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# D-Type Iterative Learning Control with Application to FNS Limb Motion Control System

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**Abstract**: A new sufficient condition proof for the convergence of D-type iterative learning control algorithm is provided and the clinical experiments applied to the control of both elbow flexion and wrist flexion with function neuromuscular stimulation (FNS) is given by means of D-type iterative learning control method. The results of clinical studies have demonstrated that D-type iterative learning control algorithm is suitable for improving the dynamic response characteristics and stabilizing the limb motion. Furthermore, the stimulated patient does not have any bad physiological reactions because the output electrical stimulation pluses vary gently.

**Key words**: iterative learning control; convergence; limb motion control; FNS; biomedical engineering
**Document code**: A

# D 型迭代学习控制及其在 FNS 肢体运动控制系统中的应用

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**摘要**: 给出了离散系统 D-型迭代学习控制算法收敛的一种充分条件,并加以证明.采用 D 型迭代学习控制算法,对基于功能性神经肌肉电刺激的曲腕和曲肘运动进行了临床实验研究,结果表明,D 型迭代学习算法改善了 FNS 肢体运动控制的跟踪性能,曲肘和曲腕运动轨迹平滑、稳定,并且刺激控制脉冲变化平缓,受试者无任何不良生理反应.

**关键词**: 迭代学习控制; 收敛性; 肢体运动控制; FNS; 生物医学工程

## 1 Introduction

The iterative learning control method is receiving increasing attention as an alternative for controlling uncertain dynamic systems in a simple manner, which is proposed by Uchiyama[1] and elaborated as a more formal theory by Arimoto[2,3] and some other researchers[4~6]. The idea is based on the use of repeated trials of tracking a preassigned trajectory. During each trial, the current trajectory difference between the real trajectory and the reference trajectory are recorded and to be exploited by a suitable algorithm in the next trial, with the aim of progressively reducing the trajectory errors. It is a recursive online control technique that relies on less calculation and requires less a priori knowledge about the system dynamics because the algorithm is independent of the plant dynamics. To a great extent, the convergence of the algorithm applied is the key to obtain desired trajectory-tracking performance. Many researchers[6~9] are devoted to give more general convergence conditions for a variety of typical algorithms for a given system. One of the main objective is to relax or eliminate the restrictions of the convergence conditions for the iterative learning control algorithms.

In this paper, another sufficient condition for the convergence of D-type iterative learning control algorithm is discussed and then the convergence proof of the algorithm is given. In comparison with the conventional method, the clinic experiments for elbow flexion and wrist flexion motion with function neuromuscular stimulation (FNS) have been conducted by means of the proposed D-type iterative learning control algorithm. The proposed method is shown to be applicable for the limb motion control based on function neuromuscular stimulation.

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## 2 Problem statement

Consider a linear discrete system

$$\begin{cases} x^{(k)}(n+1) = A(n)x^{(k)}(n) + B(n)u^{(k)}(n), \\ y^{(k)}(n) = C(n)x^{(k)}(n). \end{cases} \tag{1}$$

The D-type iterative learning control law is defined as[2,7,8]

$$u^{(k+1)}(n) = u^{(k)}(n) + \Gamma(n)(e^{(k)}(n+1) - e^{(k)}(n)). \tag{2}$$

Let $y^d(n)(0 \leqslant n \leqslant N)$ be the desired output trajectory and $\epsilon^* > 0$ a tolerance bound. Under the condition that the matrices $A(n), B(n), C(n)$ are not fully known, we wish to find a suitable control function $u^{(k)}(n)(0 \leqslant n \leqslant N)$, such that the corresponding output trajectory $y^{(k)}(n)$ of the linear discrete system (1) satisfies

$$E(y^{(k)}(n)) = \| y^d(n) - y^{(k)}(n) \| \leqslant \epsilon^*, 0 \leqslant n \leqslant N.$$

**Nomenclature**:

$\| \cdot \|$ denotes the Euclidean norm:

$$n \in \{0,1,\cdots,N\};$$

$k$ is the iteration ordinal number;

$e^{(k)}(n) = y^d(n) - y^{(k)}(n) \in \mathbb{R}^q$ is the output error vector in the $k$-th iteration;

$y^{(k)}(n) \in \mathbb{R}^q$ is the output vector in the $k$-th iteration;

$y^d(n) \in \mathbb{R}^q$ is the desirable output vector;

$x^{(k)}(n) \in \mathbb{R}^p$ is the state vector in the $k$-th iteration;

$x^{(d)}(n) \in \mathbb{R}^p$ is the desirable state vector;

$u^{(k)}(n) \in \mathbb{R}^r$ is the control vector in the $k$-th iteration;

$u^d(n) \in \mathbb{R}^r$ is the desirable control vector;

$x^{(k)}(0) \in \mathbb{R}^p$ is the initial state vector in the $k$-th iteration;

$y^{(k)}(0) \in \mathbb{R}^q$ is the initial output vector in the $k$-th iteration;

$y^{(d)}(0) \in \mathbb{R}^q$ is the desirable initial output vector;

$\Gamma(n) \in \mathbb{R}^{r \times q}$ is the iterative learning gain matrices;

$A(n) \in \mathbb{R}^{p \times p}$ is the system matrices;

$B(n) \in \mathbb{R}^{p \times r}$ is the control matrices;

$C(n) \in \mathbb{R}^{q \times p}$ is the output matrices;

$p$ is the order of the system;

$r$ is the dimension of control vector;

$q$ is the dimension of output vector.

**Theorem** Suppose that the discrete system (1) satisfies the following two conditions:

i) $\| I - B(n)\Gamma(n)C(n+1) \| \leqslant \rho \leqslant 1, 0 \leqslant n \leqslant N$;

ii) $x^{(k)}(0) = x^0, y^{(k)}(0) = y^d(0), k = 0,1,2,\cdots$.

Then, for a given desired output trajectory $y^d(n)(0 \leqslant n \leqslant N)$, the iterative control law of equation (2) guarantees that, for each $n \in \{0,1,\cdots,N\}$,

$$y^{(k)}(n) \to y^d(n) \text{ as } k \to \infty.$$

Proof Let $x^{(k)}(0)$ and $u^{(k)}(n)(0 \leqslant n \leqslant N)$ be the system initial state and control input respectively, for a given system in equation (1), whose solution can be described by

$$x^{(k)}(n) = \Phi(n,0)x^{(k)}(0) + \sum_{s=0}^{n-1} \Phi(n,s+1)B(s)u^{(k)}(s), \tag{3}$$

$$0 \leqslant n \leqslant N+1,$$

where $\Phi(n,s)$ is the state transition matrices satisfying

$$\begin{cases} \Phi(n,s) = A(n-1)A(n-2)\cdots A(s), n > s, \\ \Phi(s,s) = I. \end{cases}$$

Applying equation (3) gives

$$x^{(k+1)}(n+1) = \Phi(n+1,0)x^{(k+1)}(0) + \sum_{s=0}^{n} \Phi(n+1,s+1)B(s)u^{(k+1)}(s),$$

$$0 \leqslant n \leqslant N.$$

Denoting $\delta x^{(k)}(n) = x^d(n) - x^{(k)}(n)$ and then combining equations (1) and (2) lead to

$$u^{(k+1)}(n) = u^{(k)}(n) + \Gamma(n)(e^{(k)}(n+1) - e^{(k)}(n)) = u^{(k)}(n) + \Gamma(n)C(n+1)\delta x^{(k)}(n+1) - \Gamma(n)C(n)\delta x^{(k)}(n).$$

Therefore

$$x^{(k+1)}(n+1) = \Phi(n+1,0)x^{(k+1)}(0) + \sum_{s=0}^{n} \Phi(n+1,s+1)B(s)u^{(k)}(s) + \sum_{s=0}^{n} \Phi(n+1,s+1)B(s)\Gamma(s)C(s+1)\delta x^{(k)}(s+1) - \sum_{s=0}^{n} \Phi(n+1,s+1)B(s)\Gamma(s)C(s)\delta x^{(k)}(s),$$

$$0 \leqslant n \leqslant N.$$

Then, the state error vector in $(k+1)$ iteration can be obtained,

$$\delta x^{(k+1)}(n+1) = x^d(n+1) - x^{(k+1)}(n+1) = x^d(n+1) - x^{(k)}(n+1) - \sum_{s=0}^{n} \Phi(n+1,s+1)B(s)\Gamma(s)C(s+1)\delta x^{(k)}(s+1) + \sum_{s=0}^{n} \Phi(n+1,s+1)B(s)\Gamma(s)C(s)\delta x^{(k)}(s) = [I - B(n)\Gamma(n)C(n+1)]\delta x^{(k)}(n+1) - \sum_{s=0}^{n} \Phi(n+1,s+1)B(s)\Gamma(s)C(s+1)\delta x^{(k)}(s+1) + \sum_{s=0}^{n} \Phi(n+1,s+1)B(s)\Gamma(s)C(s)\delta x^{(k)}(s).$$

Taking norms of both sides results in

$$\delta x^{(k+1)}(n+1) \leq \| I-B(n)\Gamma(n)C(n+1) \| \cdot \| \delta x^{(k)}(n+1) \| + \sum_{s=0}^{n-1} \|\Phi(n+1,s+1)B(s)\Gamma(s)C(s+1)\| \cdot \| \delta x^{(k)}(s+1)\| + \sum_{s=0}^{n} \| \Phi(n+1,s+1)B(s)\Gamma(s)C(s) \| \cdot \| \delta x^{(k)}(s) \| \leq \rho \| \delta x^{(k)}(n+1) \| + 2k_1 \sum_{s=0}^{n-1} \| \delta x^{(k)}(s+1) \|,$$

where

$$k_1 = \sup_{0 \leq n \leq N, 0 \leq s \leq n} \| \Phi(n+1,s+1)B(s)\Gamma(s)C(s+1) \|.$$

Noting that in the above derivation the initial condition $y^{(k)}(0) = y^d(0)(k = 0,1,2,\cdots)$ has been employed. Then, multiplying by expoential $\lambda^{n+1}(0 < \lambda < 1)$ yields

$$\lambda^{n+1} \| \delta x^{(k+1)}(n+1) \| \leq \rho\lambda^{n+1} \| \delta x^{(k)}(n+1) \| + 2k_1 \sum_{s=0}^{n-1} \lambda^{n-s}\lambda^{s+1} \| \delta x^{(k)}(s+1) \| \leq \rho\lambda^{n+1} \| \delta x^{(k)}(n+1) \| + 2k_1 \sum_{s=0}^{n-1} \lambda^{n-s} \sup_{0 \leq s \leq N} \{\lambda^{s+1} \| \delta x^{(k)}(s+1) \| \} \leq \rho\lambda^{n+1} \| \delta x^{(k)}(n+1) \| + 2k_1 \frac{\lambda(1-\lambda^N)}{1-\lambda} \sup_{0 \leq s \leq N} \{\lambda^{s+1} \| \delta x^{(k)}(n+1) \| \}, \quad 0 \leq n \leq N.$$

The definition of $\| \cdot \|_\lambda$ indicates that

$$\sup_{0 \leq n \leq N} \{\lambda^{n+1} \| \delta x^{(k+1)}(n+1) \| \} \leq \tilde{\rho} \sup_{0 \leq n \leq N} \{\lambda^{n+1} \| \delta x^{(k)}(n+1) \| \}, \quad 0 \leq n \leq N,$$

where $\tilde{\rho} = \rho + 2k_1 \frac{\lambda(1-\lambda^N)}{1-\lambda}$. Because of $\rho < 1$, we can choose a sufficient small $\lambda$ in order to make $\tilde{\rho} < 1$, therefore

$$\lim_{k \to \infty} \sup_{1 \leq n \leq N+1} \{\lambda^n \| \delta x^{(k)}(n) \| \} = 0,$$

Thus

$$\| \delta x^{(k)} \|_\lambda \to 0, \ (k \to \infty).$$

We also know

$$\| e^{(k)}(n) \| \leq \| C(n) \| \cdot \| \delta x^{(k)}(n) \|.$$

Multiplying by exponential $\lambda^n (0 \leq \lambda \leq 1)$,

$$\lambda^n \| e^{(k)}(n) \| \leq \| C(n) \| \cdot \lambda^n \| \delta x^{(k)}(n) \| \leq k_2\lambda^n \| \delta x^{(k)}(n) \|, \quad 1 \leq n \leq N+1,$$

where $k_2 = \sup_{1 \leq n \leq N+1} \| C(n) \|$. By the definition of $\| \cdot \|_\lambda$, we have

$$\sup_{1 \leq n \leq N+1} \{\lambda^n \| e^{(k)}(n) \| \} \leq k_2 \sup_{1 \leq n \leq N+1} \{\lambda^n \| \delta x^{(k)}(n+1) \| \}.$$

Thus, $\lim_{k \to \infty} \sup_{1 \leq n \leq N+1} \{\lambda^n \| e^{(k)}(n) \| \} = 0$. Furthermore, the proof of the theorem can be finished by employing condition ii).

## 3 FNS limb motion control system

### 3.1 FNS system

A typical FNS system consists of several units including reference inputs, computer, stimulating map, multichannel stimulators, safeguard devices, angle sensors, signal transformer and so on, as shown in Fig. 1 and Fig. 2.

Fig. 1 FNS test instrumentation

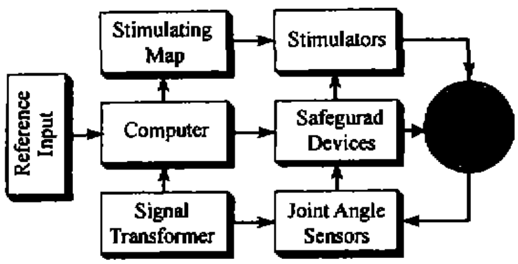



Fig. 2 Simplified block diagram of FNS system

### 3.2 Mechanism

According to the specified motion control patterns or desirable motion control strategies, the electrical pulse signals (amplitude, frequency and width are adjusted) are generated by means of computer associated with a special control algorithm. Then, the pulses are mapped, amplified and transformed into required stimulating sequences which are directly put on the multi-joint-controllable-muscles (e.g. biceps brachii, long palmar muscle etc.) via the stimulators. Consequently, a desired posture or movement of human musculoskeletal system is generated because the movement nerves are stimulated. During the movement nerves are stimulated, the advanced nerve center are also actuated through the spinal cord by the afferent neural. It is helpful to cerebral cortex to be excited after movement pattern signals are applied repeatedly. The basic functions of paralytic limbs can be improved permanently, and finally the movement functions of human body can be controlled or partly recovered.

In closed-loop FNS system, the dual-joint movement state can be measured by means of two angle sensors mounted on the motion axis centers of the elbow joint and the wrist joint. Those state signals are sent to FNS computer after having been preprocessed through signal transformer.

### 3.3 Measurement scheme of dual-joint motion angle

The task of FNS system was to specify a set of muscle stimulation parameters that can generate a desired movement of human skeletal system. In this paper, human musculoskeletal system (elbow joint and wrist joint) are controlled through both biceps brachii and long palmar muscle. The performance of FNS system based on one control algorithm is primarily evaluated by means of tracking performance of dual-joint motion angle. The measurement scheme of both the elbow joint angle and the wrist joint angle is shown in Fig.3.

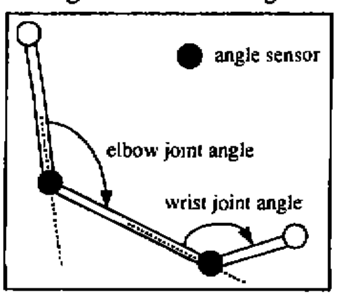
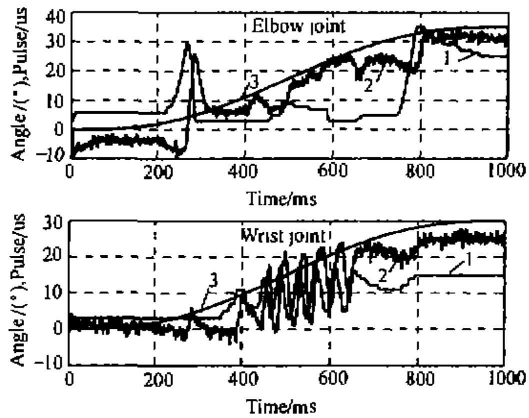



Fig. 3 Measurement scheme of two joint motion angles

## 4 Experiment results

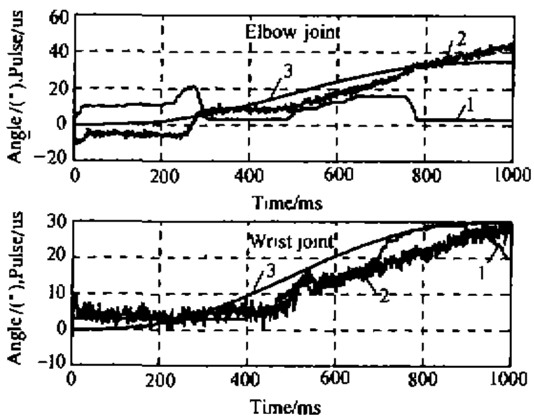
To show the validity of the proposed D-type iterative learning control algorithm, two clinical experiments applied for elbow and wrist flexion motion control with FNS are illustrated by two different control algorithms. One with the conventional control algorithm[10] and the other with the D-type interative learning control algorithm stated above.

As shown in Fig.4, it can be seen that the trajectory-following curves of the elbow and the wrist flexion motion with conventional control algorithm are characterized by sharp oscillation. Furthermore, the stimulating signal fluctuates severely. The clinical experiment shows that the patient does not feel very well and has obvious pains in the muscle when such stimulating signal obtained from conventional algrothm is stimulated to the muscle.




1—Stimulating signals
2—Real angular of elbow and wrist flexion motion
3—Desired angular of elbow and wrist flexion motion

Fig. 4 The trajectory-following curves of elbow and wrist flexion with conventional control




1—Stimulating signals
2—Real angular of elbow and wrist flexion motion
3—Desired angular of elbow and wrist flexion motion

Fig. 5 The trajectory-following curves of elbow and wrist flexion with D-type iterative learning control

The trajectory-following curves of elbow and wrist flexion with D-type iterative learning control are available in Fig. 5. Note that the proper choice of iterative learning parameter $\Gamma$ is very important for the convergence of D-type algorithm. There are a little difference of the parameter $\Gamma$ for the elbow flexion motion and the wrist flexion motion. $\Gamma$ is about 0.8 ~ 1 for desirable elbow flexion control and around 0.4 ~ 0.7 for desirable wrist flexion control. Obviously, Fig. 5 shows that the trajectory-following curves of both the elbow and the wrist flexion are very close to the desired output trajectories. Also, the stimulating signal varies smoothly. In particular, the clinical experiment shows that the patient does not have any pains while the stimulating signal generated by D-type iterative learning control algorihtm is acted on human body.

## 5 Conclusion

1) A new sufficient condition for the convergence of the D-type iterative learning control algorihtm has been given. Also the clinical experiments show that D-type iterative learning algorithm could be applied to the stable control of both the elbow flexion and the wrist flexion, which is better than any other conventional methods.

2) Further research should be directed towards the relaxation restrictions or the elimination of the assumption on the initial conditions and generality of the iterative learning algorithm for limb motion control with function neuromuscular stimulation. For the time being, the adaptive iterative learning algorithm world be a significant development.

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