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Robustness Analysis and Robust Control for a Class of Hybrid System *

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Abstract: The robust stability analysis for a class of hybrid system with discrete state uncertainty disturbance are presented. Aiming at the effect of discrete state uncertainty disturbance, a switch strategy and a sub-controller design are stated to guarantee the robust stability of the whole system. As a result, the performance of the system is improved.

Key words: hybrid system; robustness; discrete event; optimization

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一类混杂系统的鲁棒性分析与控制

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摘要: 对一类离散状态存在不确定性扰动的混杂系统进行了鲁棒稳定分析,并针对离散状态不确定性扰动对系统的影响给出了切换控制及各个子控制器的设计方案,保证了整个系统的鲁棒稳定性.结果表明整个系统的性能得到了改善.

关键词: 混杂系统; 鲁棒性; 离散事件; 优化

1 Introduction

Hybrid systems mean that systems contain both continuous and discrete variables and the two kinds of variables affect each other. It's come into being a class of coupling nonlinear systems. The continuous variables of the systems correspond usually to continuous dynamic of real systems, but the discrete variables are very manifold. The discrete variable could be produced by digital sampling from digital control systems, sample data systems or from logic switching, the change of system structure, the shift of equilibrium and so on. On the stability analysis for hybrid systems, M. S. Branicky[1] used Multiple Lyapunov Function to analyze the stability. J. L. Mancilla-Aguilar et al[2] discussed exponential stability and digital feedback stabilitzation of switching systems. The controllability, observability and stabilization of the hybrid systems had been discussed by [3 ~ 5], whose candidate systems are linear. On the use of switch to improve the performance of systems, J. H. Frommer et al[6], L. Y. Wang et al[7] designed switching control strategy and analyzed the stability of the closed-loop systems respectively. On the robust control for hybrid systems, L. Y. Wang et al[7], J. A. Ball et al[8], M. De La Sen[9] had studied respectively. But all those works did focus only on the disturbance of continuous part, not investigate the disturbance of the other important part of hybrid systems, namely the discrete dynamic part.

In this paper we analyze the effect of discrete disturbance for a class of hybrid system and based on the analysis we provided a robust control strategy for a class of hybrid system.

The paper is organized as follows: In Section 2 the problem formulation is provided, and the effect of discrete disturbance is analyzed. In Section 3 the switch strategy and sub-controller design are given. The performance and stability of the hybrid system are analyzed in Section 4. A simple simulating example is presented in Section 5 and conclusion is drawn in Section 6.

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2 Problem formulation

Consider a class of hybrid system given in Fig. 1. It includes two kinds of state variables namely continuous part and discrete part. Correspondingly, its input and output should contain the two parts too. The system can be described as

$$\begin{cases} \dot{x} = A(s) + B(s)u, \\ s(t) = D(s(t_-), \alpha), \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ is the continuous state, $s \in S$, $S = \{s_1, \cdots, s_k\}$ is the discrete state set, $u \in \mathbb{R}^m$ denotes the continuous input. α is the swithching control.

For fixed discrete state s, the system is a continuous linear system. It can be described in differential equation or transfer function. Here we use the state differential equation, $A(s) \in \mathbb{R}^{n\times n}$, $B(s) \in \mathbb{R}^{(n\times m)}$ are the system matrices for each discrete state s.

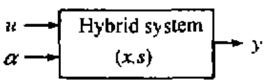

Fig. 1 Hybrid system

The change of the discrete state s can be described by DEDS, $D(\cdot)$. The change of state s may result from the switch of the system structure, or the jump of the internal states. $s(t_-)$ denotes the discrete state before the time t, $s(t)$ denotes the discrete state s at the time of t. For the switching control variable α, the shift from the old state s_{old} to the new state s_{new} is said a switching control, denote $\alpha = (s_{\text{old}}, s_{\text{new}})$. Obviously at time t, $s(t_-) = s(t)$ means no switching. The set

$$\Sigma = \{(s_i, s_j), s_i, s_j \in S, i, j = 1, \cdots, k\}$$

represents all the possible switching, namely switching control set. Clearly, it is a limit set.

Under the assumption of the system continuous variable is measurable, we only consider the feedback with continuous state in this paper.

Due to the imprecise of modeling, the aging of the equipment of real system and so on, the system is usually affected by uncertainties. With the deep study for the uncertainty robust control and H_∞ control have been greatly developed and gotten many results. Main results of them are aimed at continuous system. For hybrid system (1) in ideal state, if switching control is $\alpha = (s, s_i)$, then the corresponding continuous control is $u = k(s_i)x$, but the new discrete state maybe s_j, not s_i as the system may be affected by disturbance and other uncertain factors, therefore the closed loop system will be

$$\dot{x} = A(s_j)x + B(s_j)k(s_i)x.$$

In this case the discrete state will not match continuous control input, the primary control design can not improve the performance of system, even not guarantee the stability of subsystems.

Since there is discrete state uncertainty, we know that the system can be described as

$$\begin{cases} \dot{x} = A(s)x + B(s)u, \\ s(t) = D(s(t_-), \alpha, \delta), \end{cases} \tag{2}$$

where δ is the uncertain disturbance for the switch control α. From Fig. 2, we can see that the miss matching of the sub-systems and sub-controllers by the disturbance. The state s_i which is not affected by uncertain disturbance is said to be nominal discrete state, and name the state s_j which is affected by disturbance is said to be real discrete state. Denote

$$d(s_i, s_j) := [A(s_j) - A(s_i), B(s_j) - B(s_i)],$$

then with $\delta = (s_i \to s_j)$, the subsystems may be describe as follows:

$$\dot{x} = A(s_j)x + B(s_j)k(s_i)x =$$
$$A(s_i)x + B(s_i)k(s_i)x + d(s_j, s_i)\begin{bmatrix} I \\ k(s_i) \end{bmatrix}x. \tag{3}$$

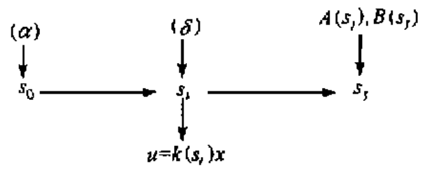

Fig. 2 Affection of discrete state disturbance

The last term of formula (3) is due to the discrete disturbance. It's evident that the "size" of affection is decided by $d(s_i, s_j)$. Would this affection damage the stability of subsystems? For discussing this affection we give some definition as follows:

Definition 1 We call subsystem (3) quadratic stable if there exist positive definite matrix P and positive real number γ for $V(x) = x^{\mathrm{T}}Px$ such that the derivative of $V(x)$ along with (3), $\frac{\mathrm{d}V(x)}{\mathrm{d}t} \leqslant -\gamma \|x\|^2 < 0$.

Definition 2 For given discrete state set $S = \{s_1, \cdots, s_k\}$, we call

$$S_i := \{s_j \mid d(s_i, s_j)d^{\mathrm{T}}(s_i, s_j) \leqslant I\}$$

the safe region of system (1).

Proposition 1 Let $\alpha = (s_{old}, s_i), \delta = (s_i, s_j), s_j \in S_i$ system (3) is quadratic stable if and only if $A(s_i) + B(s_i)k(s_i)$ is stable, and

$$\left\| \begin{bmatrix} I \\ k(s_i) \end{bmatrix} (sI - A(s_i) - B(s_i)k(s_i))^{-1} \right\|_\infty < 1.$$

Proof since

$$d(s_j, s_i)\begin{bmatrix} I \\ k(s_i) \end{bmatrix} = I \cdot d(s_j, s_i) \cdot \begin{bmatrix} I \\ k(s_i) \end{bmatrix},$$

by using Theorem 4.4.1 in [10] we can easily get the result.

3 Control strategy design

For hybrid control system (2) the control design contains two parts. One is design of the switch strategy, another is design of sub-controller. Viewing the controller as hierarchical, it is apparent that switch strategy belongs to the higher hierarchy. It represents the intelligent factors and plays a key role in system performance. Therefor we firstly present the design of switch strategy, then we deduce the corresponding conditions by the requirements of switch strategy. At last, based on these conditions, the design of sub-controller is presented. First of all, the following assumption is needed.

Assumption 1 For each $s \in S$, the subsystem $(A(s), B(s))$ is controllable.

Definition 3 For each s_i, the set

$$\bar{S}_i := \{s \mid s = D(s, \alpha, \delta, t), t \geq 0\}$$

is said to be the uncertain area of (2).

Assumption 2 The uncertain area for each s_i is known.

3.1 Switch strategy

Before designing switch strategy, it is supposed that each sub-controller has been designed. We use the performance criterion

$$J = \int_t^\infty (x^T(\tau)Qx(\tau) + u^T(\tau)Ru(\tau))d\tau$$

in [2], where both Q and R are positive definite matrices. The real discrete state is denoted by s_j, and the nominal discrete state is denoted by s_i. For simplicity, J is rewritten as $J(s_i, s_j, t)$ in this case. Obviously s_i is known, s_j is not known but its range is known, namely we know $s_j \in \bar{S}_i$, thus we can not compute the exact value of $J(s_i, s_j, t)$. However, we can know the approximate range of $J(s_i, s_j, t)$. Since $\bar{S}_i$ is a limit set, all values of $J(s_i, s_j, t), s_j \in \bar{S}_i$, can be calculated.

We denote

$$e_w(s_i, t) = \max_{s_j \in \bar{s}_i}(J(s_i, s_j, t) - J(s_i, s_i, t)),\ i = 1, \cdots, k, \quad (4)$$

$$e_b(s_i, t) = \min_{s_j \in \bar{s}_i}(J(s_i, s_j, t) - J(s_i, s_i, t)),\ i = 1, \cdots, k, \quad (5)$$

The following switch strategy is put forward then at every t, the nominal discrete state is s_i, and the new discrete state s^* can be calculated by

$$s^* = \arg\min\{J(s_i, s_i, t), J(s_p, s_p, t) + e_w(s_p, t_+) - e_b(s_i, t_-), p = 1, \cdots, k, p \neq i\}. \quad (6)$$

Formula (6) presents that

$$J(s^*, s^*, t) + e_w(s^*, t_+) \leq J(s_i, s_i, t) + e_b(s_i, t_-)$$

for each new $s^* \neq s_i$. So if nominal discrete state is s_i, the real discrete state is s_j at time t_-, and nominal discrete state is s^*, the real discrete state is s_p at time t_+,

$$J(s^*, s_p, t) \leq J(s^*, s^*, t) + e_w(s^*, t_+) \leq J(s_i, s_i, t) + e_b(s_i, t_-) \leq J(s_i, s_i, t),$$

then from (4) and (5) we get which implies that J is deduced.

It is obvious that the selection of s^* can guarantee that the performance criterion descends with s^*. That is to say that this switch strategy can improve the system performance.

3.2 Sub-controller design

From the design procedure for discrete control above we can find the necessary condition for the successful switch strategy is that $J(s_i, s_j, t)$ should be bounded. That is to say that the described system by (3) should be quadratic stable. Integrating with the foregoing analysis of system stability we can describe the design of sub-controller as the following optimization problem.

For nominal discrete state s_i, we use the linear feedback controller as $u = k(s_i)x$.

min $J(s_i, s_j, t)$

s.t.

1) $\forall s_j \in \bar{S}_i,\ s_i \in S_i$;

2) $\left\| \begin{bmatrix} I \\ k(s_i) \end{bmatrix} (sI - A(s_i) - B(s_i)k(s_i))^{-1} \right\|_\infty < 1.$

From Proposition (1), the solution $k(s_i)$ of the above formula can guarantee the stability of subsystem

$$\dot{x} = A(s_j)x + B(s_j)k(s_i)x,\ s_j \in \bar{S}_i.$$

4 Performance and stability analysis

Using the above switch strategy and sub-controller, what impact can be produced on stability and performance? Is the whole performance criterion

$$J_{\text{whole}} = \int_0^{\infty} (x^{\mathrm{T}}(\tau)Qx(\tau) + u^{\mathrm{T}}(\tau)Ru(\tau))\mathrm{d}\tau$$

improved? For these, we have the following result:

Theorem For the system on the action of switch strategy (6) the following conclusion can be drawn

1) The origin $x = 0$ of the closed loop system is globally asymptotically stable;

2) The whole performance criterion of the system on the action of switching is less than or equal to that of the system not on the action.

Proof For simplicity, we denote

$$L(x(\tau),u(\tau)) = x^{\mathrm{T}}(\tau)Qx(\tau) + u^{\mathrm{T}}(\tau)Ru(\tau).$$

We firstly prove 2). Assume the switching sequence is

$$S = \{s(t_j), j = 0,1,\cdots\},\ t_0 = 0,$$

due to the disturbance of uncertainty the real state sequence is

$$S^* = \{s^*(t_j),\ j = 0,1,\cdots,\},t_0 = 0,$$

for arbitrary positive integer $N > 0$,

$$S_N = \{s(t_j),\ j = 0,1,\cdots,N\},$$

the corresponding real state sequence is

$$S_N^* = \{s^*(t_j),\ j = 0,1,\cdots,N\}.$$

Then

$$J_N =$$
$$\int_0^{t_1} L(x(\tau),u(\tau))\mathrm{d}\tau + \cdots +$$
$$\int_{t_{N-1}}^{t_N} L(x(\tau),u(\tau))\mathrm{d}\tau + J(s(t_N),s^*(t_N),t_N) \leqslant$$
$$\int_0^{t_1} L(x(\tau),u(\tau))\mathrm{d}\tau + \cdots +$$
$$\int_{t_{N-1}}^{t_N} L(x(\tau),u(\tau))\mathrm{d}\tau + J(s(t_{N-1}),s^*(t_{N-1}),t_N) =$$
$$\int_0^{t_1} L(x(\tau),u(\tau))\mathrm{d}\tau + \cdots + \int_{t_{N-2}}^{t_{N-1}} L(x(\tau),u(\tau))\mathrm{d}\tau +$$
$$J(s(t_{N-1}),s^*(t_{N-1},t_{N-1}) \leqslant \cdots \leqslant J(s(0),s^*(0),0).$$

According to the selection of $S(t_{N+1})$ we have

$$J_N - J_{N+1} = J(s(t_N),s^*(t_N),t_{N+1}) - J(s(t_{N+1}),$$
$$s^*(t_{N+1}),t_{N+1}) \geqslant 0,$$

that is to say J_N is monotonically decreasing with N. Thus $\lim\limits_{N\to\infty} J_N$ exists and

$$J_{\text{whole}} = \lim_{N\to\infty} J_N \leqslant J(s(0),s^*(0),0).$$

2) is proved since $J(s(0),s^*(0),0)$ is the performance criterion of the system not using the switch strategy.

Due to that

$$\dot{x} = A(s^*(0))x + B(s^*(0))k(s(0))x$$

is stable, we have

$$J_{\text{whole}} \leqslant J(s(0),s^*(0),0) < +\infty.$$

Therefore $\|x(t)\| \to 0, t \to +\infty$ from both Q and R are positive definite. That is to say, $x = 0$ is globally asymptotic stable. 1) is proved.

5 Example

Consider the following system

$$\dot{x} = A_i x + B_i u,\ u = k_i x,\ i = 1,2,3,4,$$

with

$$\bar{S}_1 = \bar{S}_3 = \{s_1,s_3\},\ \bar{S}_2 = \bar{S}_4 = \{s_2,s_4\},$$

where

$$A_1 = \begin{bmatrix} -0.4093 & -0.3673 \\ -0.2560 & -1.3675 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} -1.9794 & 1.8039 \\ -0.1148 & 0.5141 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} -0.4213 & -0.2905 \\ -0.3525 & -1.1340 \end{bmatrix},$$
$$A_4 = \begin{bmatrix} -2.1236 & 1.8621 \\ -0.0845 & 0.6145 \end{bmatrix},$$
$$B_1 = B_3 = \begin{bmatrix} 2.4967 \\ 1.0316 \end{bmatrix},$$
$$B_2 = B_4 = \begin{bmatrix} 0.3580 \\ 0.5396 \end{bmatrix}.$$

The initial data is $x(0) = [1, -1]^{\mathrm{T}}$. We define the performance:

$$J_{\text{whole}} = \int_0^{\infty} (x^{\mathrm{T}}(\tau)Qx(\tau) + u^{\mathrm{T}}(\tau)Ru(\tau))\mathrm{d}\tau.$$

$Q = I_2, R = 0.001$. Under the effect of discrete state disturbance, using the strategy as above, we gained sub-controllers are

$$k_1 = [31.9053\ \ 4.9668],$$
$$k_2 = [16.6353\ \ 27.8952],$$
$$k_3 = [32.5626\ \ 3.4203],$$
$$k_4 = [16.0554\ \ 28.4114].$$

The state trajectory is shown in Fig. 3. Apparently the system is stable

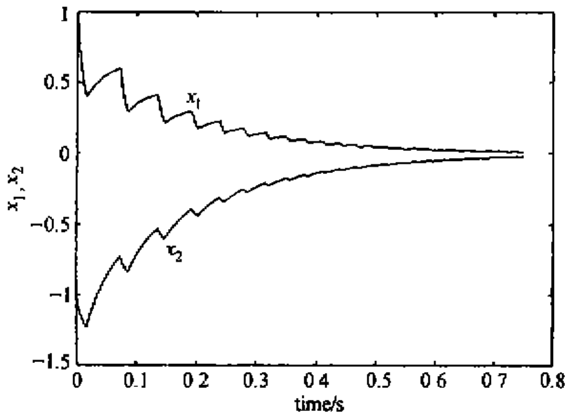

Fig. 3 System state

6 Conclusion

In this paper we analyze the robust stability for a class of hybrid system with discrete state disturbance. Aiming at the affecting of discrete state disturbance, we provide a switch strategy and a sub-controller design to guarantee the robust stability of the closed-loop system. As a result, the performance of the system is improved.

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