第 18 卷第 3 期
2001 年 6 月

控制理论与应用
CONTROL THEORY AND APPLICATIONS

Vol.18, No.3
Jun., 2001

Article ID: 1000-8152(2001)03-0433-05

Performance Analysis of Least Mean Square Algorithm for Time-Varying Systems *

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Abstract: By means of stochastic process theory, the bounded convergence of least mean square algorithm (LMS) is studied without data stationary assumption and ergodicity condition. The upper bound of the estimation error is given, and the way of choosing the convergence factor or stepsize is stated so that the upper bound of the parameter estimation error is minimized. The convergence analyses indicate that i) for deterministic time invariant systems, LMS algorithm is convergent exponentially, ii) for deterministic time-varying systems, the estimation error upper bound is minimal as the stepsize goes to unity, and iii) for time-varying or time invariant stochastic systems, the estimation error is uniformly bounded.

Key words: time-varying system; identification; parameter estimation; least mean square algorithm
Document code: A

时变系统最小均方算法的性能分析

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摘要: 在无过程数据平稳性假设和各态遍历等条件下,运用随机过程理论研究了最小均方算法(LMS)的有界收敛性,给出了估计误差的上界,论述了 LMS 算法收敛因子或步长的选择方法,以使参数估计误差上界最小.这对于提高 LMS 算法的实际应用效果有着重要意义。LMS 算法的收敛性分析表明: i) 对于确定性时不变系统,LMS 算法是指数速度收敛的; ii) 对于确定性时变系统,收敛因子等于 1,LMS 算法的参数估计误差上界最小; iii) 对于时变或不变随机系统,LMS 算法的参数估计误差一致有上界.

关键词: 时变系统; 辨识; 参数估计; LMS 算法

1 Introduction

LMS algorithm is very important in the area of adaptive signal processing and identification, and its convergence has been paid more attentions. Many papers and publications studied the general convergence of the parameter estimation error (PEE) given by the LMS algorithm only from pure mathematics theory, and the convergence conditions are very strong, for example the data stationary, ergodicity condition, M-mixed conditions and so on, and any of them can not hold for any physical system. Ref. [1 ~ 4] show that the mean squares PEE, Error, satisfies

$$\lim_{t\to\infty} \text{Error} = O\left(\mu\sigma_v^2 + \frac{\sigma_w^2}{\mu}\right), \tag{1}$$

that is, there exists constant $A < \infty$ such that

$$\lim_{t\to\infty} \text{Error} \leqslant A\left(\mu\sigma_v^2 + \frac{\sigma_w^2}{\mu}\right), \tag{2}$$

where $0 < \mu < 2$ is a convergence factor, σ_v^2 and σ_w^2 are the variances of the observation noise and parameter change rate.

Since A is unknown, the general convergence has little significance in engineering. Therefore, Ding Feng[5] has presented the bounded convergence, and the bounded covergence emphasizes the studies and estimation of the PEE upper bound so that the PEE upper bound is reduced.

The convergence analysis of identification algorithms is one of the most difficult projects in the area of control[1-6]. In this paper the bounded convergence of the LMS algorithm is studied using stochastic process theory.

* Foundation item: supported by the National Natural Science Foundation of China (60074029, 69934010) and the Foundation of Information school, Tsinghua University.
Received date: 2000-01-20; Revised date: 2000-08-28.

The analysis indicates that like forgetting factor least squares (FFLS), the LMS algorithm can track time-varying parameters, but it has less computation. For the FFLS algorithm, as the forgetting factor approaches unity, the bounds of the covariance matrix and the parameter estimation error grow without limits even for time invariant systems whose parameters are constant.

2 LMS algorithm and basic lemmas

Consider the following linear time-varying system

$$A(t,z)y(t) = B(t,z)u(t) + v(t), \quad (3)$$

where $\{u(t)\}$ and $\{y(t)\}$ are the input and output sequences of the system, respectively, $\{v(t)\}$ is a stochastic noise sequence with zero mean, and z^{-1} is the unit backward shift operator, i.e. $z^{-1}y(t) = y(t-1)$, $z^{-1}u(t) = u(t-1)$, $A(t,z)$ and $B(t,z)$ are time-varying coefficient polynomials in the unit backward shift operator z^{-1}, and

$$A(t,z) = 1 + a_1(t)z^{-1} + a_2(t)z^{-2} + \cdots + a_{n_a}(t)z^{-n_a},$$

$$B(t,z) = b_1(t)z^{-1} + b_2(t)z^{-1} + \cdots + b_{n_b}(t)z^{-n_b}.$$

Define the information vector $\varphi(t)$ and the time-varying parameter vector $\theta(t)$ as

$$\begin{cases} \theta(t-1) = [a_1(t), a_2(t), \cdots, a_{n_a}(t), b_1(t), \\ \qquad b_2(t), \cdots, b_{n_b}(t)]^T \in \mathbb{R}^n, n = n_a + n_b, \\ \varphi(t) = [-y(t-1), -y(t-2), \cdots, -y(t-n_a), \\ \qquad u(t-1), u(t-2), \cdots, u(t-n_b)]^T \in \mathbb{R}^n, \end{cases}$$

where the superscript T denotes a matrix transpose.

Then equation (3) can be written in vector form as

$$y(t) = \varphi^T(t)\theta(t-1) + v(t), \quad (4)$$

where $\theta(t) \in \mathbb{R}^n$ is the time-varying parameter vector of the system to be identified, $\varphi(t) \in \mathbb{R}^n$ is the regressive information vector consisting of the observations up to time $(t-1)$.

The LMS algorithm of estimating the time-varying parameters $\theta(t)$ is expressed as

$$\hat\theta(t) = \hat\theta(t-1) + \mu_t\varphi(t)[y(t) - \varphi^T(t)\hat\theta(t-1)], \quad (5)$$

where $\hat\theta(t)$ is the estimate of $\theta(t)$ at time t, μ_t is a convergence factor or stepsize. The analysis indicates that as long as the convergence factor satisfies $0 < \mu_t\|\varphi(t)\|^2 < 2$, the LMS algorithm is convergent. For convenience, the LMS algorithm is generally modified as

$$\hat\theta(t) = \hat\theta(t-1) + \mu_t\frac{\varphi(t)}{1+\|\varphi(t)\|^2}[y(t) - \varphi^T(t)\hat\theta(t-1)], \ 0 < \mu_t < 2. \quad (6)$$

Lemma 1 For system (4) and algorithm (6), define the transition matrix

$$\begin{cases} L(t+1,i) = [I - \mu_t\dfrac{\varphi(t)\varphi^T(t)}{1+\|\varphi(t)\|^2}]L(t,i), \\ L(i,i) = I, \ \mu_t \in (0,1], \end{cases} \quad (7)$$

μ_t is non-increasing, and assume that the following strong persistent excitation condition holds[5]:

(A1)

$$\alpha I \leqslant \frac{1}{N}\sum_{i=0}^{N-1}\varphi(t+i)\varphi^T(t+i)\varphi^T(t+i) \leqslant \beta I, \text{ a.s.}, \ t > 0,$$

$$0 < \alpha \leqslant \beta < \infty, \text{ for some } N \geqslant n.$$

Then

$$\rho_t \triangleq \lambda_{\max}[L^T(t+N,t)L(t+N,t)] \leqslant 1 - \frac{N\alpha\mu_{t+N-1}}{2(1+M)(N^2+1)}, \text{a.s.}, M \triangleq nN\beta,$$

where $\lambda_{\max}(X)$ represents the maximum eigenvalue of the matrix X.

Proof Let v_0 be the unit eigenvector corresponding to the maximum eigenvalue, ρ_t, of the matrix $L^T(t+N,t)L(t+N,t)$, and constructing difference equation[6]

$$x_{i+1} = [I - \mu_i\frac{\varphi(i)\varphi^T(i)}{1+\|\varphi(i)\|^2}]x_i = L(i+1,i)x_i, \ x_t = v_0, \quad (8)$$

using the property $L(t,i)L(i,s) = L(t,s)$, we have

$$x_{t+N} = L(t+N,t)x_t = L(t+N,t)v_0,$$

$$\|x_{t+N}\|^2 = v_0^T L^T(t+N,t)L(t+N,t)v_0 = \rho_t,$$

$$\begin{aligned} x_{i+1}^T x_{i+1} &= x_i^T[I - \mu_i\frac{\varphi(i)\varphi^T(i)}{1+\|\varphi(i)\|^2}]^2 x_i = \\ & x_i^T[I - 2\mu_i\frac{\varphi(i)\varphi^T(i)}{1+\|\varphi(i)\|^2}]^2 + \\ & \mu_i^2\frac{\|\varphi(i)\|^2\varphi(i)\varphi^T(i)}{(1+\|\varphi(i)\|^2)^2}]x_t \leqslant \\ & x_i^T[I - \frac{\mu_i(2-\mu_i)\varphi(i)\varphi^T(i)}{(1+\|\varphi(i)\|^2}]x_i = \\ & x_i^T x_i - \mu_i(2-\mu_i)\frac{\|\varphi^T(i)x_i\|^2}{1+\|\varphi(i)\|^2}. \end{aligned}$$

For any $\mu_t \in (0,2)$, equation (8) is stable. For simplifying proof, let $0 < \mu_t \leqslant 1$, we have

$$\mu_i(2-\mu_i)\frac{\|\varphi^T(i)x_i\|^2}{1+\|\varphi(i)\|^2} \leqslant \|x_i\|^2 - \|x_{i+1}\|^2,$$

$$\sum_{i=0}^{N-1}\mu_{t+i}\frac{\|\varphi^{T}(t+i)x_{t+i}\|^2}{1+\|\varphi(t+i)\|^2}\leqslant$$

$$\sum_{i=0}^{N-1}\mu_{t+i}(2-\mu_{t+i})\frac{\|\varphi^{T}(t+i)x_{t+i}\|^2}{1+\|\varphi(t+i)\|^2}\leqslant$$

$$\|x_t\|^2-\|x_{t+N}\|^2=1-\rho_t. \tag{9}$$

For any $i\in[0,N-1]$, using the formula

$$(\sum a_ib_i)^2\leqslant(\sum a_i^2)(\sum b_i^2),$$

from (8) and (9) we have

$$\|x_{t+t}-v_0\| =$$

$$\|\sum_{j=0}^{i-1}\mu_{t+j}\frac{\varphi(t+j)\varphi^{T}(t+j)}{1+\|\varphi(t+j)\|^2}x_{t+j}\|\leqslant$$

$$\sum_{j=0}^{i-1}\mu_{t+j}\frac{\|\varphi(t+j)\|\ \|\varphi^{T}(t+j)x_{t+j}\|}{1+\|\varphi(t+j)\|^2}\leqslant$$

$$[i\sum_{j=0}^{i-1}\mu_{t+j}^2\frac{\|\varphi^{T}(t+j)x_{t+j}\|^2}{1+\|\varphi(t+j)\|^2}]^{1/2}\leqslant$$

$$[i\sum_{j=0}^{i-1}\mu_{t+j}\frac{\|\varphi^{T}(t+j)x_{t+j}\|^2}{1+\|\varphi(t+j)\|^2}]^{1/2}\leqslant$$

$$\sqrt{i(1-\rho_t)}\leqslant\sqrt{N(1-\rho_t)}. \tag{10}$$

Taking the trace of condition (A1) gives

$$\|\varphi(t)\|^2\leqslant M=nN\beta,\text{a.s.},t>0. \tag{11}$$

Since μ_t is non-incereasing, pre-multiplying and post-multiplying both sides of condition (A1) by v_0 and using (9) and (10) give

$$N_a\leqslant v_0^{T}\sum_{i=0}^{N-1}\varphi(t+i)\varphi^{T}(t+i)v_0\leqslant$$

$$\frac{1+M}{\mu_{t+N-1}}v_0^{T}\sum_{i=0}^{N-1}\mu_{t+i}\frac{\varphi(t+i)\varphi^{T}(t+i)}{1+\|\varphi(t+i)\|^2}v_0\leqslant$$

$$\frac{1+M}{\mu_{t+N-1}}\sum_{i=0}^{N-1}\mu_{t+i}\frac{\|\varphi^{T}(t+i)(v_0-x_{t+i}+x_{t+i})\|^2}{1+\|\varphi(t+i)\|^2}\leqslant$$

$$\frac{2(1+M)}{\mu_{t+N-1}}[\sum_{i=0}^{N-1}\mu_{t+i}\frac{\|\varphi^{T}(t+i)(x_{t+i}-v_0)\|^2}{1+\|\varphi(t+i)\|^2}+$$

$$\sum_{i=0}^{N-1}\mu_{t+i}\frac{\|\varphi^{T}(t+i)x_{t+i}\|^2}{1+\|\varphi(t+i)\|^2}]\leqslant$$

$$\frac{2(1+M)}{\mu_{t+N-1}}[\sum_{i=0}^{N-1}\|x_{t+i}-v_0\|^2+(1-\rho_t)]\leqslant$$

$$\frac{2(1+M)}{\mu_{t+N-1}}[NN(1-\rho_t)+(1-\rho_t)]=$$

$$\frac{2(1+M)}{\mu_{t+N-1}}(N^2+1)(1-\rho_t),\ \text{a.s..}$$

The conclusion of Lemma 1 is reached.

If condition (A1) becomes[1]

$$\text{(A2)}\ \frac{1}{N}\sum_{i=0}^{N-1}\frac{\varphi(t+i)\varphi^{T}(t+i)}{1+\|\varphi t+i\|^2}\geqslant$$

$$\alpha I>0,\ \text{a.s.},\ t>0,\ \text{for some } N\geqslant n.$$

Then

$$\rho_t\leqslant 1-\frac{N_a\mu_{t+N-1}}{2(N^2+1)},\ \text{a.s..} \tag{12}$$

As $\mu_t=\mu\in(0,1)$, ρ_t does not depend on t and may be denoted as ρ, i.e.

$$\rho\triangleq\rho_t\leqslant 1-\frac{N\alpha\mu}{2(N^2+1)},\ \text{a.s..} \tag{13}$$

Lemma 2 Let non-positive sequences $\{x(t)\}$, $\{a_t\}$, $\{b_t\}$ satisfy the following relation:

$$x(t+1)\leqslant(1-a_t)x(t)+b_t,t\geqslant 0$$

and $a_t\in[0,1)$, $\sum_{t=1}^{\infty}a_t=\infty$, $x(0)<\infty$, then

$$\lim_{t\to\infty}x(t)\leqslant\lim_{t\to\infty}\frac{b_t}{a_t},$$

where it is assumed that the limit exists.

Proof See Ref.[1].

3 Convergence theorems of the LMS algorithm

Theorem 1 For the time-varying system (4) and algorithm (6), assume that the observation noise $\{v(t)\}$ and the parameter changing rate $\{w(t)=\theta(t)-\theta(t-1)\}$ are indepedent stochastic noises with zero mean and are uncorrelated with the input $\{u(t)\}$, i.e.

(A3) $E[v(t)]=0,\ E[w(t)]=0,$
$E[v(t)w(i)]=0,$

(A4) $E[v(t)v(i)]=0,$
$E[w(t)w^{T}(i)]=0, i\neq t,$

(A5) $E[v^2(t)]=\sigma_v^2(t)\leqslant\sigma_v^2<\infty,$
$E[\|w(t)\|^2]=\sigma_w^2(t)\leqslant\sigma_w^2<\infty.$

Condition (A2) holds, if the non-increasing convergence factor satisfies $\mu_t\in[\mu_0,1]$, $\mu_0>0$, then the parameter estimation error, $\hat{\theta}(t)-\theta(t)$, given by the LMS algorithm (6) satisfies

$$\lim_{t\to\infty}E[\|\hat{\theta}(t)-\theta(t)\|^2]\leqslant$$

$$2N^2\sum_{j=1}^{\infty}\Phi(i+1,j+1)[\mu^2_{N(j-1)+k+1}\sigma_v^2+\sigma_w^2]<\infty,$$

where

$$\Phi(i+1,j)=\rho_{N(i-1)+k+1}\Phi(i,j),$$

$$\Phi(i,i)=I,\ 0\leqslant k\leqslant N-1.$$

if $\mu_t=\mu\in(0,1]$, then

$$\lim_{t\to\infty}E[\|\hat{\theta}(t)-\theta(t)\|^2\leqslant k_1(\mu\sigma_v^2+\frac{\sigma_w^2}{\mu})\ \underline{\triangle}\ f(\mu),$$

where

$$k_1 = \frac{4N(N^2+1)}{\alpha}.$$

Proof Define the parameter estimation error vector

$$\tilde{\theta}(t) = \hat{\theta}(t) - \theta(t), \tag{14}$$

and assume that the $\tilde{\theta}(0)$ and $|v(t)|$ are independent, $\mathrm{E}[\|\tilde{\theta}(0)\|^2] < \infty$, using (4) and (6), we have

$$\begin{aligned}
&\tilde{\theta}(t) = \hat{\theta}(t) - [\theta(t-1) + w(t)] = \\
&\tilde{\theta}(t-1) + \mu_t \frac{\varphi(t)}{1+\|\varphi(t)\|^2}[-\varphi^{\mathrm{T}}(t)\tilde{\theta}(t-1) + v(t)] - w(t) = \\
&[I - \mu_t \frac{\varphi(t)\varphi^{\mathrm{T}}(t)}{1+\|\varphi(t)\|^2}]\tilde{\theta}(t-1) + \frac{\mu_t\varphi(t)}{1+\|\varphi(t)\|^2}v(t) - w(t) = \\
&L(t+1,t)\tilde{\theta}(t-1) + \frac{\mu_t\varphi(t)}{1+\|\varphi(t)\|^2}v(t) - w(t) = \\
&L(t+1,t-N+1)\tilde{\theta}(t-N) + \\
&\sum_{i=0}^{N-1} L(t+1,t-i+1)[\frac{\mu_{t-i}\varphi(t-i)}{1+\|\varphi(t-i)\|^2}v(t-i) - w(t-i)].
\end{aligned} \tag{15}$$

Taking the norm $\|*\|^2$ of both sides of (15) gives

$$\begin{aligned}
&\|\tilde{\theta}(t)\|^2 = \\
&\tilde{\theta}^{\mathrm{T}}(t-N)L^{\mathrm{T}}(t+1,t-N+1)L(t+1,t-N+1)\cdot \\
&\tilde{\theta}(t-N) + 2\tilde{\theta}^{\mathrm{T}}(t-N)L^{\mathrm{T}}(t+1,t-N+1)\cdot \\
&\sum_{i=0}^{N-1} L(t+1,t-i+1)[\frac{\mu_{t-i}\varphi(t-i)}{1+\|\varphi(t-i)\|^2}v(t-i) - \\
&w(t-i)] + \|\sum_{i=0}^{N-1} L(t+1,t-i+1)\cdot \\
&[\frac{\mu_{t-i}\varphi(t-i)}{1+\|\varphi(t-i)\|^2}v(t-i) - w(t-i)]\|^2 \leqslant \\
&\tilde{\theta}^{\mathrm{T}}(t-N)L^{\mathrm{T}}(t+1,t-N+1)L(t+1,t-N+1)\cdot \\
&\tilde{\theta}(t-N) + 2\tilde{\theta}^{\mathrm{T}}(t-N)L^{\mathrm{T}}(t+1,t-N+1)\cdot \\
&\sum_{i=0}^{N-1} L(t+1,t-i+1)[\frac{\mu_{t-i}\varphi(t-i)}{1+\|\varphi(t-i)\|^2}v(t-i) - \\
&w(t-i)] + N\sum_{i=0}^{N-1}\|L(t+1,t-i+1)\cdot \\
&[\frac{\mu_{t-i}\varphi(t-i)}{1+\|\varphi(t-i)\|^2}v(t-i) - w(t-i)]\|^2.
\end{aligned} \tag{16}$$

For any $i \geqslant 1$, the maximum eigenvalue of the matrix $L^{\mathrm{T}}(t+1,t-i+1)L(t+1,t-i+1)$ is equal to or less than unity. Let $T(t) = E[\|\tilde{\theta}(t)\|^2]$, taking the expectation of both sides of (16) and using Lemma 1 and conditions (A3) ~ (A5) give

$$\begin{aligned}
&T(t) \leqslant \\
&\rho_{t-N+1}T(t-N) + 0 + \\
&N\sum_{i=0}^{N-1}\mathrm{E}[\|\frac{\mu_{t-i}\varphi(t-i)}{1+\|\varphi(t-i)\|^2}v(t-i) - w(t-i)\|]^2 \leqslant \\
&\rho_{t-N+1}T(t-N) + 2N\sum_{i=0}^{N-1}[\mu_{t-i}^2\sigma_v^2(t-i) + \sigma_w^2(t-i)] \leqslant \\
&\rho_{t-N+1}T(t-N) + 2N\sum_{i=0}^{N-1}[\mu_{t-i}^2\sigma_v^2 + \sigma_w^2],
\end{aligned} \tag{17}$$

Let $t = N_i + k, 0 \leqslant k \leqslant N-1$, we have

$$\begin{aligned}
&T(t = N_i + k) \leqslant \\
&\rho_{N(i-1)+k+1}T(N(i-1)+k) + \\
&2N^2[\mu_{N(i-1)+k+1}^2\sigma_v^2 + \sigma_w^2] = \\
&\Phi(i+1,1)T(k) + \\
&2N^2\sum_{j=1}^{i}\Phi(i+1,j+1)[\mu_{N(i-1)+k+1}^2\sigma_v^2 + \sigma_w^2],
\end{aligned} \tag{18}$$

since $\mu_t \in [\mu_0, 1], \mu_0 > 0$, using (12), we have

$$\begin{aligned}
&\lim_{t\to\infty}\mathrm{E}[\|\tilde{\theta}(t)\|^2] = \\
&\lim_{t\to\infty}T(t = N_i + k) = \lim_{i\to\infty}T(t = N_i + k) = \\
&2N^2\sum_{j=1}^{i}\Phi(i+1,j+1)[\mu_{N(j-1)+k+1}^2\sigma_v^2 + \sigma_w^2].
\end{aligned} \tag{19}$$

As $\mu_t = \mu \in (0,1]$, using (13), we have

$$\begin{aligned}
&\lim_{t\to\infty}\mathrm{E}[\|\tilde{\theta}(t)\|^2] = \frac{2N^2}{1-\rho}[\mu^2\sigma_v^2 + \sigma_w^2] \leqslant \\
&\frac{2N^2 2(N^2+1)}{N\alpha\mu}[\mu^2\sigma_v^2 + \sigma_w^2].
\end{aligned} \tag{20}$$

This proves the assertion of Theorem 1.

Corollary 1 For time-varying stochastic systems

$$y(t) = \varphi^{\mathrm{T}}(t)\theta(t-1) + v(t).$$

Let $f'(\mu) = 0$ in Theorem 1 give the best convergence factor $\mu = \min(\frac{\sigma_w}{\sigma_v}, 1)$, the minimum estimation error upper bound is

$$\min f(\mu) = \min(2k_1\sigma_v\sigma_w, k_1\sigma_v^2 + k_1\sigma_w^2).$$

Theorem 2 For time-invariant systems

$$y(t) = \varphi^{\mathrm{T}}(t)\theta + v(t),$$

under the assumptions of Theorem 1, condition (A2) holds, if the convergence factor satisfies

$$\mu_t = \frac{1}{t^c}, \ 0 < c \leqslant 1.$$

Then the mean square PEE, $E[\|\hat{\theta}(t) - \theta\|^2]$, given by the LMS algorithm (6) converges to zero at the rate of $O(\frac{1}{t^c})$.

Proof Define the parameter estimation error vector

$$\tilde{\theta}(t) = \hat{\theta}(t) - \theta.$$

A similar derivation of Theorem 1 will give

$$T(t) \leqslant \rho_{t-N+1} T(t-N) + N\sum_{i=0}^{N-1} \mathrm{E}\left[\left\| \frac{\mu_{t-i}\varphi(t-i)}{1+\|\varphi(t)\|^2} v(t-i) \right\|^2\right] \leqslant \rho_{t-N+1} T(t-N) + N\sum_{i=0}^{N-1} [\mu_{t-i}^2 \sigma_v^2]. \tag{21}$$

Using (12), it gives

$$T(t) \leqslant \rho_{t-N+1} T(t-N) + N^2 \mu_{t-N+1}^2 \sigma_v^2 \leqslant \left(1 - \frac{N\alpha}{2(N^2+1)t^c}\right) T(t-N) + \frac{N^2\sigma_v^2}{(t-N+1)^{2c}}.$$

Using Lemma 2, we have

$$\lim_{t\to\infty} \mathrm{E}[\|\hat{\theta}(t) - \theta\|^2] \leqslant \lim_{t\to\infty} \frac{N^2\sigma_v^2}{t^{2c}} \frac{2(N^2+1)(t-N+1)^c}{N\alpha}.$$

This completes the proof of Theorem 2.

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