Article ID: 1000-8152(2001)03-0353-05

Stabilization of Timoshenko Beam with Dissipative Boundary Feedback*

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Abstract: The stabilization problem of a Timoshenko beam with some linear boundary feedback controls is studied. First, under the condition that the feedback coefficient matrix B is positive, the energy of the corresponding closed loop system is proven to be exponentially convergent to zero as time $t \to \infty$. Then, under the condition that rank $(B) = 1$, some necessary and sufficient conditions for the corresponding closed loop system to be asymptotically stable are derived.

Key words: Timoshenko beam; boundary feedback; C_0 semigroups; exponential stabilization
Document code: A

具有耗散边界反馈的 Timoshenko 梁的镇定

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摘要：研究了具有线性耗散边界控制的 Timoshenko 梁的边界反馈镇定问题.首先，在反馈系数矩阵 B 正定的前提下，我们证明了所论闭环系统是指数稳定的.然后，对于反馈系数矩阵 B 半正定的情况，得到了闭环系统渐近稳定的几个充要条件.

关键词：Timoshenko 梁；边界反馈；C_0 半群；指数稳定性

1 Introduction

The purpose of this paper is to study the boundary stabilization problem of Timoshenko beam. The system to be investigated is described as follows (see [1]):

$$\begin{cases} \rho\ddot{w} + K(\varphi' - w'') = 0,\ 0 < x < l, t > 0, \\ I_\rho\ddot{\varphi} - EI\varphi'' + K(\varphi - w') = 0,\ 0 < x < l, t > 0, \\ w(0,t) = \varphi(0,t) = 0, \\ K(\varphi(l,t) - w'(l,t)) = u_1(t), \\ -EI\varphi'(l,t) = u_2(t). \end{cases} \tag{1.1}$$

We apply the following linear boundary feedbacks

$$\begin{cases} u_1(t) = \alpha\dot{w}(l,t) + \beta\dot{\varphi}(l,t), \\ u_2(t) = \tau\dot{w}(l,t) + \gamma\dot{\varphi}(l,t) \end{cases} \tag{1.2}$$

as the controls to the end of the beam. Here and henceforth, the prime and the dot always denote derivatives with respect to space and time variables, respectively. The meanings of all the other variables, functions and coefficients involved in the above system are the same as those appeared in paper [1]. Set

$$F \triangleq \begin{bmatrix} \alpha & \beta \\ \tau & \gamma \end{bmatrix},\ B \triangleq \begin{bmatrix} \alpha & \frac{\beta+\tau}{2} \\ \frac{\beta+\tau}{2} & \gamma \end{bmatrix}.$$

Up to now, a lot of interesting results on the boundary feedback stabilization of Timoshenko model have been obtained by many investigators (e.g., see [1~3]). In this paper, we study on the asymptotic behavior of a Timoshenko beam with linear boundary controls. It is well known that this type of controls, under the condition of rank $(B) = 2$, can stabilize the Timoshenko beam exponentially. In the case of rank $(B) < 2$, as will be seen below, the closed loop system may no longer be asymptotically stable.

In [1], the authors proved that under the condition of $\alpha, \gamma > 0$ and $\beta = \tau = 0$, the energy corresponding to the related closed loop system of (1.1) ~ (1.2) decays uniformly to zero as time $t \to \infty$. Recently, under the condition of rank $(B) = 2$ and $\beta = \tau$, the energy of the closed loop system (1.1) ~ (1.2) with variable coeffi-

* Foundation item: supported by the National Key Project of China.
Received date: 2000-03-15; Revised date: 2000-06-02.

cients is proven to be exponentially stable in [4]. Hence, it is natural to ask, in the condition of rank $(B) \neq 2$ and $\beta \neq \tau$, whether the closed loop system (1.1) ~ (1.2) is still asymptotically stable or not. In the following sections, we will investigate this question.

2 Wellposedness of closed loop system

To begin with, we incorporate the closed loop system (1.1) ~ (1.2) into a certain function space. To this end, we define a product Hilbert space $\mathscr{H}$ as

$$\mathscr{H} = V_0^1 \times L_\rho^2(0,l) \times V_0^1 \times L_{I_\rho}^2(0,l),$$

where

$$V_0^k = \{\varphi \in H^k(0,l) \mid \varphi(0) = 0\},\ k = 1,2.$$

and $H^k(0,l)$ is the usual Sobolev space of order k. The inner product in $\mathscr{H}$ is defined as follows:

$$(Y_1, Y_2)_{\mathscr{H}} = \int_0^l K(\varphi_1 - w_k')(\varphi_2 - w_k')\mathrm{d}x + \int_0^l EI\varphi_1'\varphi_2'\mathrm{d}x + \int_0^l \rho z_1 z_2 \mathrm{d}x + \int_0^l I_\rho \psi_1\psi_2 \mathrm{d}x$$

for $Y_k = [w_k, z_k, \varphi_k, \varphi_k]^{\mathrm{T}} \in \mathscr{H},\ k = 1,2.$

We then define a linear operator A in $\mathscr{H}$

$$A\begin{bmatrix} w \\ z \\ \varphi \\ \psi \end{bmatrix} = \begin{bmatrix} z \\ -\dfrac{K}{\rho}(\varphi' - w'') \\ \psi \\ \dfrac{EI}{I_\rho}\varphi'' - \dfrac{K}{I_\rho}(\varphi - w') \end{bmatrix},\quad \begin{bmatrix} w \\ z \\ \varphi \\ \psi \end{bmatrix} \in D(A),$$

$$D(A) = \{[w,z,\varphi,\psi]^{\mathrm{T}} \in \mathscr{H} \mid w,\varphi \in V_0^2, z,\psi \in V_0^1,\ K(\varphi(l) - w'(l)) = \alpha z(l) + \beta\psi(l),\ -EI\varphi'(l) = \tau z(l) + \gamma\psi(l)\}.$$

Then the closed loop system (1.1) ~ (1.2) can be written as the following linear evolution equation in $\mathscr{H}$:

$$\frac{\mathrm{d}Y(t)}{\mathrm{d}t} = AY(t), \tag{2.1}$$

where

$$Y(t) = [w(x,t), \dot{w}(x,t), \varphi(x,t), \dot{\varphi}(x,t)]^{\mathrm{T}}.$$

Lemma 2.1 Assume $B \geq 0$, then A generates a C_0 contraction semigroup $T(t)$ in $\mathscr{H}$.

For the proof of the case of $\beta = \tau$, see [4], and in the case of $\beta \neq \tau$, the proof is similar and hence it is omitted here.

According to the linear semigroup theory, we get

Theorem 2.2 For any $Y_0 \in \mathscr{H}$, (2.1) has a unique weak solution $Y(t) = T(t)Y_0$, where $T(t)$ is the linear semigroup generated by A. Moreover, if $Y_0 \in D(A)$, $Y(t) = T(t)Y_0$ becomes the strong solution to (2.1).

3 Asymptotic behavior of the closed loop system (I)

In this section, we discuss the asymptotic behavior of the closed loop system (2.1) under the condition of $B \geq 0$ and $\beta = \tau$.

The energy corresponding to the solution of the closed loop system (2.1) is defined as

$$E(t) = \frac{1}{2}\int_0^l (EI \mid \varphi' \mid^2 + K \mid \varphi - w' \mid^2 + \rho \mid \dot{w} \mid^2 + I_\rho \mid \dot{\varphi} \mid^2)\mathrm{d}x,$$

where $Y(t) = [w(\cdot,t), \dot{w}(\cdot,t), \varphi(\cdot,t), \dot{\varphi}(\cdot,t)]^{\mathrm{T}}$ is the solution to (2.1). Let $Y_0 \in D(A)$ then

$$\dot{E}(t) = -[z(l,t), \psi(l,t)]B[z(l,t), \psi(l,t)]^{\mathrm{T}}. \tag{3.1}$$

The following proposition can be found in [4].

Proposition 3.1 Assume that $B > 0$. Then there exist positive constants M, ω such that

$$E(t) \leq M \| Y_0 \| \mathrm{e}^{-\omega t},\ \forall\, Y_0 \in \mathscr{H}.$$

Let $\rho_1 = \sqrt{\rho/K}, \rho_2 = \sqrt{I_\rho/EI}$. The following is the main result of this section.

Theorem 3.2 Assume that $\beta = \tau$. Then the energy of the closed loop system (2.1) decays asymptotically to zero for all $(\rho, I_\rho, K, EI) > 0$ if and only if rank $(B) = 2$.

Proof The sufficiency of the theorem is obvious. Now we prove the necessity. In the case of rank $(B) = 0$, the closed loop system (2.1) is conservative, and hence the assertion is trivial. Thus it remains to prove that the closed loop system (2.1) is not asymptotically stable in the case of rank $(B) = 1$ for some parameters $(\rho, I_\rho, K, EI) > 0$. We know that $0 \in \rho(A)$ and that the resolvent of A is compact. Hence for the given parameters ρ, I_ρ, K and EI, the closed loop system (2.1) does not decay asymptotically if and only if there exists $\omega \in \mathbb{R}$ such that $i\omega \in \sigma_p(A)$ (see [5]). Assume that $A\Psi = i\omega\Psi$ with $\Psi \neq 0$ and $\Psi = [w, z, \varphi, \psi]^{\mathrm{T}} \in D(A)$. It is obvious that $\omega \neq 0$. From $\dot{E}(t) \mid_{Y(t) = T(t)\Psi} = 0$, we get $[z(l), \psi(l)]B[z(l), \psi(l)]^{\mathrm{T}} = (t_1 z(l) + t_2\psi(l))^2 = 0$, where $B = [t_1, t_2]^{\mathrm{T}}[t_1, t_2]$ with two real constants t_1 and t_2, not equal to zero simultaneously. Moreover, since B is symmetric and nonnegative, we have $B(z(l), \psi(l))^{\mathrm{T}} = 0$. Hence $\varphi(l) - w'(l) = \varphi'(l) = 0$. Thus w and φ satisfy

$$\begin{cases}\frac{K}{\rho}(w''-\varphi')+\omega_w^2=0,\\ \frac{EI}{I_\rho}\varphi''-\frac{K}{I_\rho}(\varphi-w')+\omega^2\varphi=0,\\ w(0)=\varphi(0)=t_1w(l)+t_2\varphi(l)=0,\\ \varphi'(l)=\varphi(l)-w'(l)=0.\end{cases}\tag{3.2}$$

Set

$$Z(x)=[w(x),w'(x),\varphi(x),\varphi'(x)]^{\mathrm{T}}.$$

Then (3.2) can be written as

$$\begin{cases}Z'=\tilde{A}Z,\\ B_1Z(0)=0,\\ B_2Z(l)=0,\end{cases}\tag{3.3}$$

where

$$\tilde{A}=\begin{bmatrix}0&1&0&0\\ -a&0&0&1\\ 0&0&0&1\\ 0&-c&c-b&0\end{bmatrix},$$

$$B_1=\begin{bmatrix}1&0&0&0\\0&0&1&0\end{bmatrix},\ B_2=\begin{bmatrix}t_1&0&t_2&0\\0&0&0&1\\0&-1&1&0\end{bmatrix},$$

$$a=\rho_1^2\omega^2,\ b=\rho_2^2\omega^2,\ c=\frac{K}{EI}.$$

In the condition of $b\neq c$, let $Z=PZ_1$, where $P,\alpha_1,\alpha_2,\beta_1$ and β_2 are the same as those defined in [7]. Then the first equation of (3.3) becomes $Z_1'=\hat{A}Z_1$, where

$$\hat{A}\triangleq P^{-1}\tilde{A}P=\mathrm{diag}\{\alpha_1,-\alpha_1,\alpha_2,-\alpha_2\}.$$

The general solution to $Z_1'=\hat{A}Z_1$ can be written as

$$Z_1(x)=\mathrm{diag}\{e^{\alpha_1x},e^{-\alpha_1x},e^{\alpha_2x},e^{-\alpha_2x}\}\Theta,$$

where Θ is a 4×1 constant vector. Therefore, for (3.3) to have a nontrivial solution, it is necessary and sufficient that

$$\det(Q_1)=0,\tag{3.4}$$

where

$$Q_1=\begin{bmatrix}\alpha_1&\alpha_1&\alpha_2&\alpha_2\\ a\beta_1&-a\beta_1&a\beta_2&-a\beta_2\\ (\alpha_1t_1+a\beta_1t_2)e^{\alpha_1l}&(\alpha_1t_1-a\beta_1t_2)e^{-\alpha_1l}&(\alpha_2t_1+a\beta_2t_2)e^{\alpha_2l}&(\alpha_2t_1-a\beta_2t_2)e^{-\alpha_2l}\\ a\alpha_1\beta_1e^{\alpha_1l}&a\alpha_1\beta_1e^{-\alpha_1l}&a\alpha_2\beta_2e^{\alpha_2l}&a\alpha_2\beta_2e^{-\alpha_2l}\\ (a\beta_1-\alpha_1^2)e^{\alpha_1l}&(\alpha_1^2-a\beta_1)e^{-\alpha_1}&(a\beta_2-\alpha_2^2)e^{\alpha_2l}&(\alpha_2^2-a\beta_2)e^{-\alpha_2l}\end{bmatrix}.$$

We have

$$\mathrm{rank}(Q_1)=2+\mathrm{rank}\begin{bmatrix}a\beta_1t_2&\alpha_2t_1\\ \beta_1(\alpha_1\sinh l\alpha_1-\alpha_2\sinh l\alpha_2&\alpha_2(\beta_2\cosh\alpha_1l-\beta_1\cosh\alpha_2l)\\ \alpha_1(\beta_2\cosh\alpha_2l-\beta_1\cosh\alpha_1l&\beta_2(\alpha_1\sinh\alpha_2l-\alpha_2\sinh l\alpha_1l)\end{bmatrix}.$$

Therefore rank $(Q_1)<4$ if and only if the following transcendental equations on a, b and c has solution (a_0,b_0,c_0) such that $a_0,b_0,c_0>0$:

$$\begin{cases}at_2(\beta_1\cosh\alpha_2l-\beta_2\cosh\alpha_1l)=\\ t_1(\alpha_2\sinh\alpha_2l-\alpha_1\sinh\alpha_1l),\\ a\beta_1\beta_2t_2(\alpha_1\sinh\alpha_2l-\alpha_2\sinh\alpha_1l)=\\ \alpha_1\alpha_2t_1(\beta_2\cosh\alpha_2l-\beta_1\cosh\alpha_1l).\end{cases}\tag{3.5}$$

Now we prove that, under the condition of rank $(B)=1$, (3.5) has a positive solution (a_0,b_0,c_0) with $b_0\neq c_0$. Without loss of generality, we may suppose that $l=1$.

In fact, in the case of $\beta+\tau\neq0$ and $\rho_1\neq\rho_2$, denote $\xi\triangleq t_2/t_1,\sigma\triangleq\alpha_1\alpha_2t_1(a\beta_1\beta_2t_2)^{-1}$. For $\xi>0$, from the definitions of ξ and σ, it follows that (3.5) is equivalent to

$$\begin{cases}\alpha_2\sinh\alpha_2-a\xi\beta_1\cosh\alpha_2=\alpha_1\sinh\alpha_1-a\xi\beta_2\cosh\alpha_1,\\ \alpha_1\sinh\alpha_2-\beta_2\sigma\cosh\alpha_2=\alpha_2\sinh\alpha_1-\beta_1\sigma\cosh\alpha_1,\end{cases}\tag{3.6}$$

or equivalently, for $b>c$ (to be precise),

$$\begin{cases}v\sin v+a\beta_1\xi\cos v=u\sin u+a\beta_2\xi\cos u,\\ u\sin v+\beta_2\sigma\cos v=v\sin u+\beta_1\sigma\cos u,\end{cases}\tag{3.7}$$

with $\alpha_1=\mathrm{i}u,\alpha_2=\mathrm{i}v$. Set

$$\theta_1\triangleq\arccos\frac{v}{\sqrt{v^2+(a\beta_1\xi)^2}}=\arccos\frac{v}{\sqrt{v^2+(v^2-b)^2\xi^2}},$$

$$\theta_2\triangleq\arccos\frac{u}{\sqrt{u^2+(a\beta_2\xi)^2}}=\arccos\frac{u}{\sqrt{u^2+(u^2-b)^2\xi^2}}.$$

Then (3.7) can be rewritten as

$$\begin{cases}\sqrt{v^2+(v^2-b)^2\xi^2}\sin(v-\theta_1) = \\ \sqrt{u^2+(u^2-b)^2\xi^2}\sin(u+\theta_2), \\ \dfrac{\beta_2\sqrt{v^2+(v^2-b)^2\xi^2}}{v}\cos(v-\theta_1) = \\ \dfrac{\beta_1\sqrt{u^2+(u^2-b)^2\xi^2}}{v}\cos(u+\theta_2).\end{cases} \tag{3.8}$$

It is not difficult to prove that there exist two positive numbers u_0, v_0 satisfying

$$\sqrt{v_0^2+(v_0^2-b)^2\xi^2}\sin(v_0-\theta_1) = \sqrt{u_0^2+(u_0^2-b)^2\xi^2}\sin(u_0+\theta_2)$$

and $0 < v_0 - \theta_1 < \pi/2, 5\pi/2 < u_0 + \theta_2 < 3\pi$. Let a_0, b_0, c_0 be defined as above, and set

$$\beta_1 = \frac{(v_0^2-u_0^2)\tilde{v}_0\cos(v_0-\theta_1)}{v_0^2\tilde{v}_0\cos(v_0-\theta_1)-u_0^2\tilde{u}_0\cos(u_0+\theta_2)},$$

$$\beta_2 = \frac{(v_0^2-u_0^2)\tilde{u}_0\cos(u_0+\theta_2)}{v_0^2\tilde{v}_0\cos(v_0-\theta_1)-u_0^2\tilde{u}_0\cos(u_0+\theta_2)},$$

where

$$\tilde{u}_0 \triangleq \frac{u_0^2+(u_0^2-b)^2\xi^2}{u_0},\ \tilde{v}_0 \triangleq \frac{v_0^2+(v_0^2-b)^2\xi^2}{v_0}.$$

Thus it follows that (3.8) has a positive solution (a_0, b_0, c_0) with $b_0 \neq c_0$.

As for $\xi < 0$, by the same argument as above, we can prove the same assertion as that of the case of $\xi > 0$. In the case of $\rho_1 \neq \rho_2, \beta + \tau = 0$ or $\rho_1 = \rho_2$, the proof for that (3.5) has positive solution is trivial.

By the definition of a, b and c and from the (a_0, b_0, c_0) chosen above, it is easy to find parameters $\rho_0, K_0, I_{\rho 0}, EI_0$ and ω_0 such that the corresponding closed loop system (2.1) has the eigenvalue $i\omega_0 \in \sigma_p(A)$. The proof is then complete.

4 Asymptotic behavior of the closed loop system (II)

In this section, we discuss the asymptotic property of the closed loop system (2.1) for the nonsymmetrical feedback case $(\beta \neq \tau)$. Denote $\sigma \triangleq (\beta - \tau)\omega i/2$.

In the case of $B > 0$, we have known that the related closed loop system (2.1) is exponentially stable. We now discuss the case of rank $(B) = 1$.

Lemma 4.1 Let ξ, σ be defined as above. Assume that rank $(B) = 1$. Then for $\omega \neq \pm\sqrt{K/I_\rho}, i\omega \in \sigma_p(A)$ if and only if (a, b, c) is a positive solution to

$$\begin{cases}Kt_2((\sigma-a\beta_1 EI)\cosh\alpha_2 l-(\sigma-a\beta_2 EI)\cosh\alpha_1 l)) = \\ EIt_1(\alpha_1(K-\beta_2\sigma)\sinh\alpha_1 l-\alpha_2(K-\beta_1\sigma)\sinh\alpha_2 l, \\ \alpha_1\alpha_2 EIt_1(\beta_2(K-\beta_1\sigma)\cosh\alpha_2 l-\beta_1(K-\beta_2\sigma)\cosh\alpha_1 l) = \\ Kt_2(\beta_1\alpha_2(\sigma-a\beta_2 EI)\sinh\alpha_1 l-\beta_2\alpha_1(\sigma-a\beta_1 EI)\sinh\alpha_2 l),\end{cases} \tag{4.1}$$

with $b \neq c$, where

$$a = \rho_1^2\omega^2,\ b = \rho_2^2\omega^2,\ c = K/EI.$$

The proof of this lemma is similar to that of Theorem 3.2, hence it is omitted here.

By the elementary skills of analysis, we can prove the next two lemmas.

Lemma 4.2 Assume that $t_1 = 0, t_2 \neq 0$ (i.e., $\alpha = 0, \tau = -\beta$ and $\gamma \neq 0$) and $\pm i\sqrt{K/I_\rho}$ are not spectral points of A. Then the energy of the closed loop system (2.1) decays asymptotically to zero as $t \to +\infty$.

Lemma 4.3 Assume that $t_1 \neq 0, t_2 = 0$ (i.e., $\gamma = 0, \tau = -\beta$ and $\alpha \neq 0$) and $\pm i\sqrt{K/I_\rho}$ are not spectral points of A. Then the energy of the closed loop system (2.1) decays asymptotically to zero as $t \to +\infty$.

Set $\eta \triangleq Kt_2(EIt_1)^{-1}$. The following is the main result of this section.

Theorem 4.4 Assume that rank $(B) = 1, \tau \neq \beta$, $\tau + \beta \neq 0$ and $B \geqslant 0$. Then, the energy of the closed loop system (2.1) decays asymptotically to zero as $t \to +\infty$ if and only if

$$\begin{cases}\alpha_2\beta_1(\eta^2-\alpha_1^2\beta_2^2)\sinh\alpha_1 l = \\ \alpha_1\beta_2(\eta^2-\alpha_2^2\beta_1^2)\sinh\alpha_2 l, \\ \alpha_2\beta_1(\eta^2-\alpha_1^2\beta_2^2)\cosh\alpha_1 l = \\ (\eta^3+(c-\beta_2(a-b+c))\eta)\sinh\alpha_2 l, \\ \alpha_2\beta_1(\eta^2-\alpha_1^2\beta_2^2)\cosh\alpha_2 l = \\ (\eta^3+(c-\beta_1(a-b+c))\eta)\sinh\alpha_2 l\end{cases} \tag{4.2}$$

does not have positive solution (a, b, c) with $b \neq c$.

Proof According to Lemmas 4.1 ~ 4.3, it is enough to demonstrate the following two assertions:

1) (4.1) has a positive solution (a, b, c) with $b \neq c$ if and only if (4.2) does so.

2) A has no pure imaginary spectral points $\pm i\sqrt{K/I_\rho}$.

First, we prove assertion 1). We suppose that (4.1) has some positive solution (a, b, c) with $b \neq c$. Comparing the real part and the imaginary one at the both sides of (4.1) and through a demanding calculation, we obtain

$$Q_{2f} \triangleq \begin{bmatrix} \eta & -\eta & \alpha_1\beta_2 & -\alpha_2\beta_1 \\ 0 & aEI\eta(\beta_1-\beta_2) & \alpha_1(K-a\beta_1\beta_2EI) & -\alpha_2(K-a\beta_1^2EI) \\ 0 & 0 & \alpha_2\beta_1\eta^2-\alpha_1^2\alpha_2\beta_1\beta_2^2 & \alpha_1\beta_2(\alpha_2^2\beta_1^2-\eta^2) \\ 0 & 0 & f_7 & f_8 \end{bmatrix} \begin{bmatrix} \cosh\alpha_2 l \\ \cosh\alpha_1 l \\ \sinh\alpha_1 l \\ \sinh\alpha_2 l \end{bmatrix} = 0, \qquad (4.3)$$

with

$$f_7 = K\alpha_1^2\alpha_2(K-a\beta_2EI(\beta_1+\beta_2))+\alpha_2\beta_1\beta_2(aEI\eta)^2,$$

$$f_8 = K\alpha_1\alpha_2^2(a\beta_1EI(\beta_1+\beta_2)-K)-\alpha_1\beta_1\beta_2(aEI\eta)^2.$$

It is easy to see that $3 \leqslant \text{rank}\,(Q_2) \leqslant 4$ and that (4.3) is not compatible if rank $(Q_2) = 4$. Thus we know that (4.1) has positive solution (a, b, c) with $b \neq c$ if and only if $\det(Q_4) = 0$ and (4.2) holds true.

From the fact that $\cosh^2\alpha_2 l - \sinh^2\alpha_1 l = \cosh^2\alpha_1 l - \sinh^2\alpha_1 l = 1$, it follows that (4.2) implies $\det(Q_4) = 0$. Thus, assertion (1) follows.

The proof of assertion (2) is similar to that offered above.

Acknowledgement The authors would like to thank Drs Shi Dong-Hua, Li Xue-Zhi, Feng Shao-Ji and Chai Shu-Gen for their help in verifying some of the calculations involved in this paper.

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