonlinear system (1) becomes now a linear time-varying system

$$\begin{cases} x_2^{(r_2)} = u_2, \\ x_i^{(r_i)} = A_i(Y_1, u_1)\begin{bmatrix} Y_2 \\ u_2 \end{bmatrix}, i = 3,\cdots, n. \end{cases} \tag{13}$$

If system (13) can be transformed into a linear time-varying system in the form of (12) with the properties 1) and 2), then the feedback control law for u_2 can be obtained by using Lemma 1. Before proceeding to our main theorem, we first need to introduce some assumptions.

Assumption 1 $A_i(Y_1, u_1), i = 3,\cdots, n$ is not identical to zero and there exists a $p_i > 0$ such that

$$\lim_{t\to+\infty} \frac{A_i(Y_1, u_1)}{(m_0 z)^{p_i}} = D_i,$$

$$\int_0^{+\infty} \left\| \frac{A_i(Y_1, u_1)}{(m_0 z)^{p_i}} - D_i \right\| \mathrm{d}t < +\infty,$$

where $D_i = [d_{i1}\ d_{i2} \cdots d_{i(r_2+1)}]$ is a nonzero constant row vector with the same dimension of $A_i(Y_1, u_1)$, $z(t)$ is defined as in Proposition 1.

Assumption 2 $p_i \neq p_j, i, j = 3,\cdots, n; i \neq j$.

From Assumption 1, p_i can be considered as a minimum relative convergence exponent of $A_i(Y_1, u_1)$ with respect to z. Let us introduce a coordinate transformation

$$W = [W_2 \quad W_3 \quad \cdots \quad W_n]^{\mathrm{T}},$$

where $W_2 = Y_2, W_i = \dfrac{Y_i}{(m_0 z)^{p_i}}, i = 3,\cdots, n$. The time derivative of W_i yields

$$\begin{cases} \left(\dfrac{x_i^{(0)}}{(m_0 z)^{p_i}}\right)' = p_i l_0 \dfrac{x_i^{(0)}}{(m_0 z)^{p_i}} + \dfrac{x_i^{(1)}}{(m_0 z)^{p_i}}, \\ \vdots \\ \left(\dfrac{x_i^{(r_i-1)}}{(m_0 z)^{p_i}}\right)' = p_i l_0 \dfrac{x_i^{(r_i-1)}}{(m_0 z)^{p_i}} + \dfrac{x_i^{(r_i)}}{(m_0 z)^{p_i}}. \end{cases} \tag{14}$$

By (13), the last line of the above equations can be rewritten as

$$\left(\frac{x_i^{(r_i-1)}}{(m_0 z)^{p_i}}\right)' = p_i l_0 \frac{x_i^{(r_i-1)}}{(m_0 z)^{p_i}} + \frac{A_{i2}(Y_1, u_1)}{(m_0 z)^{p_i}}\begin{bmatrix} Y_2 \\ u_2 \end{bmatrix}. \tag{15}$$

Decompose $\dfrac{A_{i2}(Y_1, u_1)}{(m_0 z)^{p_i}}$ as $D_{i2} + \left(\dfrac{A_{i2}(Y_1, u_1)}{(m_0 z)^{p_i}} - D_{i2}\right)$. Then Eq. (15) becomes

$$\left(\frac{x_i^{(r_i-1)}}{(m_0 z)^{p_i}}\right)' = p_i l_0 \frac{x_i^{(r_i-1)}}{(m_0 z)^{p_i}} + D_i \begin{bmatrix} Y_2 \\ u_2 \end{bmatrix} + \left(\frac{A_{i2}(Y_1, u_1)}{(m_0 z)^{p_i}} - D_i\right) \begin{bmatrix} Y_2 \\ u_2 \end{bmatrix}. \quad (16)$$

Denote $\left(\frac{A_{i2}(Y_1, u_1)}{(m_0 z)^{p_i}} - D_i\right)$ as

$$\left(\frac{A_{i2}(Y_1, u_1)}{(m_0 z)^{p_i}} - D_i\right) = [\, h_{i1}(t) \; h_{i2}(t) \; \cdots \; h_{i(r_2+1)}(t) \,],$$

then system (13) can be transformed into the form:

$$W' = (A_0 + A_1(t))W + (B_0 + B_1(t))u_2, \quad (17)$$

where

$$A_0 = \begin{bmatrix}
0 & 1 & \cdots & 0 & & & & & & & & & \\
\vdots & \ddots & \ddots & \vdots & & & & & & & & & \\
0 & \cdots & 0 & 1 & & & & & & & & & \\
0 & \cdots & 0 & 0 & & & & & & & & & \\
 & & & & p_3 l_0 & 1 & \cdots & 0 & & & & & \\
 & & & & 0 & \ddots & \ddots & \vdots & & & & & \\
 & & & & \vdots & \ddots & p_3 l_0 & 1 & & & & & \\
d_{31} & d_{32} & \cdots & d_{3r_2} & 0 & \cdots & 0 & p_3 l_0 & & & & & \\
 & & & & & & & & \ddots & & & & \\
\vdots & \vdots & & \vdots & \vdots & & & & & p_n l_0 & 1 & \cdots & 0 \\
\vdots & \vdots & & \vdots & \vdots & & & & & 0 & \ddots & \ddots & \vdots \\
 & & & & & & & & & \vdots & \ddots & p_n l_0 & 1 \\
d_{n1} & d_{n2} & \cdots & d_{nr_2} & 0 & & & & & 0 & \cdots & 0 & p_n l_0
\end{bmatrix},$$

$$B_0 = [\, 0 \; \cdots \; 1 \; 0 \; \cdots \; d_{3(r_2+1)} \; \cdots \; 0 \; \cdots \; d_{n(r_2+1)} \,]^{\mathrm{T}},$$

$$A_1(t) = \begin{bmatrix}
0 & 0 & \cdots & 0 & & & & & & & & & \\
\vdots & \ddots & \ddots & \vdots & & & & & & & & & \\
0 & \cdots & 0 & 0 & & & & & & & & & \\
0 & \cdots & 0 & 0 & & & & & & & & & \\
 & & & & 0 & 0 & \cdots & 0 & & & & & \\
 & & & & \vdots & \ddots & \ddots & \vdots & & & & & \\
 & & & & 0 & \cdots & 0 & 0 & & & & & \\
h_{31}(t) & h_{32}(t) & \cdots & h_{3r_2}(t) & 0 & \cdots & 0 & 0 & & & & & \\
 & & & & & & & & \ddots & & & & \\
\vdots & \vdots & & \vdots & & & & & & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & & \vdots & & & & & & \vdots & \ddots & \ddots & \vdots \\
 & & & & & & & & & 0 & \cdots & 0 & 0 \\
h_{n1}(t) & h_{n2}(t) & \cdots & h_{nr_2}(t) & & & & & & 0 & \cdots & 0 & 0
\end{bmatrix},$$

$$B_1(t) = [\, 0 \; \cdots \; 0 \; 0 \; \cdots \; h_{3(r_2+1)}(t) \; \cdots \; 0 \; \cdots \; h_{n(r_2+1)}(t) \,]^{\mathrm{T}}.$$

It is easy to see that matrices $A_1(t), B_1(t)$ satisfy property 1).

It can be verified that under Assumption 2, (A_0, B_0) is a controllable pair if and only if

$$\sum_{j=0}^{r_2} d_{i(j+1)} [p_i l_0]^j \neq 0, \; i = 3, \cdots, n. \quad (18)$$

It is easy to show that there always exists an l_0 such that (18) holds. Indeed, l_0 can be chosen as any positive real number except the following values:

$$\frac{y_{i1}}{p_i}, \cdots, \frac{y_{is_i}}{p_i}, \; i = 3, \cdots, n,$$

where $y_1, \cdots, y_{si}$ are s_i positive real roots of the following polynomial equation

$$d_{i(r_2+1)} y^{r_2} + \cdots + d_{i2} y + d_{i1} = 0. \quad (19)$$

Note that if $s_i = 0$, i.e., Eq. (19) does not have any positive real roots, then l_0 can be chosen as any positive real number.

Now we are standing at a position to state our main result.

Theorem 1 System (1) under Assumptions 1 and 2 can be exponentially stabilized by (4) and the following control law:

$$u_2 = -K_2\left[x_2^{(0)} \cdots x_2^{(r_2-1)} \frac{x_3^{(0)}}{(m_0 z)^{p_3}} \cdots \frac{x_3^{(r_3-1)}}{(m_0 z)^{p_3}} \cdots \frac{x_n^{(0)}}{(m_0 z)^{p_n}} \cdots \frac{x_n^{(r_n-1)}}{(m_0 z)^{p_n}}\right]^{\mathrm{T}}, \quad (20)$$

where the feedback gain vetor K_2 is chosen to make $(A_0 - B_0 K_2)$ be a Hurwitz matrix.

Proof From Proposition 1 we know that x_1 subsystem of (1) can be exponentially stabilized by (4) and Y_1, u_1 can be made the same convergent rate $z = e^{-l_0 t}$. From the aforegoing discussions we know that through a coordinate transformation $W = \left[Y_2 \frac{Y_3}{(m_0 z)^{p_3}} \cdots \frac{Y_n}{(m_0 z)^{p_n}}\right]^{\mathrm{T}}$ system (13) can be transformed into system (17) which is linear time-varying and satisfies the 1) and 2) properties under Assumptions 1, 2. By virtue of Lemma 1, system (17) can be exponentially stabilized by the feedback $u_2 = -K_2 W$, i.e. (20). Thus, $Y_2, \frac{Y_3}{(m_0 z)^{p_3}}, \cdots, \frac{Y_n}{(m_0 z)^{p_n}}$ converge exponentially to zero which implies that $Y_2, Y_3, \cdots, Y_n$ converge exponentially to zero. Hence, the result follows.

Remark 2 From Eq. (6) ~ (9), we can deduce that $m_0 z$ equals to $n_{11} x_0(t) + \sum_{j=2}^{r_1+1} n_{1j} x_1^{(j-2)}(t)$. For the convenience of physical realization of control system, $m_0 z$ can be substituted by the latter.

3 Example

In this section we show how our method is applied to the extended power form system[10]

$$\begin{cases} x_1^{(2)} = u_1, \\ x_2^{(2)} = u_2, \\ x_i^{(1)} = \dfrac{1}{(i-2!)} x_1^{i-2} x_2^{(1)}, i = 3, \cdots, n. \end{cases} \quad (21)$$

which is a special case of the vector power form.

According to Theorem 1, smooth time-varying feedback laws can be used to exponentially stabilize the system at the origin. Here we have $r_1 = r_2 = 2, r_i = 1, p_i = i-2, i = 3, \cdots, n$ and

$$A_{i2}(Y_1, u_1) = \left[0 \ \frac{1}{(i-2)!} x_1^{i-2} \ 0\right], i = 3, \cdots, n.$$

By (11), we know that $\lim_{t \to +\infty} \frac{x_1}{m_0 z} = -l_0$. We get

$$D_{i2} = \lim_{t \to +\infty} A_{i2} = \left[0 \ \frac{(-1)^{i-2}}{(i-2)!} l_0^{i-2} \ 0\right], i = 3, \cdots, n.$$

The controllability condition (18) is now reduced to $l_0 \neq 0$, which is obvious. Thus, the control laws for extended power form system are obtained.

A practical example which can be described in the extended power form is the knife edge system using steering and pushing inputs, whose motion equation can be given by[11]

$$\ddot{x} = \lambda \sin\phi + \tau_1 \cos\phi,$$
$$\ddot{y} = -\lambda \cos\phi + \tau_1 \sin\phi,$$
$$\ddot{\phi} = \tau_2.$$

Define the variables

$$\begin{cases} x_1 = x\cos\phi + y\sin\phi, \\ x_2 = \phi, \\ x_3 = x\sin\phi - y\cos\phi, \\ u_1 = \tau_1 - \tau_2(x\sin\phi - y\cos\phi) - \dot{\phi}^2(x\cos\phi + y\sin\phi), \\ u_2 = \tau_2, \end{cases}$$

so that the reduced differential equations are given by

$$x_1^{(2)} = u_1, \ x_2^{(2)} = u_2, \ x_3^{(1)} = x_1 x_2^{(1)}. \quad (22)$$

A simulation with knife edge system has been conducted. The initial condition was chosen as

$$(x(0), y(0), \phi(0), \dot{x}(0), \dot{y}(0), \dot{\phi}(0)) = (0, 1, 1, -1, 0, -1),$$

i.e.

$$(x_1(0), x_2(0), x_3(0), x_1^{(1)}(0), x_2^{(1)}(0)) = (0.84, 1.0, -0.54, -2.39, -1.0).$$

Select

$$K_1 = [6.72 \quad 11.92 \quad 6.2],$$

then we get $l_0 = 1, l_1 = 2.4, l_2 = 2.8$. Select $x_0(0) = 2$ and we get $m_0 = 6.12$. Matrices A_0, B_0 are given by

$$A_0 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 1 \end{bmatrix}, \ B_0 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$

The gain vector K_2 is synthesized by using the pole placement technique with the eigenvalues of $A_0 - B_0 K_2$

specified as $\lambda_1 = -1.5, \lambda_2 = -1.2, \lambda_3 = -0.8$. Then we obtain the gain vector $K_2 = [-1.44\ 4.5\ -9.9]$. Fig. 1 shows the response of variables (x, y, ϕ) of knife edge system. Fig. 2 shows the time history of the control torques (τ_1, τ_2). Fig. 3 shows the path of knife edge system.

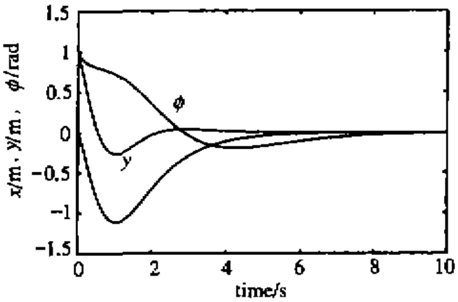

Fig. 1 The response of variables (x, y, ϕ)

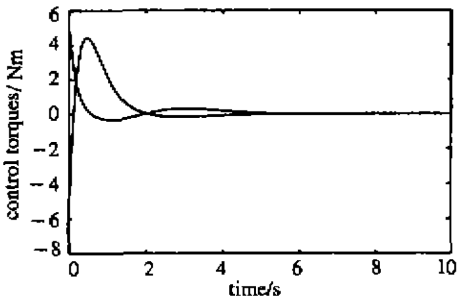

Fig. 2 Time history of control torques (τ_1, τ_2)

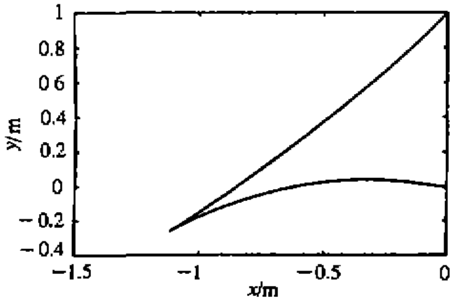

Fig. 3 Path of knife edge system

4 Conclusion

In this paper, a smooth time-varying feedback control law with exponential convergence rate for nonholonomic systems in the vector power form has been developed. Such systems can be transformed into linear time-varying control system by introducing a proper assistant state variable. Thus, asymptotic stability with exponential convergence is achieved by using a smooth time-varying feedback control law. Besides the advantage of design simplicity, the convergence rate of each state can be specified a priori. The idea of introducing an assistant state variable, which is the integral of some state, is curcial in deriving time-varying, exponential stabilizing control laws for nonholonomic systems.

References

[1] Brockett R W. Asymptotic stabilty and feedback stabilization [A]. Differential Geometric Control Theory [M]. Boston: Birkhauser, 1983, 181 - 191

[2] Samson C. Velocity and torque feedback control a Nonholonomic Cart [A]. Int. Workshop on Adaptive and Nonlinear Control [C], Grenoble, 1990, 125 - 151

[3] Pomet J B. Explicit design of time-varying stabilizing control laws for a class of controllable systems without drift [J]. Systems and Control Letters, 1992, 18(2): 147 - 158

[4] M'Closkey R T and Murray R M. Nonholonomic systems and exponential convergence: some analysis tools [A]. Proc. IEEE Conf. Decision and Control [C], San Antonio, 1993, 943 - 948

[5] Sørdalen O J and Egeland O. Exponential stabilization of nonholonomic chained systems [J]. IEEE Trans. Automatic Control, 1995, 40(1): 35 - 49

[6] Astolfi A. Discontinuous control of nonholonomic systems [J]. Systems and Control Letters, 1996, 27(1): 37 - 45

[7] Luo J and Tsiotras P. Exponentially convergent control laws for nonholonomic systems in power form [J]. Systems and Control Letters, 1998, 35(2): 87 - 95

[8] Tian Y P and Li S. Smooth exponential stabilization of nonholonomic systems via time-varying feedback [A]. Proc. IEEE Conf. Decision and Control [C], Sydney, 2000, 1912 - 1917

[9] Kolmanovsky I and McClamroch N H. Developments in nonholonomic control problems [J]. IEEE Control System Magazine, 1995, 15 (6): 20 - 36

[10] Kolmanovsky I, Reyhanoglu M and McClamroch N H. Switched feedback control laws for nonholonomic systems in extended power form [J]. Systems and Control Letters, 1996, 27(1): 29 - 26

[11] Bloch A M. Reyhanoglu M and McClamroch N H. Control and stabilization of nonholonomic dynamic Systems [J]. IEEE Trans. Automatic Control, 1992, 37(11): 1746 - 1756

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